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ON REDUCING DISCRETE EXTREMAL PROBLEMS TO THE PROBLEM OF DECODING AND FINDING THE MAXIMUM UPPER LIMIT OF DISCRETE MONOTONE FUNCTIONS

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ABSTRACT. The article examines algorithms for solving individual classes of discrete extremal problems for finding the exact optimum by decoding and finding the maximum upper zero of discrete monotone functions. Theorems on reducing the studied classes of discrete extremal problems to the problem of decoding and finding the maximum solution of discrete monotone functions are proved.

1. INTRODUCTION

It is known that to construct algorithms for solving individual classes of discrete extremal problems for finding the exact optimum, procedures are used for decoding and finding the maximum upper zero of discrete monotone functions, and methods are given for solving problems using procedures for decoding and finding the maximum upper zero of discrete monotone functions. Methods for solving problems of decoding and finding the maximum upper zero of discrete

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monotone functions of multivalued functions are investigated. A class of problems is considered that can be reduced to decoding a monotone function defined on a finite structure, or finding the maximum upper zero of discrete monotone functions. Theorems are proved and criteria are given for reducing to problems of decoding and finding the maximum upper zero of discrete monotone functions.

2. Statement of the problem

Let $E_\rho = \{0, \ldots, \rho - 1\}$ and $T_1^*, \ldots, T_\omega^*$ be rectangular tables with elements from E_ρ, containing n columns $m_1, \ldots, m_\omega$ of rows each:

$$T_{nm}^0 = \bigcup_{i=1}^{\omega} T_i^*, \ \ \sum_{i=1}^{\omega} m_i = m.$$

We'll call rows elements, and columns attributes. By defining ω tables, we define a certain set of elements and divide them into ω classes: $K_1, \ldots, K_\omega$. Thus, the element a_{ij}^l, l -y table is the value of the j-th feature in l the object l class K_l. It is assumed that each reference is contained in only one class. Let $S_1, \ldots, S_m$ be the standard and be $x_1, \ldots, x_n$ the attributes. Then $x_j(S_i) = \alpha_{ij}$ - the value on S_i.

It is known that a set $M = \{x_{i_1}, \ldots, x_{i_r}\}$ is called a tester, and for any pair of standards $S_i, \ S_j$ belonging to different classes, there is a sign $x_t \in M$ such that $x_t(S_i) \neq x_t(S_j)$. Obviously, in the case when $m_1 = \ldots = m_\omega = 1$ the definitions of the test and testers tables are the same. A tester is called a dead end if, after removing any attribute from it, it ceases to be a tester. A tester is called minimal if it contains the minimum number of attributes among all testers in the tables.

Let us assume that $F(x_1, \ldots, x_n)$ an arbitrary partial function of k-valued logic $\mathfrak{M} = \{f_1, \ldots, f_m\}$ is a system of non-everywhere defined Boolean functions $f_i(x_1, \ldots, x_n), \ i = \overline{1, m}$.

It is known [1–3] that a set of variables $M = \{x_{i_1}, \ldots, x_{i_k}\}$ is called essential for $F(\tilde{x})(\mathfrak{M})$, if there is not everywhere a certain function $\varphi = \{x_{i_1}, \ldots, x_{i_k}\}$ of k-valued logic $\mathfrak{M}' = \{\varphi_1, \ldots, \varphi_n\}$ (a system of Boolean functions $\varphi(x_{i_1}, \ldots, x_{i_k})$, $i = \overline{1, m}$) such that $F = \varphi, \ \left(\varphi_i = f_i, \ i = \overline{1, m}\right)$.

An essential set M for $F(\mathfrak{M})$ is called a dead end if, after removing any variable from it, it ceases to be essential for $F(\mathfrak{M})$. A deadlock M for $F(\mathfrak{M})$ is called minimal if it contains the minimum number of variables among all sets of variables essential for $F = \varphi,\ \left(\varphi_i = f_i,\ i = \overline{1,\ m}\right)$.

