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ON CORPORATE DOMINATION IN GRAPHS

S. Padmashini[1] and S. Pethanachi Selvam

ABSTRACT. In this paper, we initiate the concept of corporate domination in graphs. We have found the corporate dominating set and corporate domination number for some standard graphs.

1. INTRODUCTION

All graphs $G = (V, E)$ are considered in this paper are finite, simple and undirected with vertex set V and edge set E. For all the graph-theoretic terminology and notations we follow F. Harray [4] & T.W. Haynes et al. [5]. M. A. Henning [6] initiated the idea about domination for regular graphs and A. Gayathri et.al [3] discussed the Study of various Dominations. The perfect domination was discussed by Michael R. Fellows and Mark N. Hoover [2]. D. Bange et.al [1] discussed the perfect edge, perfect edge covering, and perfect edge vertex dominating sets. The study of a new domination parameter, namely Corporate domination where the set of vertices or edges or corporate of both, dominate the vertices of G is studied in [7]. In this paper, we determine the corporate domination number for cycle and path.

[1]*corresponding author*

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2. Preliminaries

Definition 2.1. *A subset S of $V(G)$ is said to be a perfect dominating set if for each vertex v not in S, v is adjacent to exactly one vertex of S. The minimum cardinality of perfect dominating set is called perfect domination number and is denoted by $\gamma_{pf}(G)$.*

Definition 2.2. *A set F of edges of a graph G is said to be a perfect ev-dominating set if every vertex of a graph is m-dominated by exactly one edge in F. The perfect ev-domination number of G, denoted by $\gamma_{pev}(G)$ is the minimum number of edges of any perfect ev-dominating set.*

Definition 2.3. *A set $S \subseteq V$ is called an efficient and total efficient dominating set if $\Big|N[v] \cap S\Big| = 1$ and $\Big|N(v) \cap S\Big| = 1$ for every $v \epsilon V$ respectively.*

3. Corporate dominating set

Here, we define the corporate domination number with example. Also, we state some basic results on corporate domination.

Definition 3.1. *Let $G(= V, E)$ be a graph. Let $C = V_1 \cup E_1 (\subseteq V \cup E)$. Take $P = \{u \epsilon V(G[E_1]) / |N(u) \cap N(w)| \leq 1$ for all $w(\neq u) \epsilon V(G[E_1])\}$ where $V(G[E_1])$ denote the vertex set of an edge induced subgraph $G[E_1]$ and $Q = \{v \epsilon V_1 / N(v) \cap N(w) = \phi$ for all $w(\neq v) \epsilon V_1\}$. A subset C is said to be a corporate dominating set if every vertex $v \notin P \cup Q$ is adjacent to exactly one element of $P \cup Q$. The minimum cardinality of elements in C is called corporate domination number of G and is denoted by $\gamma_{cor}(G)$.*

Example 1. *For a graph G which is given in Figure 1, let $E_1 = \{v_2v_3\}$. Then $V(G[E_1]) = \{v_2, v_3\}$. Let $V_1 = \{v_6\}$. Then $C = \{v_2v_3, v_6\}$ and $\gamma_{cor}(G) = 2$.*

Proposition 3.1. *Let G be a graph. Then $\gamma_{cor}(G) = 1$ if and only if one of the following holds.*

(i) There exists a full degree vertex in G.

(ii) There exists an edge uv in G such that uv does not lie on any triangle and $d(u) + d(v) = n - 2$.

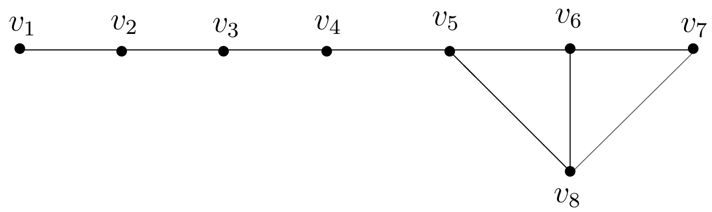

Figure 1

Remark 3.1. *Corporate domination number need not exist for all graphs.*

Proposition 3.2.
(a) *For any complete graph* $K_n(n \geq 3)$, $\gamma_{cor}(K_n) = 1$.
(b) *For any star graph* $K_{1,n}, n \geq 2$, $\gamma_{cor}(K_{1,n}) = 1$.
(c) *For any wheel graph* $W_n(n > 3)$, $\gamma_{cor}(W_n) = 1$.

