Advances in Mathematics: Scientific Journal **10** (2021), no.5, 2361–2380
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.5.6

MODELLING OF OIL PRICE VOLATILITY USING ARIMA-GARCH MODELS

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ABSTRACT. In this paper, the behavior of the oil price series named OIL is examined. The non-stationarity on average and variance, with the non-normality of the OIL series distribution, indicate the volatility of the series. The study is based on a combination of the Box-Jenkins methodology with the GARCH processes (Engle and Bollerslev). The first part models the lnOIL series in which, by applying the first difference the series becomes DlnOIL. Then the Box-Jenkins methodology is applied. The choice of the model was made on basis of minimization of criterion -Akaike (AIC), Shwarz (SIC)- and maximization of log likelihood (LL). Of the four models identified, ARMA (3.1) is retained. According to the statistical indicators of the ARMA model (3,1), the nature of the residuals and other tests, it is shown that the series of squares of the residuals follows a conditionally heteroscedastic ARCH model. The second part is devoted to a symmetrical and asymmetrical GARCH modelling. The model used for predicting volatility is the EGARCH model (1,2). The data available relates to 3652 daily values of the change in OIL, from 01/01/2019 to 12/31/2019. The forecast is made for the first three months of 2020; the result concludes that the predicted values and the current values are very close, and that the model ARIMA (3,1,1) + EGARCH (1,2) is the best forecast model.

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2020 *Mathematics Subject Classification.* 62H11, 62P10.

Key words and phrases. ARIMA Model, Oil price volatility, Heteroscedasticity, Asymmetric GARCH Model, Forecasting.

Submitted: 08.04.2021; *Accepted:* 26.04.2021; *Published:* 04.05.2021.

1. Introduction

For more than a century, petroleum has been the most important source of energy on which all of western civilization has built its development. It is the most consumed source of energy in the world. That is why oil plays a huge role in the global economy. In fact, during the last twenty years, the world oil market has undergone major changes, moving from a simple physical oil trading market to a sophisticated market (financial market). Therefore, the international hydrocarbon market is considered to be one of the most important commodity markets globally.

Oil consumption is driven by the needs of emerging countries, in particular in the transport and petrochemicals sector. Petroleum reserves are mainly held by the OPEC countries, which are the world's largest producers. Despite the fact that it sometimes goes through long periods of stability, the hydrocarbons market remains volatile due to the violence of the shocks and counter shocks that pass through it. The peculiarity of the spectacular reversals in oil prices lies in the extent of the variations compared to the initial prices.

The price of crude oil fell from \$ 9 per barrel in December 1998 to \$ 145 in July 2008. It then fell to \$ 32 in December 2008, before rising again in 2009 and reaching the end of the year a level of 90 dollars. This conjunction of an upward trend and high volatility is likely to continue in the coming years. Of course, experience has shown how difficult it is to predict the evolution of the price of oil.

Hedging strategies are based on the correct estimation of price volatility. Therefore, it is necessary to adequately model the volatility of oil prices. Univariate GARCH-type processes allow an acceptable estimate of price volatility because they accurately describe the aggregation characteristics of volatility and asymmetry. Several models have been suggested for capturing special features of financial data, and most of these models have the property that the conditional variance depends on the past. The frequently applied models to estimate exchange rate volatility are the autoregressive conditional heteroscedastic (ARCH) improved by R.F. Engle [12]; and generalized autoregressive conditional heteroscedastic (GARCH), developed independently by Bollerslev [7], Taylor [30], Yoon and Lee [34], Hamadu and Adeleke [16]. Ng and