Let T_{nm} be a binary table containing n columns and m rows and ρ_i be the weight i of the row $\rho_i > 0,\ i = \overline{1,\ m}$. We assume that rows $i_1, \ldots, i_k$ form a column cover T_{nm} if j-s row T_{nm} there is at least one row for any column $i \in \{i_1, \ldots, i_k\}$ such that the element α_{ij} in the table T_{nm} is a single one. $M = \{i_1, \ldots, i_k\}$ table coverage T_{nm} is called deadlocked if it ceases to be coverage after removing row R from it T_{nm}. M A table cover T_{nm} is called minimal if $\sum_{j=1}^{k} \rho_{i_j}$-is the minimum number among all sums of cover weights T_{nm}.

Let an alphabet of Boolean variables $\{x_1, \ldots, x_n\}$. be given let us Consider the set Ω of all ECS from k variables $(0 \le k \le n,\ n \ge 1)$. Denote by the interval (under the cube) in the cube E_n^k, corresponding to the e.q. $N_{\mathfrak{A}}$.

For an arbitrary $\mathfrak{A} = 1$ e.c. in Ω (interval $N_{\mathfrak{A}}$), we assign a set $(\alpha_1, \ldots, \alpha_n)$ such that

$$\alpha_i = \begin{cases} \sigma_i, & \text{if } i \in \{i_1, \ldots, i_k\} \\ & 2 \text{ otherwise} \end{cases},$$

moreover, the e.q. $\mathfrak{A} = 1$ corresponds to the set $(2, \ldots, 2)$. Consider a structure S^n, where $S = \{0,\ 1,\ 2\},\ 0 < 1,\ 0 < 2$. Let's encode the elements of the set $S:\ 0 \to 2,\ 1 \to 1,\ 2 \to 0$. We have $2 < 1,\ 2 < 0$. This order induces a partial order in the set S^n :

$$\tilde{\alpha} = (\alpha_1, \ldots, \alpha_n) \le \tilde{\beta} = (\beta_1, \ldots, \beta_n) \text{ at } \tilde{\alpha}_i \le \tilde{\beta}_i,\ i = \overline{1,\ n}.$$

It is easy to see that the sets U_j of the structure level S^n correspond to an e.q. of rank j to a set Ω, and chains $\{\tilde{\alpha}^{i_1}, \ldots, \tilde{\alpha}^{i_k}\}$ in the set $\mathfrak{M}_{ij},\ j = \overline{1,\ k}$ of e.q. such that $N_{\mathfrak{M}_{j_1}} \subset N_{\mathfrak{M}_{j_2}} \subset \ldots \subset N_{\mathfrak{M}_{j_k}}$.

Let $f\,(x_1, \ldots, x_n)$ be functions of the logic algebra and $F\,(x_1, \ldots, x_n)$ be a partial Boolean function defined using sets $M_1, M_2,\ \ M_1 \cap M_2 \ne \emptyset$.

We introduce functions $g\,(y_1, \ldots, y_n)\ \tilde{g}\,(y_1, \ldots, y_n)\ y \in \{0,\ 1,\ 2\}$, defined on sets $\tilde{\alpha}$of sets S^n. Let $\tilde{\alpha}$ corresponds to the e.q. $\mathfrak{A}$in Ω :

$$g\,(\tilde{\alpha}) = \begin{cases} 0, \text{ if } & - \text{invalide.c.for } f, \\ 1\,, \text{ otherwise,} \end{cases}$$

$$g\,(\tilde{\alpha}) = \begin{cases} 1, \text{ if } N_L \cap M_1 \ne \emptyset,\ N_L \cap M_2 \ne \emptyset, \\ 0, \text{ otherwise.} \end{cases}$$

