Advances in Mathematics: Scientific Journal **10** (2021), no.4, 2095–2104
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.4.24

IMPACTS OF HEAT SOURCE/SINK AND ELECTROMAGNETIC FIELD ON HEAT TRANSFER IN FERROFLUID FLOW

V. Loganayagi and Peri K. Kameswaran[1]

ABSTRACT. The inspiration driving this article is to explore the impacts of electromagnetic field and heat source/sink on two dimensional nanofluid flow over a stretching cylinder. The effectively controlled nanofluid model utilized in the current assessment incorporates the nanoparticle fraction model. The nanoparticles specified here are Cobalt Co and Gadolinium Gd. Cobalt, Co is a shunt material in the thermomagnetic generator. Gadolinium, Gd is the material first used practically in the thermomagnetic generator. Motor Oil has taken as a base fluid. The governing system of equations solved numerically utilizing MATLAB. The effect of electric field parameter on velocity profile, Heat source/sink on temperature and heat transfer profile, has examined. Further, we noticed that Gadolinium is the most suitable material for the thermomagnetic generator. Its temperature is close to room temperature.

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 76, 80.
Key words and phrases. Heat transfer, Magnetohydrodynamics, Electric field, Heat Source/sink.
Submitted: 11.03.2021; *Accepted:* 01.04.2021; *Published:* 14.04.2021.

1. INTRODUCTION

The thermomagnetic generator is an active device that used to convert heat energy into electrical energy. It is one of the solid-state thermal energy harvesters, and it works with no fuel and doesn't give any destructive discharge. Ferrofluid is a colloidal mix of engaging nanoparticles in a transporter liquid, conventionally water, or oil. Ferrofluid has utilized in numerous applications, for example, loudspeakers, rotational seals and pumps, and so on. Gholinia et al. [1] inquired about ethylene glycol with two nanofluids flow on a vertical porous cylinder in the presence of a magnetic field. They discovered that the velocity distribution enhances for more substantial nanoparticle volume fraction, mixed convection parameter, permeability parameter, and curvature parameter. Guestal et al. [2] examined the impacts of heat transfer by buoyancy convection of nanofluids in a partially heated closed cylinder. In their examinations, they discovered that as the nanoparticle volume fraction increases, heat transfer increases. Nagaraju and Mahesh [3] analyzed the impact of magnetized couple stress, heat mass transfer, cross-diffusion, and chemical reaction on a stretching cylinder with the convective boundary condition. They found that with an ascent in curvature parameter, velocity, temperature, and concentration profiles enhanced. Qasim et al. [4] inspected MHD boundary layer ferrofluid slip flow and heat transfer with prescribed thermal flux along a stretching cylinder. They found that surface shear stress and coefficient of heat transfer at the surface increases as the curvature parameter enhances. Hayat et al. [5] inspected the hydromagnetic boundary layer flow of Williamson fluid in the presence of ohmic dissipation and thermal radiation. They found that the local nusselt number raises for a larger electric parameter, suction parameter, Prandtl number and unsteadiness parameter.

An enormous number of the evaluations gave a record on Heat Source/Sink. Elbashbeshy et al. [6] studied numerically the boundary layer steady flow on a stretching horizontal cylinder under the effect of uniform magnetic field with suction/injection and heat source/sink. They found that, their results indicate that the shear stress at the surface can be minimized by increasing the stretching cylinder radius, the suction/injection velocity and decreasing the magnetic field strength. Dash and Behera [7] explored the heat transfer characteristics of laminar natural convective viscoelastic liquid from an isothermal cylinder with heat source/sink. They graphically compared the difference between the heat transfer coefficient of combined momentum and single momentum equation for viscoelastic liquids.

Disregarding of the Heat source/sink, it's essential to consider the electric field impact on the stretching cylinder is significant in Thermomagnetic generator. Feng et al. [8] studied numerical examination on simulated pool boiling curves with heat transfer in the presence of a uniform applied electric field by using the lattice Boltzmann (LB) model. They deduced that increasing gradually the electric field strength, intensity could enhance the heat transfer of boiling liquid. Based on the above showed assessments, at present, we assessed the consolidated impacts of the magnetic field, electric field, and heat source/sink effects on ferrofluid fluid over-stretching cylinder with convective heat transfer. We consider here, Cobalt, Co and Gadalonium, Gd nanoparticles with base fluid as motor oil. The combined effects of all the parameters in this study are significant in the thermomagnetic generator.