Proposition 3.3. *Let* C *be a corporate dominating set with* $C = V_1$. *Then*
(i) *every corporate dominating set is the dominating set.*
(ii) *every corporate dominating set is the perfect dominating set. But the converse need not be true.*

4. Main Results

In the present section, the corporate domination number of Path and Cycle are determined.

Theorem 4.1. *For any cycle* C_n *with* $n \geq 3$, *we have* $\gamma_{cor}(C_n) = \lceil \frac{n}{4} \rceil$.

Proof. Let C_n be any cycle with n vertices and n edges. We consider the following cases.

Case 1: Let n be even and let $n \equiv o(\mod 4)$. Then $n = 4k,\ k = 1, 2, \ldots$. For $1 \leq i \leq k$, let C=$\{v_{4i-2}v_{4i-1}\}$. As $C = E_1$, let $E_1 = \{v_{4i-2}v_{4i-1}\}$ and $V_1 = \phi$.

Let $P = \{u \epsilon V(G[E_1])/|N(u) \cap N(w)| \leq 1$ for all $w(\neq u) \epsilon V(G[E_1])\}$, where $V(G[E_1])$ is the vertex set of an edge induced subgraph $G[E_1]$ and $Q = \{v \epsilon V_1/N(v) \cap N(w) = \phi$ for all $w(\neq v) \epsilon V_1\}$. Here $P = \{v_{4i-2}, v_{4i-1}\}$ and $Q = \phi$. Clearly, $|P| = \frac{n}{2}$. Since for any $u \epsilon (P \cup Q)^c, N(u) \cap (P \cup Q) = \{w\}$, where $w \epsilon\ P \cup Q$, C is the corporate dominating set. Since $P(= (P \cup Q))$ contains $\frac{n}{2}$ vertices and n is even, C has $\frac{n}{4}$ elements. We claim that C is the minimum, let C' be any other corporate dominating set and P', Q' be the sets corresponding to C'such that every vertex in $(P' \cup Q')^c$ is adjacent to exactly one vertex in $P' \cup Q'$. Furthermore, the set C' will be in one of the following forms.

$$(i)\ C' = V_1' \qquad (ii)\ C' = E_1' \qquad (iii)\ C' = V_1' \cup E_1'$$

If (i) holds, then $P' = \phi$ and $Q' \neq \phi$. This exists only if $n = 4$ & $n \equiv 0 (\mod 3)$, as $n \equiv 0 (\mod 4)$. This implies that $2 \leq |Q'| \leq \frac{n}{2}$. Suppose $n \not\equiv 0 (\mod 3)$ (except $n = 4$). Then there exist at most two vertices in $(Q')^c$ which are adjacent to none of the vertices in Q'. This is a contradiction to our hypothesis. Thus C' contains at most $\frac{n}{2}$ vertices. Hence $|C'| \geq |C|$.

If (ii) holds, then $P' \neq \phi$ and $Q' = \phi$. Let $|P'| \geq |P|$ with $2 \leq |P'| \leq n - 2$ and $|Q'| = 0$. Then C' contains at most $n - 3$ edges. Hence $|C'| \leq n - 3$ and $|C'| \geq |C|$. Suppose $|P'| < |P|$. Then there is some vertex $v \epsilon\ (P')^c (= (P' \cup Q')^c)$ which is adjacent to none of the vertices in P'. This implies that there exist some vertex v_i such that $v_i \epsilon Q'$. Thus $|Q'| \geq 1$, which is a contradiction, since $|Q'| = 0$.

If (iii) holds, then $P' \neq \phi$ and $Q' \neq \phi$.

