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FRW UNIVERSE WITH GENERALIZED GHOST DARK ENERGY MODEL

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ABSTRACT. In this paper we consider a correspondence between generalized ghost dark energy density of the form $\rho = \alpha H + \beta H^2$ and inhomogeneous equation of state of form $p = \omega\rho + \Lambda(t)$ with equation of state parameter ω is constant and $\omega(\rho) = A_0\rho^{\delta-1}$ for viscous model of non -perfect fluid by avoiding the introduction of exotic dark energy. We consider the first model in terms of deceleration parameter q has a viscosity of the form $\zeta = \zeta_0 + (\zeta_1 - \zeta_2 q)H$. In this framework we find the solutions of field equations and investigate the behaviour of cosmological parameters.

1. INTRODUCTION

A new model of dark energy has been proposed by Urban and Zhitnitsky [1, 5] and Ohta [6] under the name of Veneziano ghost dark energy (GDE). The Veneziano GDE plays a crucial role in the resolution of the $U(1)_A$ problem on the basis of Veneziano ghost chromodynamics (QCD) called GDE [7]. Cai et.al [8] presented a generalized GDE of the form $\alpha H + \beta H^2$ based on the vaccum energy of the QCD ghost field and investigated a dark energy problem.

In this paper we extend our previous work of Khadekar and Deepti [9] by considering the above inhomogeneous equation of state with the assumption of

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generalized GDE density of the form $\rho = \alpha H + \beta H^2$ and presented the above bulk viscous cosmological model and find the solutions of the field equations analytically and numerically.

The paper is organized as follows. In section 2 we obtain the explicit solution of the model using Generalized GDE and calculate analytically and numerically the corresponding cosmic quantities for $\omega(\rho) = A_0\rho^{\delta-1}$. We present our conclusion in the last section.

2. SECOND SECTION: THE MODEL AND THE FIELD EQUATIONS

We consider the viscous fluid energy momentum tensor as defined by Xin Meng [10],

$$T_{\mu\nu} = \rho U_\mu U_\nu + (p - \zeta\theta)h_{\mu\nu}, \tag{2.1}$$

where ρ is the energy density, p is the pressure, U^μ is the four velocity of the fluid in comoving coordinates, $h_{\mu\nu}$ is the projection tensor, θ is the scalar expansion and ζ is the bulk viscosity.

Here we consider the Friedman-Robertson-Walker(FRW) universe with line element,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\phi^2\right], \tag{2.2}$$

where $k = -1, 0, 1$. Here $\theta = 3H$ and $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Therefore the corresponding Friedman equation, considering the case of flat space-time, is of the form [9]

$$H^2 = \frac{8\pi G}{3}\rho, \tag{2.3}$$

$$\dot{H} + H^2 = \frac{4\pi G}{3}(\rho + 3\bar{p}), \tag{2.4}$$

where (.) represents the time derivative and $\bar{p} = p - \zeta\theta$ is the effective pressure.

The conservation equation for a complete dynamics system $(T^\mu_\nu); \mu = 0$ gives

$$\dot{\rho} + (\rho + \bar{p})\theta = 0. \tag{2.5}$$

We consider the EoS of the form

$$p = \omega\rho + \Lambda(t), \tag{2.6}$$

where $\omega(\rho) = A_0\rho^{\delta-1} - 1$.

The generalized GDE dark energy proposed by Cai et.al [8] is of the form

$$\rho = \alpha H + \beta H^2. \tag{2.7}$$

After combining Eq. (2.5), Eq. (2.6) and Eq. (2.7) we get,

$$\left(\frac{\alpha}{H_0} + 2\beta\right) H\dot{H} + 3A_0(\alpha H + \beta H^2)^\delta H + 3\Lambda(t)H - 9\zeta H^2 = 0. \tag{2.8}$$

The dimensionless equation can be written as

$$\left(\frac{\alpha}{H_0} + 2\beta\right)\frac{\dot{h}}{H_0} + \left[\frac{3A}{H_0^2}(\alpha h H_0 + \beta h^2 H_0^2)\right]^\delta + \frac{3\Lambda_0 h}{H_0^2} - \xi h = 0, \tag{2.9}$$

where $\Lambda(t) = \Lambda_0 h$, $h = \frac{H}{H_0}$, $\xi = \frac{9\zeta}{H_0^2}$. The above equation in terms of dash (') can be written as

$$\left(\frac{\alpha}{H_0} + 2\beta\right) h' + 3A\beta^\delta H_0^{2\delta-2}h^{2\delta-1} + 3A\beta^{\delta-1}H_0^{2\delta-3}h^{2\delta-2} = \left[\xi - \frac{3\Lambda_0}{H_0^2}\right], \tag{2.10}$$

where $\frac{d}{dt} = \frac{\dot{a}}{a}\frac{d}{dlna}$ and dash (') denote the differential respect to conformal time lna.