2. Mathematical Formulation

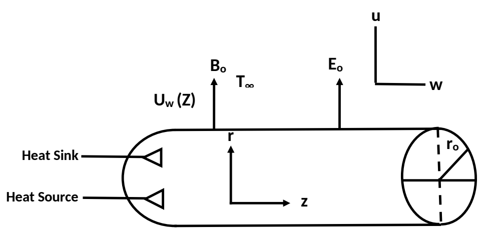

FIGURE 1. Physical model and coordinate system

A steady, two dimensional, laminar, viscous incompressible nanofluid flow over-stretching cylinder with radius r_0 is presented in Fig. 1. The coordinate system is chosen, such that z axis is measured along the axis of the tube and r axis is measured in the radial direction. The velocity components in the directions of r, z are u, w, p is fluid pressure. The constant and ambient temperatures at the cylinder's surface are T_w, T_∞ such that $T_w > T_\infty$. It is assumed that the uniform magnetic field with intensity of $B = (0, B_0, 0)$ acts in the radial direction and the effect of the induced magnetic field is negligible, which is valid only for small Reynolds number. The electromagnetic body force or Lorentz force is given by $J \times B$, where $J = \sigma (E + V \times B)$ is the Joul current, σ is the electrical conductivity, $V = (u, v)$ is the fluid velocity and $E = (0, 0, -E_0 z)$ is electric field vector. Under the above considerations, the governing equations can be expressed as (Tiwari and Das [9])

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0 \tag{2.1}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + \frac{\sigma}{\rho_{nf}}\left(E_0 B_0 z - B_0^2 w\right) \tag{2.2}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) \tag{2.3}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{Q}{(\rho C_p)_{nf}}(T - T_\infty). \tag{2.4}$$

The boundary conditions for Eqns. (2.1)-(2.4) are

$$\begin{aligned} &u = 0, \quad w = w_e = \frac{U_0 z}{l}, \quad T = T_w, \text{ at } r = R \\ &w \to 0, \quad T \to T_\infty, \quad p \to p_\infty, \text{ as } r \to \infty. \end{aligned} \tag{2.5}$$

The effective dynamic viscosity, density and thermal diffusivity of nanofluid are

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}. \tag{2.6}$$

The heat capacitance of the nanofluid is given by

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s. \tag{2.7}$$

The thermal conductivity of the nanofluids is approximated by the Maxwell-Garnetss (MG) model ([10] and [11])

$$k_{nf} = k_f\left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\right]. \tag{2.8}$$

Here, the subscripts nf, f, and s represent the thermophysical properties of the nanofluid, base fluid, and nanosolid particles, respectively (Table 1). Then introduce the new variable $\eta = \sqrt{\frac{U_0}{\nu_f l}}\left(\frac{r^2 - R^2}{2R}\right)$. Then, the velocity and temperature components are

$$u = -\sqrt{\frac{\nu_f U_0}{l}}\frac{R}{r}f(\eta), \quad w = \frac{U_0 z}{l}f'(\eta), \quad T = T_\infty + \Delta T\,\theta(\eta). \tag{2.9}$$

Substituting Eqns. (2.6)-(2.9) in Eqns. (2.2)-(2.5), we get

$$(1+2\gamma\eta)\,f''' + 2\gamma f'' + \frac{\phi_1}{\phi_2}M\left(E_1 - f'\right) - \phi_1\left(f'^2 - ff''\right) = 0 \tag{2.10}$$

$$(1+2\gamma\eta)\left(P' - \frac{\phi_2}{\phi_1}f''\right) + \phi_2\frac{R^2}{r^2}\gamma f^2 - \phi_2 ff' = 0 \tag{2.11}$$

$$(1+2\gamma\eta)\,\theta'' + 2\gamma\theta' + \left(\frac{k_f}{k_{nf}}\right)Pr\left(\phi_3 f\theta' + \delta_1\theta\right) = 0 \tag{2.12}$$

$$f(0) = 0, \quad f'(0) = 1, \; f'(\infty) \to 0 \tag{2.13}$$

$$\theta(0) = 1, \; \theta(\infty) \to 0 \tag{2.14}$$

$$P(\infty) \to 0 \tag{2.15}$$

The non-dimensional constants in Eqns. (2.10)-(2.12) are the curvature parameter γ, magnetic parameter M, electric parameter E_1, Prandtl number Pr, heat source/sink parameter δ_1 and pressure term p. These parameters are defined as follows:

$$\gamma = \sqrt{\frac{\nu_f l}{U_0 R^2}}, \quad M = \frac{\sigma B_0^2 l}{\rho_f U_0}, \quad E_1 = \frac{E_0 l}{B_0 U_0}, \quad Pr = \frac{\nu_f (\rho C_p)_f}{k_f},$$
$$\delta_1 = \frac{Ql}{\left(\rho C_p\right)_f U_0}, \quad p = p_\infty - \rho_f \frac{U_0}{l}\nu_f P(\eta), \quad \phi_1 = (1-\phi)^{2.5}\left[1 - \phi + \phi\left(\frac{\rho_s}{\rho_f}\right)\right],$$
$$\phi_2 = 1 - \phi + \phi\left(\frac{\rho_s}{\rho_f}\right), \quad \phi_3 = 1 - \phi + \phi\frac{\left(\rho_{Cp}\right)_s}{\left(\rho_{Cp}\right)_f}.$$