It is not difficult to prove that $g(y_1, \ldots, y_n)$, $(\tilde{g}(y_1, \ldots, y_n))$ is is a monotone function, i.e., for any sets $\tilde{\alpha}$ and $\tilde{\beta}$ in S^n such that $\tilde{\alpha} \leq \tilde{\beta}$ is valid: $g(\tilde{\alpha}) \leq g\left(\tilde{\beta}\right)$, $\left(\tilde{g}(\tilde{\alpha}) \leq \tilde{g}\left(\tilde{\beta}\right)\right)$, the lower units $g(\tilde{\alpha})$, $\left(\tilde{g}\left(\tilde{\beta}\right)\right)$ correspond to the maximum intervals N in N_f, $(E_n^2 \backslash M_2,\ N \cap M_2,\ N \cap M_1 \neq \emptyset)$. Moreover, the set of all lower units $g(\tilde{y})$, $(\tilde{g}(\tilde{y}))$ defines the set of all maximum intervals N in N_f, $(E_n^2 \backslash M_2,\ N \cap M_1 \neq \emptyset)$.

Consider the discrete problem $Z_s \in \{Z_s\}$, $(Z'_s \in \{Z'_s\})$ of finding all the extrema (global extremum) F of an object's functional $S \in \{S\}$. It is obvious that the set $\{Z_s\}$, $(\{Z'_s\})$ corresponds one-to-one to the population $\{S\}$. For example, if S there is a partially-defined function of k-valued logic $F(x_1, \ldots, x_n)$, then $Z_F \in \{Z_F\}$, $(Z'_F \in \{Z'_F\})$ it is formed as a problem of finding all sets (minimum set) of variables essential for $F(x_1, \ldots, x_n) \in \{F_n\}$, where $\{F_n\}$-is the set of all partially-defined functions of k- valued logic that depend on n-variables. Let the object S correspond to a monotone function $\varphi_s(y_1, y_2, \ldots, y_n) \in M$, defined on a finite structure $\tilde{M}$. Moreover, the upper zeros of the function $\varphi_s$correspond one-to-one to the extrema F of the object's functional S. We will assume that the problem $Z_s$corresponds to the problem of decoding $\varphi_s$and Z'_S searching for M. V. N. φ_s We will talk about the complete set management $\{Z_s\}$ problem of decoding if for any function f from M_n there exists M_n from $\{Z_S\}$ such that $\varphi_S = f$. We denote by $\{f_\alpha\}$ the set (class) of monotone functions from M_n, which $\tilde{\alpha} \in M$ have a maximum upper zero. We assume that the problem Z'_s corresponds to a class $\{f_{\tilde{\alpha}}\}$ if $\varphi_s \in \{f_{\tilde{\alpha}}\}$. In the case when for any set $\tilde{\alpha} \in M_n$ there exists Z'_s such that $\varphi_s \in \{f_{\tilde{\alpha}}\}$, then we will talk about the complete reduction of the set $\{Z'_s\}$ to the search for the m.v.n. functions in M_n. Otherwise, the data is considered incomplete.

3. Criteria for reducing discrete monotone functions to problems of decoding and finding the maximum upper zero

In this section, we prove the criteria for reducing to problems of decoding and finding the maximum upper zero of discrete monotone functions problems about optimal coverage of columns of a binary table by rows, finding minimum tests, table testers, optimal continuation of partially defined logical functions, and so on. Let $\varphi(n)$ be the Shannon function [17–19] for solving the problem of decoding monotone functions in a class σ. The theorem [5–16] holds.

Theorem 3.1. *If there is a complete reduction of the population* $\{Z_S\}$ *to the problem of decoding* M_n*, then the reduction of the population* $\{Z'_S\}$ *to the search for m.v.n. functions in* M_n *is complete.*

Proof. Obviously, the complete reduction of the population $\{Z_s\}$ to the decoding problem means that for any function from M_n, there exists Z_s from $\{Z_s\}$, for which $\varphi_s = f$.