Now we consider the bulk viscosity in terms of decelerating parameter of the form

$$\zeta = \zeta_0 + (\zeta_1 - \zeta_2 q)H, \tag{2.11}$$

in which $q = -\ddot{a}/aH^2$ is referred as the decelerating parameter and ζ_1, ζ_2, ζ_3, are constants respectively.

By following the procedure of Xin Meng [10] the transformation $\zeta_0 = \frac{H_0^2\xi_0}{12\pi G}$, $\zeta_1 = \frac{H_0}{12\pi G}(\xi_1 - \xi_2)$, $\zeta_2 = \frac{H_0\xi_2}{12\pi G}$ we obtain the dimensionless form of viscosity:

$$\xi = \xi_0 + \xi_1 h + \xi_2 h'. \tag{2.12}$$

After substituting above value, Eq. (10) reduces to the form

$$\left(\frac{\alpha}{H_0} + 2\beta - \xi_2\right) h' + 3A\beta^\delta H_0^{2\delta-2}h^{2\delta-1} + 3A\beta^{\delta-1}H_0^{2\delta-3}h^{2\delta-2} - \xi_1 h = X_0, \tag{2.13}$$

where $X_0 = \xi_0 - \frac{3\Lambda_0}{H_0^2}$.

2.1. **Case I** $\beta = 0$. In this case $\beta = 0$ Eq. (2.13) reduces to

$$\left(\frac{\alpha}{H_0} - \xi_2\right) h' - \xi_1 h = X_0. \tag{2.14}$$

By using the initial condition $h(a_0) = 1$ we get the analytical solutions as

$$h(a) = \frac{-X_0}{\xi_1} + \left[\frac{\xi_1 + X_0}{\xi_1}\right]\left[\frac{a}{a_0}\right]^{\frac{-\xi_1}{\frac{\alpha}{H_0}-\xi_2}}. \tag{2.15}$$

After integration we obtain the corresponding scale factor,

$$a(t) = a_0 \left[\frac{-\xi_1}{X_0} exp\left(\frac{X_0 H_0 (t-t_0)}{\frac{\alpha}{H_0} - \xi_2}\right) - \frac{\xi_1}{X_0} + 1\right]^{\frac{\frac{\alpha}{H_0}-\xi_2}{\xi_1}}. \tag{2.16}$$

Hence the Hubble parameter with respect to t gives

$$H = \frac{H_0 X_0}{-\xi_1}\left[\frac{exp\left(\frac{X_0 H_0 (t-t_0)}{\frac{\alpha}{H_0}-\xi_2}\right)}{exp\left(\frac{X_0 H_0 (t-t_0)}{\frac{\alpha}{H_0}-\xi_2}\right) - \frac{\xi_1 + X_0}{\xi_1}}\right], \tag{2.17}$$

and the decelerating parameter q is

$$q = -1 + \left[\frac{\xi_1 + X_0}{\frac{\alpha}{H_0} - \xi_2}\right] exp\left(-\frac{X_0 H_0 (t-t_0)}{\frac{\alpha}{H_0} - \xi_2}\right). \tag{2.18}$$

Using Eq. (2.17), Eq. (2.18), we obtain the bulk viscosity from Eq. (2.11) as

$$\begin{aligned}\zeta &= \frac{H_0^2}{12\pi G}\left[\frac{-3X_0}{2\xi_1} + X_0\left(\frac{\xi_1 + X_0}{\xi_1}\right)\right] \\ &\times \left[\left(\frac{\xi_1}{\frac{\alpha}{H_0} - \xi_2}\right)\frac{1}{exp\left(\frac{X_0 H_0 (t-t_0)}{\frac{\alpha}{H_0}-\xi_2}\right)} - \frac{\xi_1 + X_0}{\xi_1}\right].\end{aligned} \tag{2.19}$$

2.2. **Case II:** $\beta \neq 0$. In this case, for simplicity we consider $\delta = \frac{3}{2}$, then Eq. (2.13) reduces to

$$h' = \left[Y_1 h^2 + Y_2 h + Y_3\right], \tag{2.20}$$

where $Y_1 = \frac{-3A\beta^{\frac{3}{2}} H_0}{X_1}$, $Y_2 = \frac{\xi_1 - \frac{9}{2}\alpha A \beta^{\frac{1}{2}}}{X_1}$, $Y_3 = \frac{X_0}{X_1}$ and $X_1 = \frac{\alpha}{H_0} + 2\beta - \xi_2$.