The Skin friction and dimensionless nusselt modulus coefficients are characterized as

$$C_f\,(1-\phi)^{2.5}\sqrt{Re_z} = f''(0), \quad Nu_z Re_z^{-1/2}\left(\frac{k_f}{k_{nf}}\right) = -\theta'(0), \tag{2.16}$$

where $Re = \frac{U_0}{\nu_f l}$.

TABLE 1. Thermo-physical properties of motor oil, Cobalt and Gadolinium nanoparticles are

Properties $\to$	$\rho\ (kg/m^3)$	$C_p\ (J/kgK)$	$k\ (W/mK)$
Oil (Motor Oil 10W30)	856	1990	0.158
Cobalt (Co)	8860	423	84.935
Gadolinium (Gd)	7870	230	8.786

3. Pressure gradient equation

The pressure gradient equation is partially decoupled from the momentum equation

$$P'(\eta) = \frac{\phi_2}{\phi_1} f'' + \phi_2 \frac{R^2}{r^2} f f' - \phi_2 \frac{R^4}{r^4} \gamma f^2 \tag{3.1}$$

4. Validation of Results

In pursuance of confirming the precision of the current numerical results, we've examined our outcomes with those of Elbashbeshy et al. [6], Ishak and Nazar [12], Grubka and Bobba [13]. The outcomes of these comparisons are as shown in Table 2.

Table 2. Examination of $-\theta'(0)$ for Different Measures of Pr in the exclusion of $M = E_1 = \gamma = \delta_1 = 0$

Pr	Present Results	Elbashbeshy et al. [6]	Ishak and Nazar [12]	Grubka and Bobba [13]
1	0.5820	0.5820	0.5820	0.5820
10	2.3080	2.3080	2.3080	2.3080

5. Discussion of Results

Figure 2 addresses the impact of the electric field parameter E_1 on the velocity profile. From the figure, we presume that the velocity decrease when it enhances the electric field parameter, because of Lorentz force acts perpendicular to the nanoparticles, and the drag force on the nanofluids reduces fluid flow motion. The density of Cobalt Co is more than Gadolinium Gd. It is more attracted by an electric field so that the velocity profile is diminishing more in Gadolinium Gd nanoparticles compare to Cobalt Co nanoparticles.

Figure 3 displays the fluctuations of heat source parameters on the temperature profile. A heat source produces thermal energy that carries the working substance to a high-temperature state. If the heat source parameter δ_1 enhances, the temperature of nanoparticles also increases. We see that the temperature profile enhances more in Gadolinium Gd than Cobalt Co with an enhance in the heat source parameter. It can be clarified that, Gadolinium Gd nanoparticles heat rapidly in the light of the fact that it has the least value of specific heat and low density than Cobalt Co nanoparticles.

Figure 4 illustrates the impact of the heat sink parameter on the temperature profile. A heat sink parameter used to transfer heat from a device to the surroundings, to prevent the device from overheating. If the heat sink parameter enhances, then the temperature profile also increases. Because of high thermal conductivity, Cobalt Co nanoparticles have minimal temperature than Gadolinium Gd nanoparticles.

Figure 5 displays the effect heat source/sink parameter δ_1 on the heat transfer coefficient with increasing the nanoparticle volume fraction. This figure shows that the heat transfer coefficient $-\theta'(0)$ diminishes very high in Gadolinium Gd nanoparticles than Cobalt Co nanoparticles for increasing values of heat source/sink parameter. From Figs. 3, and 4, we observe that the temperature gradient increases more in Gadolinium Gd nanoparticles compared to Cobalt Co nanoparticles with heat source/sink. Furthermore, with an increase in nanoparticle volume fraction, temperature increases in both cases of nanofluids. That's why the heat transfer coefficient decreases for both the nanofluids.