(a) Let $|P'| \leq |P|$ and $|Q'| > |Q|$ with $2 \leq |P'| \leq \frac{n}{2}$ and $1 \leq |Q'| \leq \lfloor \frac{n-4}{3} \rfloor$. Thus C' has at most $\frac{n-2}{2}$ edges and $\lfloor \frac{n-4}{3} \rfloor$ vertices. Hence $|C'| \geq |C|$.

(b) Suppose $|P'| > |P|$ and $|Q'| > |Q|$ with $\frac{n+2}{2} \leq |P'| \leq n - 5$ and $1 \leq |Q'| \leq \lfloor \frac{n-6}{6} \rfloor$, $(n > 8)$. It follows that $\frac{n}{2} \leq |E_1'| \leq n - 6$ and $1 \leq |V_1'| \leq \lfloor \frac{n-6}{6} \rfloor$. Thus $|C'| \leq n - 6 + \lfloor \frac{n-6}{6} \rfloor$ and $|C'| \geq |C|$.

Case 2: Let $n \equiv 2 (\mod 4)$. Then $n = 4k + 2, k = 1, 2, \ldots$. For $k = 1$, let $C = \{v_2, v_5\}$. Here $P = \phi$ and $Q = \{v_2, v_5\}$. It is easy to see that, C is the corporate dominating set. For $1 < i \leq k$, let $C = \{v_2, v_5, v_{4i}v_{4i+1}\}$. As $C = V_1 \cup E_1$, let $P = \{v_{4i}, v_{4i+1}\}$ and $Q = \{v_2, v_5\}$. Clearly, $|Q| = 2$. Since for any $u \epsilon (P \cup Q)^c, N(u) \cap (P \cup Q) = \{w\}$, where $w \epsilon\ P \cup Q$, C is the corporate dominating set. Since $|Q| = 2$ and $|P| = \frac{n-6}{2}$, $|P \cup Q| = \frac{n-2}{2}$. Therefore, $|C| = \frac{n+2}{4} = \lceil \frac{n}{4} \rceil$.

To prove C is minimum, let C' be any other corporate dominating set (as in Case 1). If $C' = V_1'$ holds, then $P' = \phi$ and $Q' \neq \phi$. This exists only if $n \equiv 0 (\mod 3)$, as $n \equiv 2 (\mod 4)$. This implies that $|Q'| = \frac{n}{3} = |C'|$. Suppose $n \not\equiv 0 (\mod 3)$. Then proceed as in Case 1, C_n does not have a corporate dominating set. Hence $|C'| \geq |C|$. If $C' = E_1'$ holds, then $P' \neq \phi$ and $Q' = \phi$. Let $|P'| > |P|$ with $4 \leq |P'| \leq n - 2$ and $|Q'| = 0$. Then C' contains at most $n - 3$ edges. Suppose $|P'| \leq |P|$. As in the Case 1, we get a contradiction. If $C' = V_1' \cup E_1'$ holds, then $P' \neq \phi$ and $Q' \neq \phi$.

(a) Let $|P'| \leq |P|$ and $|Q'| \geq |Q|$ with $2 \leq |P'| \leq \frac{n-6}{2}$ and $3 \leq |Q'| \leq \lfloor \frac{n-4}{3} \rfloor$, $(n > 6)$. Hence $|C'| \leq \frac{n-8}{2} + \lfloor \frac{n-4}{3} \rfloor$. Thus, $|C'| \geq |C|$. Suppose $|Q'| < |Q|$. Then there exist some vertex $v \epsilon \ (P')^c (= (P' \cup Q')^c)$ which is adjacent to none of the vertices in P', which is a contradiction.

(b) Let $|P'| > |P|$ and $|Q'| \leq |Q|$ with $3 \leq |P'| \leq n-5$ and $1 \leq |Q'| \leq 2$. Hence C' contains at most $n-6$ edges and two vertices. Thus $|C'| \leq n-4$. Therefore $|C'| \geq |C|$. Suppose $|Q'| > |Q|$ with $7 \leq |P'| \leq n-11$ and $3 \leq |Q'| \leq \lfloor \frac{n}{6} \rfloor$, $(n > 14)$. Hence C' contains at most $n-12$ edges and $\lfloor \frac{n}{6} \rfloor$ vertices. Thus $|C'| \geq |C|$.