For the case $\beta \neq 0$, the solution of Eq. (2.14) depends on the sign of $\Delta = \left[\frac{Y_2}{1-\xi_2}\right]^2 - \frac{4Y_3 Y_1}{1-\xi_2}$.

For $\Delta < 0$:

In the case $\Delta < 0$, when $\frac{4Y_1}{1-\xi_2} + \frac{2Y_2}{1-\xi_2} \neq 0$ the evolution of the universe is

$$h = \frac{-4Y_3 - Y_2}{2Y_1 + Y_2} + (1 - \frac{-4Y_3 - Y_2}{2Y_1 + Y_2})\frac{1}{1 - \frac{2Y_1+Y_2}{2\sqrt{-\Delta}}}[tan(\frac{\sqrt{-\Delta}}{2})]ln(\frac{a}{a_0}). \tag{2.21}$$

For $\Delta > 0$:

In the case $\Delta > 0$, when $\frac{4Y_1}{1-\xi_2} + \frac{2Y_2}{1-\xi_2} \neq 2\sqrt{\Delta}$ and $Y_1 \neq 0$ an alternative expression is yield as

$$\frac{2\sqrt{\Delta}(1-\xi_2) - 2Y_2}{4Y_1} + \frac{(1-\xi_2)\sqrt{\Delta}}{Y_1}\frac{4Y_1 - 2\sqrt{\Delta}(1-\xi_2) + 2Y_2}{4Y_1 + 2\sqrt{\Delta}(1-\xi_2) + 2Y_2}$$
$$\left[\left(\frac{a}{a_0}\right)^{\sqrt{\Delta}} - \frac{4Y_1 - 2\sqrt{\Delta}(1-\xi_2) + 2Y_2}{4Y_1 + 2\sqrt{\Delta}(1-\xi_2) + 2Y_2}\right]^{-1},$$

and for simplicity we rewrite this formula as

$$h = 1 - \frac{\bar{Y_1}\bar{Y_2}}{\bar{Y_1} - 1} + \frac{\bar{Y_1}\bar{Y_2}}{\bar{Y_1} - \left(\frac{a}{a_0}\right)^{\sqrt{\Delta}}}, \tag{2.22}$$

where $\bar{Y_1} = \frac{4Y_1 - 2\sqrt{\Delta}(1-\xi_2)+2Y_2}{4Y_1+2\sqrt{\Delta}(1-\xi_2)+2Y_2}$ and $\bar{Y_2} = \frac{(1-\xi_2)\sqrt{\Delta}}{Y_1}$.

Using $\frac{\dot{a}}{a} = hH_0$ an expression of a(t) is obtained by integration, which is

$$\begin{aligned} \frac{[a(t)]^{\bar{Y_1} - \bar{Y_1}\bar{Y_2} - 1}}{\bar{Y_1} - a(t)^{\bar{Y_2}(\bar{Y_1}-1)}} &= C_1(\bar{Y_1} - \bar{Y_1}\bar{Y_2} - 1) \\ &\cdot \exp\left[\frac{(\bar{Y_1} - \bar{Y_1}\bar{Y_2} - 1)(\bar{Y_1} - \bar{Y_2} - 1)}{(\bar{Y_1} - 1)}H_0(t - t_0)\right], \end{aligned} \tag{2.23}$$

where C_1 is an constant of integration determined by the condition $a(t_0) = a_0$. When $a(t)$ grows with time flying so that $a(t)^{\bar{Y_2}} >> \bar{Y_1}$, Eq. (2.24) reduces to

$$a(t) \cong a_0 exp\left[Y_4 H_0(t - t_0)\right], \tag{2.24}$$

where $Y_4 = \frac{(\bar{Y_1} - \bar{Y_1}\bar{Y_2} - 1)(\bar{Y_1} - \bar{Y_2} - 1)}{(\bar{Y_1} - 1)(\bar{Y_1} + \bar{Y_2} - 2\bar{Y_1}\bar{Y_2})}$.