Therefore, for any set $\tilde{\alpha} \in M_n$, there exists such Z'_s that $\varphi_s \in \{f_{\tilde{\alpha}}\}$. Consequently, it is possible $\{Z'_s\}$ to assert a complete reduction of the functions in M_n.

The theorem is proved. □

Let $f(x_1, \ldots, x_n)$ be a Boolean function, $\{Z_f\}$ and be a class Z of problems for constructing abbreviated mathematical functions for all Boolean functions $f(x)$. Consider a monotone function $g(y_1, \ldots, y_n)$ in S^n, corresponding to $f(x_1, \ldots, x_n)$.

Theorem 3.2. *Reducing the class* $\{Z\}$ *to a decryption task is incomplete.*

Proof. It follows from the fact that for a monotone function $\psi(y_1, \ldots, y_n)$ in S^n such that

$$\psi(\tilde{\alpha}) = \begin{cases} 1, \text{ if } \tilde{\alpha} \in \bigcup\limits_{i=j}^{n} U_i, \;\; i < j < n \\ 0 - \text{otherwise} \end{cases}$$

there Z_f is no out of $\{Z_f\}$ that $\psi \equiv \varphi_f$. □

Similarly, it is proved that the reduction of the class $\{Z_f\}$ of problems Z_f for constructing abbreviated partial functions of F, k-valued logic to the decoding problem is incomplete. Let $F(x_1, \ldots, x_n)$ be an arbitrary non universally defined function of k-valued logic, Z'_f let be the problem of finding the minimum set of variables essential for F and φ_F let be a monotone Boolean function corresponding F to. Let's assume that $\alpha_{i_1}, \ldots, \alpha_{i_k}$- zero coordinates of the set $(\alpha_1, \ldots, \alpha_n) \in E_n^2$. Let's put

$$\varphi_T = \begin{cases} 0, \text{ if } \{x_{i_1}, \ldots, x_{i_k}\} \text{ - test table} \\ 1 - \text{otherwise} \end{cases}$$

Theorem 3.3. *For the Shannon function $\mu(n)$ of searching for the maximum upper zero in the class $\{\varphi_F\}_n$ of all monotone Boolean functions φ_F, corresponding $F \in \{F\}_n$ to is valid $\mu(n) = C_n^{[n/2]+1}$.*

Proof. It follows from the statement of theorem 3.1 and [5–15] that for any $f \in M_n$ there is such $F(x_1, \ldots, x_n)$, that $\varphi_F = f$. It is easy to see that the statement of theorem 4 is also valid in relation to the problem of finding the minimum k- values of tables. □

Let $T_{nm} = \|\alpha_{ij}\|_{\min}$ be a binary table consisting of m rows and n columns. Task Z'_T in $\{Z'_T\}$, corresponding $\{T\}$ to, is to find the minimum test of the table. Let φ_T be a monotone Boolean function corresponding T. Let's assume that $(\alpha_{i_1}, \ldots, \alpha_{i_k})$the null coordinates of the set $(\alpha_{i_1}, \ldots, \alpha_{i_k}) \in E_n^2$ are. Let

$$\varphi_T = \begin{cases} 0, \ if \ \{x_{i_1}, \ldots, x_{i_k}\} \text{ - test of the table } T \\ 1 - \text{ otherwise} \end{cases}$$

Theorem 3.4. *The reduction Z'_T to the search for the maximum upper zero functions in M_n is incomplete.*

Proof. It is known [17–19] that if $x_{i_1}, \ldots, x_{i_k}$is a table test T, then $t \geq]\log_2 m[+1$. Therefore, if for a set $\tilde{\alpha}$ in E_n^2 norm $|\tilde{\alpha}| > n -]\log_2 m[- 1$, then there is $\{Z'_{\tilde{T}}\} \in \{Z'_T\}$ no such set that, $\varphi_T \in \{f_{\tilde{\alpha}}\}$. Consequently, the reduction $\{Z'\}$to the search for the m.v.n of functions in M is incomplete. □

