Advances in Mathematics: Scientific Journal **10** (2021), no.4, 2131–2139
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.4.27

DISTANCE DOMINATION ON TOPOLOGICAL GRAPHS AND ITS APPLICATIONS

R. Veerasivaji and S. Meenakshi

ABSTRACT. In this paper we introduce topological graph as bus topological graph, ring topological graph, star topological graph, mesh topological graph and hybrid topological graph. We extend the result that if T is a tree and it has maximum degree m then there exist at least m pendant vertices in to if T is a tree except bus topological graph and it has maximum degree m then there exist exactly m pendant vertices.

1. INTRODUCTION

There are several papers and books regarding domination [4] in graph theory. In 1975, domination was extended to distance domination by Meir and Moon [5]. Some of the results based on distance dominating set were extended by P.J. Slater in his paper R-domination in graphs in 1976 [8]. The concept of perfect dominating set was introduced by Marilynn Livingston and Quentin F. Stout in 1990 [6].

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 68R10, 90C35, 94C15, 05C69, 05C45.
Key words and phrases. Distance Domination, Bus, Star, Ring, Mesh, Hybrid Topological Graphs and Block.
Submitted: 26.03.2021; *Accepted:* 14.04.2021; *Published:* 17.04.2021.

In this paper we will discuss the application of graph theory particularly the applying the distance domination concept in to some of network topology. In the field of computer science there are lots of network based results available now a day, these networks are very useful to reduce the cast, time, comfort, design and connectivity. We have to choose some network topology as bus topology, ring topology, star topology, mesh topology and hybrid topology. These networks have some merits and demerits. So, all such a networks are converting in to topological graph and applying the distance domination concept to such the networks.

A dominating set is a subset D of V in $G = (V, E)$ such that every vertex not in D is adjacent to at least one member of D [1, 7]. The different type of trees rolled as network topology and more results discussed in distance dominating set on tree [9]. A distance dominating set is a subset D of V such that every vertex u not in D, the distance $d(u, v) \leq k$ for some v in D. Sometimes it is called as k-distance dominating set or kd-dominating set [9].

2. Topological graphs

We introduce topological graph as bus topological graph, ring topological graph, star topological graph, mesh topological graph and hybrid topological graph. These topological graphs definitions, characterizations, applications and some examples are discussed as below.

2.1. Bus Topological Graph.

Definition 2.1. *A graph is said to be* ***bus topological graph*** *if each adjacent vertex of pendent vertex has exactly degree three.*

Example 1. *Any degree sequence of bus topological graph is always*

$$(1, 1, \underbrace{1, 1, ..., 1}_{m\ times}, \underbrace{3, 3, ..., 3}_{m\ times}).$$

Example 2. *Bus topological graph on 16 vertices.*

The bus topological graph is a special type of tree, its characterization are it has 2 unique pendent vertices plus m pendent vertices their adjacent vertices are degree exactly 3. It has at least one of the degrees 3, it has cut vertex, it has no perfect matching, but its maximum matching is $\dfrac{(n-2)}{2}$ *and it has always 2 unsaturated*

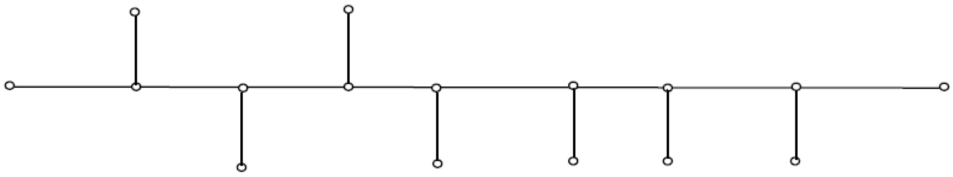

FIGURE 1.

vertices, its edge chromatic number always 3, its independent number either $\frac{n}{2}$ *plus 1 if* n *is multiple of 4 or* $\frac{n}{2}$.

2.2. **Ring Topological Graph.**

Definition 2.2. *A graph is said to be* ***ring topological graph*** *if each vertex has degree two.*

Example 3. *Any ring topological graph has degree sequence* $(2, 2, 2, ..., 2)$.