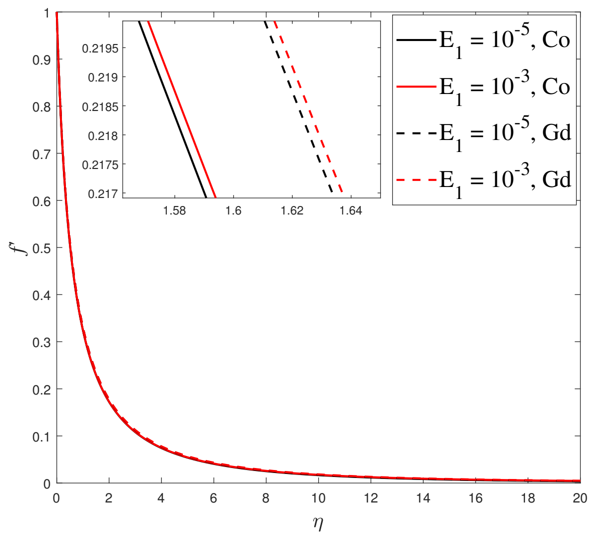

FIGURE 2. Variations of velocity profile for different values of E_1, Co, and Gd nanoparticles, when $Pr = 1531.92, M = 0.5, \gamma = 1.2, \delta_1 = 0.2, \phi = 0.1$

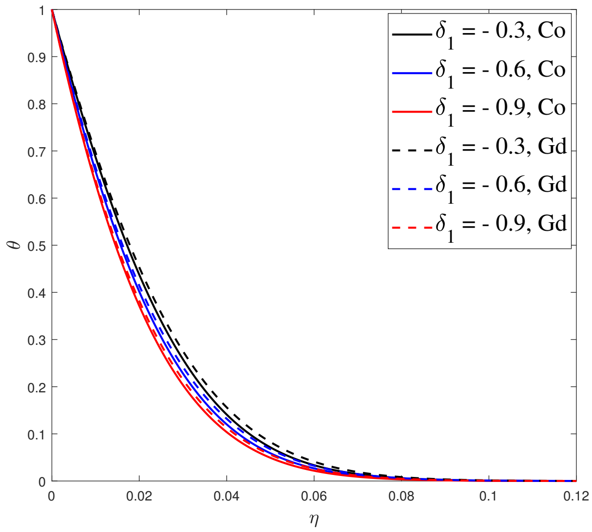

FIGURE 4. Variations of temperature profile for different values of δ_1 (negative), Co, and Gd nanoparticles, when $M = 0.5, Pr = 1531.92, \gamma = 1.2, E_1 = 10^{-3}, \phi = 0.1$

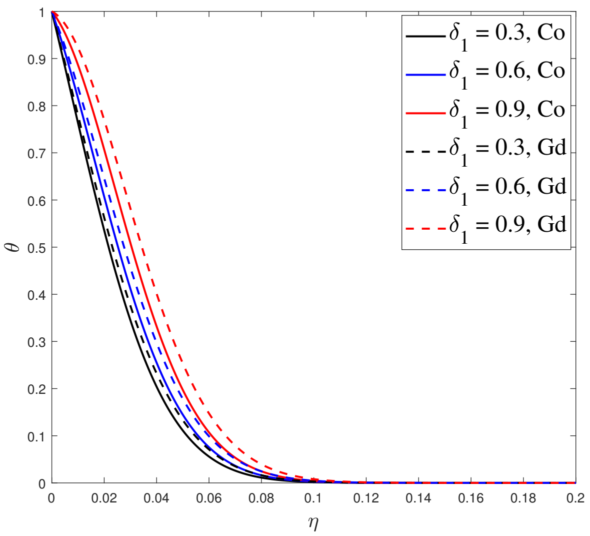

FIGURE 3. Variations of temperature profile for different values of δ_1 (positive), Co, and Gd nanoparticles, when $M = 0.5, Pr = 1531.92, \gamma = 1.2, E_1 = 10^{-3}, \phi = 0.1$