The theorem is proved. Let the problem Z'_T consist in finding the minimum coverage of matrix columns $E = \|\alpha_{ij}\|_{\min}$ by rows and φ_T be a monotone Boolean function such that

$$\varphi_T(\alpha_1, \ldots, \alpha_m) = \begin{cases} 0, \text{ if } \{i_1, \ldots, i_k\} \text{ - th rows forms colums coverage } T \\ 1 - \text{ otherwise} \end{cases}$$

Theorem 3.5. *The reduction Z'_T to the search for the maximum upper zero functions in M_n is complete.*

Proof. Let $\tilde{\alpha}$ be an arbitrary set E_m^2 and $i_1, \ldots, i_k$ be zero coordinates $\tilde{\alpha}$. Let's build a binary table $T_{\tilde{\alpha}}$. Let $n = m$'s say. $U_1, \ldots, U_m$ Select the table rows in such a way that □

$$\begin{array}{l} U_{i_1} = (1 \ldots 0 \ldots 00 \ldots 0) \\ U_{i_2} = (0 \ldots 1 \ldots 00 \ldots 0) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ U_{i_k} = (0 \ldots 0 \ldots 01 \ldots 0) \,. \end{array}$$

Moreover, the remaining lines $T_{\tilde{\alpha}}$ are pairwise different and the first coordinates take the value zero. It is easy to see that the set $\tilde{\alpha}$ is the m.v.n. of the function $\varphi_{T_{\tilde{\alpha}}}$ and $\varphi_{T_{\tilde{\alpha}}} \in M_m$. So for any $\tilde{\alpha} \in E_m^2$ there exists a table $T_{\tilde{\alpha}}$such that $\varphi_{T_{\tilde{\alpha}}} \in \{f_{\tilde{\alpha}}\}$. The theorem is proved. Let $\mathfrak{M} = \{f_1, \ldots, f_m\}$ be a system of functions $f_i\left(_1, \ldots, _n\right),\ i = \overline{1,\ m}$ of the logic algebra. The problem Z'_m is to find the minimum set of variables essential for $\mathfrak{M}$ and φ_m- a monotone Boolean function corresponding $\mathfrak{M}$ to. For example, $\alpha_{i_1}, \ldots, \alpha_{i_k}$-zero coordinates of the set $(\alpha_1, \ldots, \alpha_m) \in E_m^2$. Let's put

$$\varphi_m\left(\tilde{\alpha}\right) = \begin{cases} 0,\ if\ \ \{x_{i_1}, \ldots, x_{i_k}\} - \text{existsfor}\ , \\ 1\ \text{otherwise.} \end{cases}$$

Theorem 3.6. *The reduction Z'_m to the search for the maximum upper zero functions in M_n is complete.*

Proof. Let $\tilde{\alpha}_1$ be an arbitrary set E_n^2 $i_1, \ldots, i_k$ of zero coordinates $\tilde{\alpha}_1$ and $\tilde{\alpha}_2, \ldots, \tilde{\alpha}_m$ be pairwise distinct sets of the interval $N_{\tilde{\alpha}_1}$ in E_n^2 spanned by the sets $\tilde{\alpha}_1$. We construct a system $\mathfrak{M} = \{f_1, \ldots, f_m\}$ of functions $f_i\left(_1, \ldots, _n\right),\ i = \overline{1,\ m}$ such that on all sets $\tilde{\beta} \in E_n^2$, containing $(k-1)$ zeros, $f_i\left(\tilde{\beta}\right) = 0,\ f_i\left(\tilde{\alpha}\right) = 1,\ i = \overline{1,\ m}.$ On the other sets E_n^2, the system functions $\mathfrak{M}$ are not defined. It is not difficult to notice that $\{x_{i_1}, \ldots, x_{i_k}\}$ is the minimal set essential for $\mathfrak{M}$, and the set $\tilde{\alpha}_1$ of is the m.v.n. of a functionφ_m, so $\varphi_m \in \{f_{\tilde{\alpha}_1}\}$in M_n. The theorem is proved. Let $f_c = \mathfrak{A}_1 \vee \ldots \vee \mathfrak{A}_m$ be an abbreviated d.n.f. of a Boolean function $f\left(_1, \ldots, _n\right)$ The task Z'_f is to construct the shortest d.n.f. $f\left(_1, \ldots, _n\right)$ Let φ_f be a monotone function M_n corresponding $f\left(_1, \ldots, _n\right)$ to and $\alpha_{i_1}, \ldots, \alpha_{i_k}$coordinates of $(\alpha_{i_1}, \ldots, \alpha_{i_k}) \in E_n^2$:

$$\varphi_f = \begin{cases} 0,\ \text{if}\ \left(f \to \bigvee\limits_{j=1}^{k} \right) \equiv 1, \\ 1\ \text{otherwise.} \end{cases}$$

□

Theorem 3.7. *Information $\{Z'_f\}$ for the finding maximum upper zero functions in M_n is incomplete.*

Proof. It is known [17–19] that for the maximum value $I_k(n)$- the length of the shortest d.b.f. of Boolean functions $f = \{x_1, x_2, \ldots, x_n\}$- the equality holds $I_k(n) = 2^{n-1}$. Therefore, if for any set $\tilde{\alpha}$ and E_n^2 $|z| < 2^{n-1}$, then there is no problem Z'_f such that $\varphi_f \in \{f_Z\}$. □

Consequently, the reduction $\{Z'_f\}$to the search for the m. v. n. of the function in M_n is incomplete. The theorem is proved. Let $\mathfrak{M}$ be a system of $\mathfrak{M}$ inequalities. The task Z'_m is to find the maximum joint subsystem of the system $\mathfrak{M}$. Let be a monotone function of the logic algebra corresponding to. Let's assume that the unit coordinates of the set are.

Theorem 3.8. *The reduction Z'_m to the search for the maximum upper zero functions in M_n is complete.*

Proof. Let the system $\mathfrak{M}_q,\ (1 \le q \le m)$ have the form $x_i \ge 0,\ i = \overline{1,\ q}$, $x_1 + \ldots + x_q = -C_j,\ j = \overline{1,\ m-q+1}$, where $\dot{N}_1, \ldots, \dot{N}_{m-q+1}$ are real positive integers, and for $q = m,\ \mathfrak{M}_m = \left\{x_i \ge 0,\ i = \overline{1,\ m}\right\}$.

It is easy to see that if for a set $\tilde{\alpha} \in E_m^2$ $|\tilde{\alpha}| \in q$, then for the problem $Z'_{\mathfrak{M}_q}$ of finding the maximum joint subsystem $\mathfrak{M}_q$, the occurrence occurs $\varphi_{\mathfrak{M}} \in \{f_{\tilde{\alpha}}\}$. So $q = \overline{1,\ m}$, somehow for sets $\tilde{\alpha}_1, \ldots, \tilde{\alpha}_m \in E_m^2,\ |\tilde{\alpha}| = i$ you can build systems $\mathfrak{M}_1, \ldots, \mathfrak{M}_m$, for which $\varphi_{\mathfrak{M}} \in \{f_{\tilde{\alpha}}\} \in M_n,\ i = \overline{1,\ m}$. □

The theorem is proved. Thus, from the statements of the proved theorems, it follows that solutions of the listed discrete extremal problems can be reduced to solving problems of decoding or searching for monotone functions M_n.

4. The concept of a test and its relation to a system of Boolean equations

Let's define a table 1 elements consisting of rows (objects) and columns (attributes), and $\alpha_{ij} \in \{0, 1, \ldots, k-1\},\quad k \ge 1,\quad j = \overline{1,\ n},\quad i = \overline{1,\ m}$.