Example 4. *Ring topological graph on 6 vertices.*

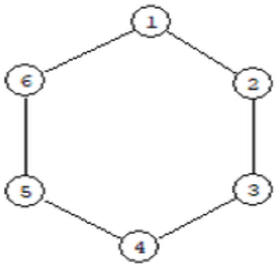

FIGURE 2.

The ring topological graph is a circuit graph, its characterization are any vertices has degree two, it is 2 regular graph, its degrees are equal, it has no cut vertex, it has two path between any two vertices, it has perfect matching, its maximum matching is $\frac{n}{2}$, *its edge chromatic number always 2, its independent number* $\left\lfloor \frac{n}{2} \right\rfloor$.

2.3. **Star Topological Graph.**

Definition 2.3. *A graph is said to be* ***star topological graph*** *if* m *be the maximum degree then there is exactly* m *pendent vertex.*

Example 5. *Any star topological graph has degree sequence* $(\underbrace{1, 1, ..., 1}_{m\ times}, m)$.

Example 6. *Star topological graph on 6 vertices.*

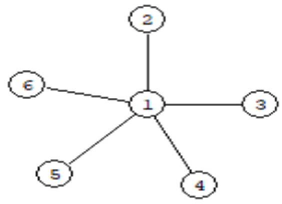

FIGURE 3.

The star topological graph is a graph, its characterization are it has m pendent vertex if and only if its maximum degree is m, it has a cut vertex, it is a biggest connected graph for getting more number of component when remove the cut vertex, it has no perfect matching, but its maximum matching is 1 and it has always $n-2$ unsaturated vertices, its edge chromatic number always $n-1$, its independent number is $n-1$, its covering number is 1.

2.4. **Mesh Topological Graph.**

Definition 2.4. *A graph is said to be **Mesh topological graph** if sum of the degree is $n(n-1)$, where n is number of vertices.*

Example 7. *Any mesh topological graph has degree sequence $(n-1, n-1, ..., n-1)$.*

Example 8. *Mesh topological graph on 6 vertices*

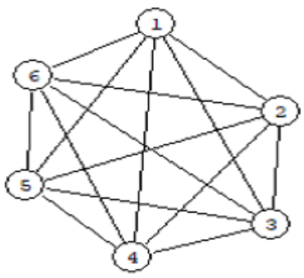

FIGURE 4.

The mesh topological graph is a complete graph, its characterization are it has n vertices then it is called $n-1$ regular graph, that is, each vertex of degrees are same, moreover we cannot add an edge in it, it has neither cut vertex nor cut vertex *set, it has perfect matching, but its maximum matching is* $\frac{n}{2}$*, its edge chromatic number always* $\frac{n}{2}$*, its independent number is 1, its covering number is* $n-1$.

2.5. **Hybrid Topological Graph.**

Definition 2.5. *A graph is said to be* ***hybrid topological graph*** *if it is mixed of bus, ring, star and mesh topological graph.*

Example 9. *Any hybrid topological graph has degree sequence cannot predict before.*

Example 10. *Hybrid topological graph on 12 vertices*

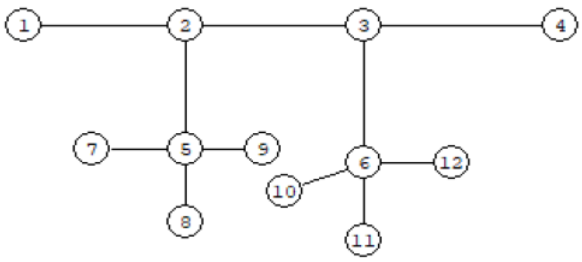

FIGURE 5.

The hybrid topological graph is a graph, its characterization is not predicting same for all graph.

3. DISTANCE DOMINATION ON TOPOLOGICAL GRAPHS

In field of computer science the bus topology, star topology, ring topology, mesh topology, hybrid topological are common network connections, there are some demerits in this networks. The concept of distance domination helps to analysis and alternatives connectivity for topological graphs, if M be the value of dm in any degree sequence of tree T [2, 3], then T has M pendant vertices, but bus topological graph is like tree which is not satisfied the Theorem 4.1 in [9]. Alternatively we say that if m be the maximum value of degree sequence of tree T then there are m pendent vertices in tree T.