Case 3: Let n be odd and let $n \equiv 1 \pmod 4$. Then $n = 4k+1$, $k = 1, 2, \ldots$. For $k = 1$, consider C_5. Let $C = \{v_i v_{i+1}, v_{i+2}\}$, $1 \leq i \leq 3$. It is easy to see that, C is the corporate dominating set. For $k = 2$, let $C = \{v_2, v_5, v_8\}$. Then $P = \phi$ and $Q = \{v_2, v_5, v_8\}$. Clearly, C is the corporate dominating set of C_9. For $3 \leq i \leq k$, let $C = \{v_2, v_5, v_8, v_{4i-1}v_{4i}\}$. As $C = V_1 \cup E_1$, let P=$\{v_{4i-1}, v_{4i}\}$ and $Q = \{v_2, v_5, v_8\}$. Clearly $|Q| = 3$. Since for any $u \epsilon (P \cup Q)^c, N(u) \cap (P \cup Q) = \{w\}$, where $w \epsilon\ P \cup Q$, C is the corporate dominating set. Since $|Q| = 3$ and $|P| = \frac{n-9}{2}$, $|P \cup Q| = \frac{n-3}{2}$ and $|E_1| = \frac{n-9}{4}$. Hence C contains 3 vertices and $\frac{n-9}{4}$ edges. Therefore, $|C| = \frac{n+3}{4} = \lceil \frac{n}{4} \rceil$.

Now, we show that C is minimum.

As in Case 1, let C' be any other corporate dominating set. If $C' = V_1'$ holds, then $P' = \phi$ and $Q' \neq \phi$. This exists only if $n \equiv 0 \pmod 3$, as $n \equiv 1 \pmod 4$. This implies that $|Q'| = \frac{n}{3} = |C'|$.

If $C' = E_1'$ holds, then $P' \neq \phi$ and $Q' = \phi$. Let $|P'| > |P|$ with $3 \leq |P'| \leq n-2$ and $|Q'| = 0$. Then C' contains at most $n-3$ edges. Hence $|C'| \leq n-3$ and $|C'| \geq |C|$. Suppose $|P'| \leq |P|$. As in Case 1, we get a contradiction. If $C' = V_1' \cup E_1'$ holds, then $P' \neq \phi$ and $Q' \neq \phi$.

(a) Let $|P'| \leq |P|$ and $|Q'| \geq |Q|$ with $3 \leq |P'| \leq \frac{n-9}{2}$ and $4 \leq |Q'| \leq \lfloor \frac{n-5}{3} \rfloor$, $(n > 9)$. Hence $|C'| \leq \frac{n-11}{2} + \lfloor \frac{n-5}{3} \rfloor$.Thus $|C'| \geq |C|$. Suppose $|Q'| < |Q|$. Then for some $v_j \notin P' \cup Q'$ such that $N(v_j) \cap (P' \cup Q') = \phi$, which is a contradiction. Hence $|Q'| < |Q|$ is impossible.

(b) Let $|P'| > |P|$ and $|Q'| \leq |Q|$ with $4 \leq |P'| \leq n-5$ and $1 \leq |Q'| \leq 3$. Hence $|C'| \leq n-6+3 = n-3$. Suppose $|Q'| > |Q|$ with $\frac{n-7}{2} \leq |P'| \leq n-14$ and $4 \leq |Q'| \leq \lfloor \frac{n+3}{6} \rfloor$ $(n > 17)$. Hence $|C'| \leq n-15+\lfloor \frac{n+3}{6} \rfloor$.Therefore $|C'| \geq |C|$.