To describe and construct tests, it is convenient to use the logic algebra tool. Let $T = \{i_1, \ldots, i_t\}$ be a certain test. Consider $S_i,\ S_j$ from table 1. Since is a T test, there is a feature $x_{i_l} \in T,\ (1 \le l \le t)$ such that $\alpha_{ii_l} \ne \alpha_{ji_l}$. This feature, therefore, is included in $T_{ij}-$the set of all the features on which objects

$S \backslash x$	x_1	x_2	$\cdots$	x_n
S_1	a_{11}	a_{12}	$\cdots$	a_{1n}
S_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
S_m	a_{m1}	a_{m2}	$\cdots$	a_{mn}

TABLE 1. Description of the table

S_i and S_j differ. Thus, T is the result of selecting the features of all sets T_{ij}, where $i, j = \overline{1,\ m},\ (i \neq j)$. It should be noted that the choice principle used in education T complicates the construction of the test.

Let's start with the description of building tests. To do this, we use the apparatus for solving systems of logical equations. In fact, let $T_{ij} = \left\{x_1^{ij}, \ldots, x_{k_{ji}}^{ij}\right\}$. Let's write the set T_{ij} as an equation

$$g\left(x_1, \ldots, x_n\right) = 1,\ \text{where } g\left(x_1, \ldots, x_n\right) = x_1^{ij} \vee \ldots \vee x_{k_{ji}}^{ij}. \tag{4.1}$$

It is clear that solutions $(\alpha_1, \ldots, \alpha_n)$ of equation 4.1, which $\left(\alpha_{i_1}, \ldots, \alpha_{i_k}\right),\ k < t$ has unit coordinates, will mean that the features $x_{i_1}, \ldots, x_{i_k}$ belong to a set T_{ij}, i.e. it distinguishes objects $S_i,\ S_j$. Let's construct a system of logical equations 4.2:

$$\begin{array}{llll} x_1^{12} & \vee x_2^{12} & \vee \ldots & \ldots \vee x_{k_{12}}^{12} = 1 \\ x_1^{13} & \vee x_2^{13} & \vee \ldots & \ldots \vee x_{k_{13}}^{13} = 1 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ x_1^{1m} & \vee x_2^{1m} & \vee \ldots & \ldots \vee x_{k_{1m}}^{1m} = 1 \\ x_1^{23} & \vee x_2^{23} & \vee \ldots & \ldots \vee x_{k_{23}}^{23} = 1 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ x_1^{2m} & \vee x_2^{2m} & \vee \ldots & \ldots \vee x_{k_{2m}}^{2m} = 1 \\ x_1^{(m-1)m} & \vee x_2^{(m-1)m} & \vee \ldots & \ldots \vee x_{k_{(m-1)m}}^{(m-1)m} = 1 \end{array} \tag{4.2}$$

where $x_i^{ij} \in T_{ij}$. We assume that x_i^{ij} are variables of the algebra of logic. Then the system of equations 4.2 is a system of logical equations.

It is easy to see that system 4.2 is compatible, so to solve it, we use

$$\bigwedge_{i \neq j} \left(x_1^{ij} \vee \ldots \vee x_{i_{k_{ij}}}^{ij}\right) = 1,\ i, j = \overline{1,\ m}. \tag{4.3}$$

Assuming $x_l^{ij} \wedge x_l^{ij} = x_l^{ij}, \quad x_l^{ij} \vee Ax_l^{ij} = x_l^{ij}$ we reduce expression 4.3 to the form

(4.4) $$\bigvee_{j=1}^{q} \left(x_{i_1} \wedge \ldots \wedge x_{i_{k_j}}\right) = 1$$

moreover, the sum will not contain any extra terms.