Under the same approximation the Hubble parameter with respect to t is constant,

$$H(t) \cong Y_4 H_0. \tag{2.25}$$

The energy density ρ from Eq.(2.3) with the help of Eq. (2.26) can be expressed as

$$\rho = \frac{3}{8\pi G} Y_4^2 H_0^2. \tag{2.26}$$

Avoiding higher powers δ Eq. (2.13) takes the form as

$$X_1 h' + 3A\beta^{\delta} H_0^{2\delta-2} h^{2\delta-1} - \xi_1 h = X_0. \tag{2.27}$$

For simplicity, we consider $\delta = \frac{5}{4}$. Eq. (2.28) reduces to the form

$$h' + \frac{3X_2}{X_1} h^{\frac{3}{2}} - \frac{\xi_1}{X_1} h - \frac{X_0}{X_1} = 0, \tag{2.28}$$

where $X_2 = A\beta^{\frac{5}{4}} H_0^{\frac{1}{4}}$.

In order to solve Eq. (2.29) we use the method proposed by Saadat and Pourhassan (2013),

$$h = \frac{A}{t^2} + \frac{E}{t} + dt + Ce^{bt}, \tag{2.29}$$

where constants A, E, d, C, and b should be determined. Substituting Eq. (2.30) in Eq. (2.29) gives the coefficients $d = \frac{X_0}{X_1}$, $A = \frac{4}{9}\frac{X_1^2}{X_2^2}$,$E = \frac{2}{9}\frac{\xi_1 X_1}{X_2^2}$, $C = \frac{2}{27}\frac{\xi_1^2 X_1}{X_2} - \frac{27}{25}\frac{X_2^6}{\xi_1^2 X_1^2}$ and $b = \frac{\frac{\xi_1}{3X_1}\left(\frac{3\xi_1 X_2}{X_1^2}\right)\left(\frac{X_0 X_2}{X_1^2} - \frac{3X_0^3}{2\sqrt{3}X_1^3}\right)}{\frac{8\sqrt{3}X_2}{X_1}\left[\frac{\sqrt{3}\xi_1^4}{81X_1^4} - \left(\frac{3\sqrt{3}X_2}{128X_1}\right)^7\right]} + \frac{\left[\frac{27}{32}\left(\frac{\sqrt{3}X_2}{X_1} - \frac{7}{8}\right) + \frac{1}{144}\frac{\xi^4}{X_1^2} + O(r^n)\right]}{\frac{8\sqrt{3}X_2}{X_1}\left[\frac{\sqrt{3}\xi_1^4}{81X_1^4} - \left(\frac{3\sqrt{3}X_2}{128X_1}\right)^7\right]}$.

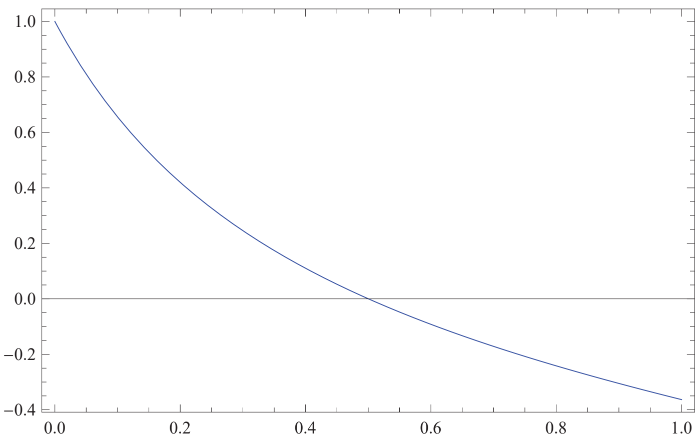

FIGURE 1. Relation between a and t for Model I with $\xi_1 = \xi_2 = H_0 = \alpha = X_0 = 1$