We can choose the vertices from bus topological graph, it is clear that some of its degrees are three, suppose there are m such a vertices and any two of them should dominate all other vertices at the distance $\left[\dfrac{m+1}{2}\right]$, if m is odd, $\left[\dfrac{m}{2}\right]$, if m is even, such the vertices are called middle distance dominating vertices and they are adjacent such the vertices are sometimes called middle distance dominating path and also divider distance dominating path [10].

Let T be a tree, if it has maximum degree m, then there exist at least m pendant vertices in [9]. We consider the example on tree has 6 vertices, both are having maximum degree are 3, but Figure 6 having 4 pendant vertices, Figure 7 having 3 pendant vertices.

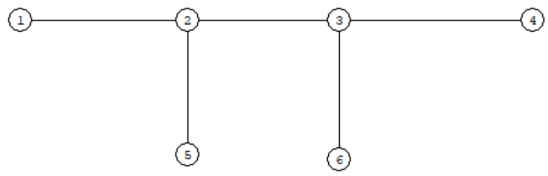

FIGURE 6.

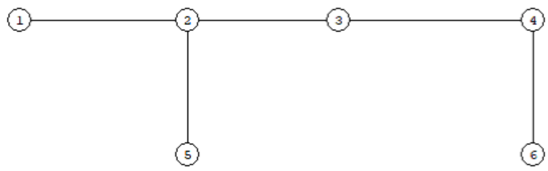

FIGURE 7.

Therefore we have the following theorem

Let R be a ring topological graph has n vertices, if $v \in R$ then $\{v\}$ is at most $\left[\dfrac{n+1}{2}\right]$ distance dominating set, such the vertex is called middle distance dominating vertex and we choose another vertex is adjacent to v such the vertices are sometimes called middle distance dominating path and also not divider distance dominating path [10]. Let $v \in R$, v is removing from R then R is still connected graph, by the same time is non-adjacent to v and removes u then R is disconnected. If R has n vertices then removes u, v then R has two components if they are same length (h) it is clear that either $\{u\}$ or $\{v\}$ is middle distance dominating set. Therefore $\{u\}$ or $\{v\}$ is $(2+h)$ middle distance dominating set.

The two components of same length of R is called middle distance dominating path and also not divider distance dominating path. R has perfect distance dominating set [11].

Let S be a star topological graph has n vertices, if $v \in S$ then $\{v\}$ is at most 1distance dominating set, such the vertex is called middle distance dominating vertex and we choose another vertex is adjacent to v such the vertices are sometimes called middle distance dominating path and also divider distance dominating path [10]. S has perfect distance dominating set [11].

Let M be a mesh topological graph has n vertices, if $v \in M$ then $\{v\}$ is at most 1 distance dominating set, such the vertex is called middle distance dominating vertex and we choose another vertex is adjacent to v such the vertices are sometimes called middle distance dominating path and also not divider distance dominating path [10]. M has perfect distance dominating set [11], since mesh topological graph is a complete graph.

The characterization on n vertices in M:

(i) $n-1$-regular graph
(ii) Cannot add an edge
(iii) Neither cut vertex nor cut vertex set
(iv) 1-distance domination
(v) The number of edge is defined by $\epsilon = \dfrac{n(n-1)}{2}$.

Theorem 3.1. *Let T be a tree except bus topological graph, if it has maximum degree m, and then there exist exactly m pendant vertices.*

Proof. Let T be a tree except bus topological graph, let m be a maximum degree of T, there are three possible cases arrives on degree of T. If $m = 1$, then k_2 is only possible and we just leave it because k_2 not consider as tree. If $m = 2$, then T has no branch, clearly there are two pendent vertices. If $m = 3$, then T has 3 pendent vertices if and only if T has only one vertex is degree three. Suppose that more than one vertex is degree three then T become a bus topological graph which is contradiction. □

Theorem 3.2. *Let T be a bus topological graph, if T has n vertices then T has $\frac{n}{2}$ block.*