Case 4: Let $n \equiv 3(\mod 4)$. Then $n = 4k+3,\ k = 0,1,2,\ldots$. For $k=0$, let $C = \{v_i\}$, $1 \le i \le 3$. Clearly C is the corporate dominating set. For $1 \le i \le k$, let $C = \{v_2, v_{4i+1}v_{4i+2}\}$. As $C = V_1 \cup E_1$, let P=$\{v_{4i+1}, v_{4i+2}\}$ and $Q = \{v_2\}$. Clearly $|Q| = 1$. Since for any $u\epsilon(P \cup Q)^c, N(u) \cap (P \cup Q) = \{w\}$, where $w\epsilon\ P \cup Q$, C is the corporate dominating set. Since $|Q| = 1$ and $|P| = \frac{n-3}{2}$, $|P \cup Q| = \frac{n-1}{2}$ and $|E_1| = \frac{n-3}{4}$. Therefore, $|C| = 1 + \frac{n-3}{4} = \lceil \frac{n}{4} \rceil$. Now, we shall prove that C is minimum. As in Case 3, if $C' = V_1'$ holds, then $P' = \phi$ and $Q' \ne \phi$. This exists only if $n \equiv 0(\mod 3)$, as $n \equiv 3(\mod 4)$. This implies that $|Q'| = \frac{n}{3} = |C'|$. If $C' = E_1'$ holds, then $P' \ne \phi$ and $Q' = \phi$.

Let $|P'| > |P|$ with $5 \le |P'| \le n-2$ and $|Q'| = 0$. Then $|C'| \le n-3$. Hence $|C'| \ge |C|$. Suppose $|P'| \le |P|$. Proceed as in Case 1, we get a contradiction. If $C' = V_1' \cup E_1'$ holds, then $P' \ne \phi$ and $Q' \ne \phi$.

(a) Let $|P'| \le |P|$ and $|Q'| \ge |Q|$ with $2 \le |P'| \le \frac{n-3}{2}$ and $1 \le |Q'| \le \lfloor \frac{n-4}{3} \rfloor$. Hence $|C'| \le \frac{n-5}{2} + \lfloor \frac{n-4}{3} \rfloor$. Thus $|C'| \ge |C|$.

(b) Suppose $|P'| > |P|$ and $|Q'| \ge |Q|$ with $\frac{n-1}{2} \le |P'| \le n-5$ and $1 \le |Q'| \le \lfloor \frac{n-3}{6} \rfloor$. Then $|C'| \le n - 6 + \lfloor \frac{n-3}{6} \rfloor$. Thus $|C'| \ge |C|$. From the above cases, C is the minimum and $\gamma_{cor}(C_n) = \lceil \frac{n}{4} \rceil$. □

Illustration 4.1.

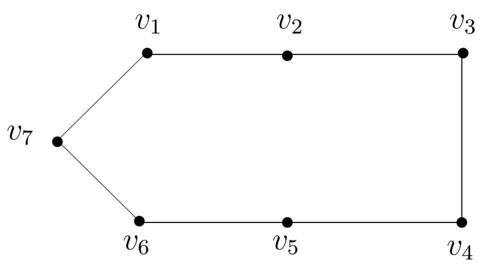

Figure 2

In Figure 2, let $C = \{v_2, v_5v_6\}$. By using case 3 of Theorem 4.1, C is the corporate dominating set and $\gamma_{cor}(C_7) = 2$.

Theorem 4.2. *For any path P_n with $n \ge 3$, we have $\gamma_{cor}(P_n) = \lceil \frac{n}{4} \rceil$.*

Proof. Let P_n be any path with n vertices and $n-1$ edges. We consider the following cases.

Case 1: Let n be even and let $n \equiv o(\mod 4)$. Then $n = 4k, k = 1, 2, \ldots$. Proceed as in Case 1 of Theorem 4.1, C is the corporate dominating set and $|C| = \frac{n}{4}$.

We claim that C is the minimum.

Let C' be any other corporate dominating set. Since $|P| = \frac{n}{2}$ and $|Q| = 0$. As in Theorem 4.1, if $C' = V_1'$ holds, then $P' = \phi$ and $Q' \neq \phi$. Thus C' contains at most $\frac{n}{2}$ vertices. Hence $|C'| \geq |C|$.

If $C' = E_1'$ holds, then $P' \neq \phi$ and $Q' = \phi$. Thus C' contains at most $n - 3$ edges. Hence $|C'| \leq n - 3$ and $|C'| \geq |C|$. If $C' = V_1' \cup E_1'$ holds, then $P' \neq \phi$ and $Q' \neq \phi$.