3. Conclusion

In this paper based on vacuum energy of the QCD ghost field we have constructed bulk viscous model by considering inhomogeneous equation of state of the form $p = \omega\rho + \Lambda(t)$ where $\omega(\rho)$ depends on energy density ρ under the assumption that our universe is bound more tightly. The presence of inhomogeneous term in the EoS leads either to compression of the universe in the evolution process or to a Quasi-periodical change in the energy density and the Hubble parameter and also the appearance of singularities [11]. It is easy to see that the effective value of EoS parameter may easily be adjusted so as to approximately equal to -1 in the first case. At present it corresponds to current observational bounds. By adding a cosmological constant term Λ in the inhomogeneous EOS led as to the conclusions that at some time $t \to t_0$, a future singularity forms, in the sense that the energy density and Hubble parameter simultaneously approaches to infinity as $(t - t_0) \to \infty$. We find an exact solutions of this model and discuss the fate of the universe evolution. It is observed that the energy density is positive throughout the evolution of the universe. Therefore, the bulk viscosity varies with respect to not only conventional momentum term H but also accelerating status $\dot{H}$ for model .

Brevik [12] proposed a little rip cosmology as a purely viscosity effect. From Eq. (2.17) it is seen that when $\frac{\xi_1 + X_0}{\xi_1} > 1$ and $t \to \infty$ we get $H \to \infty$ which is a case of Little rip. Similarly when $\frac{\xi_1 + X_0}{\xi_1} < 1$ and $t \to \infty$ we get $H \to constant$ which is a case of Pseudo rip. Thus we get both Little and Pseudo rip in presence of viscosity for the model. From Eq. (2.27) it is observed that ρ is constant and hence in this case Hubble parameter is also constant. Also from Eq. (2.25) we get the evolution of the scale factor in the form of $a(t) \cong a_0 exp\left(X_6 H_0 (t - t_0)\right)$ which is the de-Sitter universe.

We have drawn the behaviour of scale factor $a(t)$ versus time t. It is observed that there are two phases during the evolution: An exponentially inflationary scenario at the beginning of the universe followed by decelarating phase as shown in Fig 1.

Finally, we would like stress that in fact there have not been any precise calculations showing that the vacuum energy density of the Veneziano ghost of QCD in an FRW universe is of the form $\alpha H + \beta H^2$, because the vacuum energy calculation of the Veneziano ghost is quiet difficult in both flat and curved space-times due to the intrinsic difficulties of QCD, and strongly interacting fields in general [1, 5].

REFERENCES

[1] F.R. URBAN, A.R. ZHITNITSKY: *The cosmological constant from the QCD Veneziano ghost*, Phys. Lett. B **688** (2010), 9-12.

[2] F.R. URBAN, A.R. ZHITNITSKY: *Cosmological constant from the ghost: A toy model*, Phys. Rev. D **80** (2009), 063001.

[3] F.R. URBAN, A.R. ZHITNITSKY): *Cosmological constant, violation of cosmological isotropy and CMB.* J. Cosmol. Astropart. Phys, **09** (2009), 018.

[4] F.R. URBAN, A.R. ZHITNITSKY: *The QCD nature of dark energy*, Nucl. Phys. B **835** (2010), 135-173.

[5] A.R. ZHITNITSKY: *Entropy, contact interaction with horizon, and dark energy* , Phys. Rev. D **84** (2011), 124008.

[6] N. OHTA: *Dark energy and QCD ghost* , Phys. Lett. B **695** (2011), 41-44.

[7] G. VENEZIANO: *U(1) without instantons* , Nucl. Phys. B **159** (1979), 213-224.

[8] R.G. CAI, Z.L. TUO, H.B. ZHANG, Q. SU: *More on QCD ghost dark energy*, Phys. Rev. D **86** (2012), 023511-1 to 023511-8.

[9] G.S. KHADEKAR, D. RAUT: *FRW viscous fluid cosmological model with time-dependent inhomogeneous equation of state* , Int. Jour. Geom. Meth. in Mod. Phys **15** (2018), 1830001-1 to 1830001-14

[10] X.H. MENG, Z.Y. MA: *Rip/singularity free cosmology models with bulk viscosity*, Eur. Phys. J C **72** (2012), 2053.

[11] I. BREVIK, O. GORBUNOVA, A.V. TIMOSHKIN: *Dark energy fluid with time-dependent, inhomogeneous equation of state* , Euro. Phy. J C **51** (2007), 179-183.

[12] I. BREVIK, E. ELIZALDE, S. NOJIRI, S.D. ODINTSOV: *Viscous little rip cosmology* , Phy. Rev. D **84** (2011), 103508-1 to 103508-6.

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