It is easy to see that each term $x_{i_1}, \ldots, x_{i_j}$of equation 4.4 on a binary set $(\alpha_1, \ldots, \alpha_n)$, $\left(\alpha_{i_1}, \ldots, \alpha_{i_{k_j}}\right)$with unit coordinates takes the value of one, so such a set $(\alpha_1, \ldots, \alpha_n)$ is a solution of equation 4.4 and system 4.2.

Theorem 4.1. *Let $K = \left\{x_{i_1}, \ldots, x_{i_{k_j}}\right\}$ be a sum and statement of equation 4.4. Then the set K of features forms a dead-end test (table 1), and the number of terms of the statement in 4.4 is equal to the number of all dead-end tests. The validity of the theorem follows from the fact that the term contains elements from each bracket $\left(x_1^{ij} \vee \ldots \vee x_{i_{k_{ij}}}^{ij}\right)$ 4.3.*

Note 1. *The question of finding tests is reduced to constructing a set that has at least one element in common with each set in expression 4.3.*

Note 2. *The term statement in equation 4.4, which contains the minimum number of elements, corresponds to the minimum test in table 1.*

5. Testers and Boolean equation systems

Let $E_\rho = \{0, 1, \ldots, \rho - 1\}$, $T_1^*, T_2^*, \ldots, T_\omega^*$ be rectangular tables with elements from E_ρ. Let $T = \{x_{i_1}, x_{i_2}, \ldots, x_{i_t}\}$ be a certain tester. Consider $S_i = k_l$. Since T- is a tester, there is an attribute $x_i \in T$ such that $\alpha_{ii_l} = \alpha j i_l$. Let's denote by the set of all attributes on which objects S_i and S_j classesk_ρ and k_l differ, respectively. Obviously, there T is a result of choosing features from all sets $T_{ij}^{l\rho}$, where $i = \overline{1, m_\rho}$.

To form testers, we use the device for solving systems of logical equations. We assume that $x_1, x_2, \ldots, x_n$ they are Boolean variables.

Let's say $T_{ij}^{l\rho} = \{x_{j_1}, x_{j_2}, \ldots, x_{j_n}\}$, $(q \leq t)$. As in the case of tests, we write the set as a logical equation $T_{ij}^{l\rho}$ as a logical equation where $g_{ijl\rho}(x_1, x_2, \ldots, x_n) = 1, \quad g_{ijl\rho}(x_1, x_2, \ldots, x_n) = x_{j_1} \vee x_{j_2} \vee \ldots \vee x_{j_q}$ Let be a system of logical equations $g_{ijl\rho}(x_1, x_2, \ldots, x_n) = 1$ where $i = 1, m_l, j = 1, m_\rho, l\rho = \overline{1, \omega}, i \neq j, \rho \neq 1$.

Similarly to tests, the system L is reduced to the form

$$\bigvee_{i=1}^{q} \tilde{g}\left(x_1, x_2, \ldots, x_n\right) = 1 \tag{5.1}$$

Let equation 5.1 be represented as

$$\bigvee_{t=1}^{q} \left(x_{i_1} \vee x_{i_2} \vee \ldots \vee x_{ik_j}\right) = 1 \tag{5.2}$$

Where $x_{ij} \in (x_1, x_2, \ldots, x_n)$. Then the following occurs.

Theorem 5.1. *Aggregates $L_j = \left\{x_{i_1}, x_{i_2}, \ldots, x_{i_{k_j}}\right\}, j = \overline{1, q}$ form dead-end testers of the reference table, and their number is equal to the number of terms $i = \overline{1, p}$ in equation 5.2.*

Conclusion

Solutions of some classes of discrete extremal problems are investigated. Theorems are proved on reducing the studied classes of discrete extremal problems, such as optimal coverage of columns of a binary table by rows, search for minimal tests, table testers, optimal continuation of partially defined logical functions, etc.to the problem of decoding and finding the maximum number of discrete monotone functions.

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