(a) Let $|P'| \leq |P|$ and $|Q'| > |Q|$ with $2 \leq |P'| \leq \frac{n}{2}$ and $1 \leq |Q'| \leq \lfloor \frac{n-3}{3} \rfloor$, $(n > 4)$. Thus $|C'| \leq \frac{n-2}{2} + \lfloor \frac{n-3}{3} \rfloor$. Hence $|C'| \geq |C|$.

(b) Suppose $|P'| > |P|$ and $|Q'| > |Q|$ with $\frac{n+2}{2} \leq |P'| \leq n - 3$ and $1 \leq |Q'| \leq \lceil \frac{n-4}{6} \rceil$, $(n > 4)$ for $n \cong 2(\mod 3)$ & $1 \leq |Q'| \leq \lfloor \frac{n-4}{6} \rfloor$, otherwise. It follows that $1 \leq |E_1'| \leq n - 4$ and $1 \leq |V_1'| \leq \lfloor \frac{n}{8} \rfloor$. Thus $|C'| \leq n - 4 + \lfloor \frac{n}{8} \rfloor$ and $|C'| \geq |C|$.

Case 2: Let $n \equiv 2 \ (\mod 4)$. Then $n = 4k+2, k = 1, 2, \ldots$. Proceed as in Case 2 of the Theorem 4.1, C is the corporate dominating set and $|C| = \frac{n+2}{4} = \lceil \frac{n}{4} \rceil$.

To prove that C is minimum, let C' be any other corporate dominating set. Since $|P| = \frac{n-6}{2}$ and $|Q| = 2$. If $C' = V_1'$ holds or if $C' = E_1'$ holds, then proceed as in Case 2 of Theorem 4.1, $|C'| \geq |C|$.

If $C' = V_1' \cup E_1'$ holds, then $P' \neq \phi$ and $Q' \neq \phi$.

(a) Let $|P'| \leq |P|$ and $|Q'| \geq |Q|$ with $2 \leq |P'| \leq \frac{n-6}{2}$ and $2 \leq |Q'| \leq \lceil \frac{n-3}{3} \rceil$for $n \equiv 2(\mod 3)$ &$2 \leq |Q'| \leq \lfloor \frac{n-3}{3} \rfloor$ otherwise, $(n > 6)$. Hence $|C'| \leq \frac{n-8}{2} + \lceil \frac{n-3}{3} \rceil$. Thus, $|C'| \geq |C|$.

(b) Let $|P'| > |P|$ and $|Q'| \leq |Q|$ with $2 \leq |P'| \leq n-3$ and $1 \leq |Q'| \leq 2$. Hence C' contains at most $n - 4$ edges and two vertices. Thus $|C'| \leq n - 2$. Therefore $|C'| \geq |C|$.

Suppose $|Q'| > |Q|$ with $\frac{n-4}{2} \leq |P'| \leq n - 10$ and $3 \leq |Q'| \leq \lfloor \frac{n+4}{6} \rfloor, (n > 14)$. Hence C' contains at most $n - 11$ edges and $\lfloor \frac{n+4}{6} \rfloor$ vertices. Thus $|C'| \geq |C|$.

Case 3: Let n be odd and $n \equiv 1(\mod 4)$. Then $n = 4k + 1$, $k = 1, 2, \ldots$. Proceed as in Case 3 of Theorem 4.1,C is the corporate dominating set and $|C| = \frac{n+3}{4} = \lceil \frac{n}{4} \rceil$.

Now, we claim that C is minimum. Since $|P| = \frac{n-9}{2}$ and $|Q| = 3$.If $C' = V_1'$ holds or if $C' = E_1'$ holds, then proceed as in Case 2 of Theorem 4.1, $|C'| \geq |C|$. Suppose $C' = V_1' \cup E_1'$ holds, then $P' \neq \phi$ and $Q' \neq \phi$.