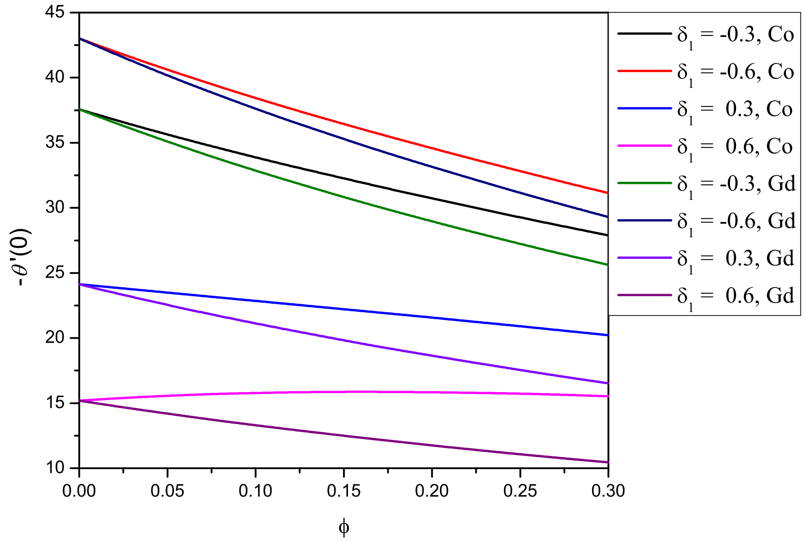

FIGURE 5. Heat transfer coefficient as a function of nanoparticle volume fraction ϕ for different values of heat source/sink parameter δ_1, Co, and Gd nanoparticles, when $Pr = 1531.92, \gamma = 1.2, E_1 = 10^{-3}, M = 0.5$

REFERENCES

[1] M. GHOLINIA, S. GHOLINIA, KH. HOSSEINZADEH, D.D. GANJI: *Investigation on ethylene glycol Nano fluid flow over a vertical permeable circular cylinder under effect of magnetic field*, Results in Physics, **9** (2018), 1525–1533.

[2] M. GUESTAL, M. KADJA, M. TON HOANG: *Study of heat transfer by natural convection of nanofluids in a partially heated cylindrical enclosure*, Case Studies in Thermal Eng. **11** (2018), 135–144.

[3] N. GAJJELA, M. GARVANDHA: *The influence of magnetized couple stress heat, and mass transfer flow in a stretching cylinder with convective boundary condition, cross-diffusion, and chemical reaction*, Thermal Science and Engineering Progress, **18**(1) (2020), 100517.

[4] M. QASIM, Z. H. KHAN, W. A. KHAN, I. A. SHAH: *MHD Boundary Layer Slip Flow and Heat Transfer of Ferrofluid along a Stretching Cylinder with Prescribed Heat Flux*, PLOS ONE, **9**(1) (2014), e83930.

[5] T. HAYAT, A. SHAFIQ, A. ALSAEDI: *Hydromagnetic boundary layer flow of Williamson fluid in the presence of thermal radiation and Ohmic dissipation*, Alexandria Engineering Journal, **55**(3) (2016), 2229–2240.

[6] E. M. A. ELBASHBESHY, T. G. EMAM, M. S. EL-AZAB, K. M. ABDELGABER: *Effect of Magnetic Field on Flow and Heat Transfer over a Stretching Horizontal Cylinder in the Presence of a Heat Source/Sink with Suction/Injection*, Journal of Applied Mechanical Engineering **1**(1) (2012), 1000106 (1-6).

[7] G. C. DASH, S. C. BEHERA: *Laminar free convection stagnation heat transfer of a viscoelastic liquid from an isothermal cylinder with an internal source and/or sink*, Wear, **81**(2) (1982), 197–208.

[8] Y. FENG, H. LI, K. GUO, X. LEI, J. ZHAO: *Numerical study on saturated pool boiling heat transfer in presence of a uniform electric field using lattice Boltzmann method*, International Journal of Heat and Mass Transfer, **135** (2019), 885–896.

[9] R.K. TIWARI, M.K. DAS: *Heat transfer augmentation in a twosided lid-driven differentially heated square cavity utilizing nanofluids*, International Journal of Heat and Mass Transfer, **50** (2007), 2002–2018.

[10] J.C. MAXWELL GARNETT: *Colours in metal glasses and in metallic films*, Philosophical Transactions of the Royal Society of London. A, **203** (1904), 385–420.

[11] C.A. GUERIN, P. MALLET, A. SENTENAC: *Effective-medium theory for finite-size aggregates*, Journal of the Optical Society of America A, **23**(2) (2006), 349–358.

[12] A. ISHAK, R. NAZAR: *Laminar Boundary layer flow along a stretching cylinder*, European Journal of Scientific Research, **36**, (2009) 22–29.

[13] L.J. GRUBKA, K.M. BOBBA: *Heat transfer characteristics of a continuous stretching surface with variable temperature*, ASME Journal of Heat Transfer, **107** (1985) 248–250.

DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
VELLORE INSTITUTE OF TECHNOLOGY
VELLORE, 632014,
INDIA.
Email address: loganayagi2008@gmail.com

DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
VELLORE INSTITUTE OF TECHNOLOGY
VELLORE, 632014,
INDIA
Email address: perikamesh@gmail.com