Proof. Let T be a bus topological graph, let n be number of vertices in T, since the characterization of T that it has unique 2 pendent vertices plus m pendent vertices their adjacent vertices are degree exactly 3. We have

$$n = 2 + m \text{ pendent vertices their adjacent vertices are degree exactly } 3$$
$$= 2 + 2 \text{ times of } m \text{ pendent vertex} = 2 + 2m$$
$$\frac{n}{2} = (1 + m)$$

Since $K_2 = (1 + m)$ and K_2 is a block. Therefore T has $\frac{n}{2}$ block. □

Corollary 3.1. *Every block of bus topological graph is k_2.*

Theorem 3.3. *Let R be a ring topological graph, if R has n vertices then R has n block.*

Proof. Let R be a ring topological graph, let n be the number of vertices in R, it is clear that any path has two vertices in R then there are two length of its path, one of the length is 1 and another is $n - 1$. Since all vertices of degree are two. Therefore, The number of block $= 1 + n - 1 = n$. □

Corollary 3.2. *Let R be a ring topological graph, if R has n vertices then R has n block are K_2.*

Theorem 3.4. *Let S be a star topological graph, if S has n vertices then S has $n - 1$ block.*

Proof. Let S be a ring topological graph, let n be the number of vertices in S, there exist a maximum degree m to the vertex v and m vertices are adjacent to v. Remove v from S then getting $n - 1$ component. Since m pendent vertices are incident with v. Therefore, The number of block $= m = n - 1$. □

Corollary 3.3. *Let S be a star topological graph, if S has n vertices then S has $n - 1$ block are K_2.*

Theorem 3.5. *Let M be a mesh topological graph, if M has n vertices then M has $\frac{n(n-1)}{2}$ block.*

Proof. The proof immediately from its characterization on n vertices of M. The number of block = $\frac{n(n-1)}{2}$. □

Corollary 3.4. *Let M be a mesh topological graph, if M has n vertices then M has $\frac{n(n-1)}{2}$ block are K_2.*

CONCLUSION

In this paper, analysis the demerits of topological graph as bus topological graph, ring topological graph, star topological graph, mesh topological graph except hybrid topological graph. It will be very useful to connect a network and other network problem in the field of computer science and it can be extend to all network fields like social network, electrical network, ego network and others.

REFERENCES

[1] J.A. BONDY, U.S.R. MURTY: *Graph Theory with Applications*, North-Holland, New York, Amsterdam, Oxford, 1976.

[2] A. CAYLEY: *A theorem of trees*, Quart. j. Math., **23**, 376–78.

[3] P. ERDOS, T. GALLAI: *Graphs with prescribed degrees of vertices (Hungarian)*, Mat. Lapok, **11** (1960), 264–274.

[4] T.W. HAYNES, S.T. HEDETNIEMI, AND P.J. SLATER: Fundamentals of Domination in Graphs, Marcel Dekker, New York, NY, USA, 1998.

[5] A. MEIR, J.W. MOON: *Relation between packing and covering number of a tree*, Pacific J. Math. **61**(1) (1975), 225–233.

[6] MARILYNN LIVINGSTON, QUENTIN F. STOUT: *Perfect Dominating Sets*, In Congressus Numerantium **79** (1990), 187–203.

[7] NARSINGH DEO: *Graph Theory with Applications to Engineering and Computer Science*, Prentice-Hall, Englewood Cliffs, N.J., 1974.

[8] P.J. SLATER: *R-domination in graphs*, Journal of Association for Computer Machinery, **23**(3) (1976), 446–450.

[9] R. VEERASIVAJI, S. MEENASHI: *Distance Dominating on Tree*, Journal of Xidian University, **14**(11) (2020), 640–646.

[10] R. VEERASIVAJI, S. MEENASHI: *Middle Distance Dominating Set*, GORTERIA Journal, **34**(1) (2021), 209–218.

[11] R. VEERASIVAJI, S. MEENASHI: *Perfect Distance Dominating Set*, Advances in Mathematics: Scientific Journal **10**(3) (2021), 1153–1159.

DEPARTMENT OF MATHEMATICS, VISTAS, CHENNAI - 600 117, INDIA.
Email address: vracvag@gmail.com

DEPARTMENT OF MATHEMATICS, VISTAS, CHENNAI - 600 117, INDIA.
Email address: meenakshikarthikeyan@yahoo.co.in