(a) Let $|P'| \leq |P|$ and $|Q'| \geq |Q|$ with $2 \leq |P'| \leq \frac{n-9}{2}$ and $3 \leq |Q'| \leq \lfloor \frac{n-3}{3} \rfloor$ $for\ n \cong 1(\mod 3)\&3 \leq |Q'| \leq \lceil \frac{n-3}{3} \rceil$ otherwise, $(n > 5)$. Hence $|C'| \leq \frac{n-11}{2} + \lfloor \frac{n-3}{3} \rfloor$. Thus, $|C'| \geq |C|$.

(b) Let $|P'| > |P|$ and $|Q'| \leq |Q|$ with $2 \leq |P'| \leq n-3$ and $1 \leq |Q'| \leq 3, (n > 5)$. Hence C' contains at most $n-4$ edges and three vertices. Thus $|C'| \leq n-1$. Therefore $|C'| \geq |C|$. Suppose $|Q'| > |Q|$ with $5 \leq |P'| \leq n-12$ and $4 \leq |Q'| \leq \lfloor \frac{n+7}{6} \rfloor for\ n \cong 1(\mod 3)$ $\&4 \leq |Q'| \leq \lceil \frac{n+7}{6} \rceil$ $otherwise, (n > 13)$. Hence C' contains at most $n-13$ edges and $\lceil \frac{n+7}{6} \rceil$ vertices. Thus $|C'| \geq |C|$.

Case 4: Let $n \equiv 3(\mod 4)$. Then $n = 4k+3,\ k = 0, 1, 2, \ldots$. As in Case 4 of Theorem 4.1,C is a corporate dominating set and $|C| = \frac{n+1}{4} = \lceil \frac{n}{4} \rceil$.

We claim that C is minimum. Since $|P| = \frac{n-3}{2}$ and $|Q| = 1$. If $C' = V_1'$ holds or if $C' = E_1'$ holds, then apply the similar argument which is used in Case 3 of Theorem 4.1,$|C'| \geq |C|$.

Suppose $C' = V_1' \cup E_1'$ holds, then $P' \neq \phi$ and $Q' \neq \phi$.

(a) Let $|P'| \leq |P|$ and $|Q'| \geq |Q|$ with $2 \leq |P'| \leq \frac{n-3}{2}$ and $1 \leq |Q'| \leq \lfloor \frac{n-3}{3} \rfloor$ for $n \equiv 1(\mod 3)$ & $1 \leq |Q'| \leq \lceil \frac{n-3}{3} \rceil$ otherwise. Hence $|C'| \leq \frac{n-5}{2} + \lceil \frac{n-3}{3} \rceil$. Thus, $|C'| \geq |C|$.

(b) Suppose $|P'| > |P|$ and $|Q'| \geq |Q|$ with $3 \leq |P'| \leq n-3$ and $1 \leq |Q'| \leq \lfloor \frac{n-1}{6} \rfloor$ for $n \equiv 2(\mod 3)$ & $1 \leq |Q'| \leq \lceil \frac{n-1}{6} \rceil$, otherwise. Hence C' contains at most $n-4$ edges and $\lceil \frac{n-1}{6} \rceil$vertices. Therefore $|C'| \geq |C|$. From the above cases, C is minimum and $\gamma_{cor}(P_n) = \lceil \frac{n}{4} \rceil$. □

Conclusion and Scope

We have obtained $\gamma_{cor}(G)$ for some standard graphs. We also determined the corporate domination number of the Cartesian product of a cycle and path. Further $\gamma_{cor}(G)$ for the Cartesian product of two paths can be found. The following problems arise.

Problem 1. Characterize graphs G of order $n, (n > 2)$ for which $\gamma_{cor}(G) + \gamma(G) = n$.

Problem 2. Characterize graphs G for which $\gamma_{cor}(G) < \gamma_p(G)$.

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THE STANDARD FIREWORKS, RAJARATNAM COLLEGE FOR WOMEN, SIVAKASI, TAMILNADU, INDIA.

Email address: padmashini1992@gmail.com

THE STANDARD FIREWORKS, RAJARATNAM COLLEGE FOR WOMEN, SIVAKASI, TAMILNADU, INDIA.

Email address: pethanachi-mat@sfrcollege.edu.in