McAleer [26] used simple GARCH (1,1) and TARCH(1,1) models for testing estimation and forecasting the volatility of daily returns in S&P 500 Composite Index and the Nikkei 225 Index. They concluded that TARCH (1,1) was the best performing model with S&P 500 data, whereas the GARCH(1,1) model was better in some cases with Nikkei 225. Ramzan and al. [28] modelled exchange rate dynamics in Pakistan, using the GARCH family models, on the monthly data from July 1981 to May 2010. The study results showed that GARCH (1, 2) was better than EGARCH (1, 2) model. However, the GARCH(1, 2) model was used to remove the persistence in volatility while EGARCH (1, 2) successfully overcame the leverage effect in the exchange rate returns. Moreover they concluded that the GARCH family of models captures the volatility and leverage effect in the exchange rate returns, giving fairly good forecasting performance for the model. Abdalla, S.Z.S. [1] considered the (GARCH) approach in modelling exchange rate volatility, in a panel of nineteen of the Arab countries. Using daily observations over the period of 1st January 2000 to 19th November 2011, he applied both symmetric and asymmetric models that capture most common stylized facts about exchange rate returns; such as volatility clustering and leverage effect. Based on the GARCH(1,1) model, the results showed that for ten out of nineteen currencies, the sum of the estimated persistent coefficients exceed one, implying that volatility is an explosive process, in contrast, it was quite persistent for seven currencies; a result which is required to have a mean reverting variance process. Furthermore, the asymmetrical EGARCH (1,1) results provided evidence of leverage effect for majority of currencies, indicating that negative shocks imply a higher next period volatility than positive shocks. In [37] H. Zeghdoudi and al. studied ARCH models and their applications to the Value-At-Risk. They gave an extensive bibliographic overview of the developments of the (GARCH) models and its applications. They made an application relates to the exchange rate volatility of Algerian dinar against the EURO and the U.S. Dollar for the period from June 2009 at May 2011, where they compared the models resulting from various standard processes ARCH (ARCH, GARCH, IGARCH, EGARCH, TGARCH, and APARCH) and over various periods. M.A.Thorlie and al. [31] examined the accuracy and forecasting performance of volatility models for the Leones/USA dollars exchange rate return, including the (ARMA), (GARCH), and Asymmetric GARCH models with normal and non-normal ($student$'s t and $skewed$ Student t) distributions. In fitting these models to the monthly exchange rate returns data over the period January 2004 to December 2013, they found that, the Asymmetric (GARCH) and GARCH model better fits under the non-normal distribution rather than the normal distribution, and improve the overall estimation for measuring conditional variance. Efimova, and Serletis [10] investigated the empirical properties of oil, natural gas, and electricity price volatilities using a range of univariate and multivariate GARCH models.They get results confirmed by several studies. Zeghdoudi and Bouseba [35] added another profit of GARCH models including an application relates to exchange rate volatility of oil price.A. Guerouahand al. [15] examined the relationship between stock and oil markets. In addition, they evaluated the performance of each model with a range of diagnostic and forecast erformance tests using univariate GARCH(1,1) and bivariate BEKK GARCH(1,1), DCC GARCH(1,1) models. In [25] authors have built a model to forecast the exchange rate of Bangladesh. A study on Monthly average exchange rates of Bangladesh for the period from August, 2004 to April, 2019. They have selected ARIMA (1,1,1) as a mean model for this study. Then they tried to model the volatility of exchange rate using ARCH, GARCH, EGARCH, IGARCH and TARCH models. ARIMA (1,1,1)-GARCH (1,1) is selected as a best model compared to others since it has the lower values of RMSE, MAE, MAPE and TI than other models. However, to the authors´ best knowledge, very few publications can be found on volatility of the oil price by symmetric and assymetric models that capture most common stylized facts about oil price such as volatility clustering and leverage effect. The main objective is to study the caracteristics of volatility of the oil price on a sample of daily data of 3652 observations, over the period of 1^{st} January 2010 to 31 December. The remainder of the paper is organized as follows: Section 2 is devoted to a brief theoretical presentation of GARCH models and its extensions, while section 3 presents data and the empirical results. Section 4 concludes this paper.

2. METHODOLOGY

2.1. **ARIMA Modelling.**

2.1.1. $ARIMA$ *Models.*

Autoregressive model of order p, $AR(p)$.

The process $(X_t)_{t\in\mathbb{Z}}$ is an autoregressive model of order p if it satisfies the equation:

$$\Phi(B)X_t = \varepsilon_t, \tag{1}$$

where $\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$. The process $AR(p)$ is stationary if all the roots of $\Phi(B)$ are outside the unit circle. $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a centered white noise process with variance $\sigma^2 < \infty$. An AR process is invertible, that is to say it can always be written as an MA process.

Moving average model of order q $MA(q)$.

The $MA(q)$ process is a stationary process given by the following equation:

$$X_t = \Theta(B)\varepsilon_t, \tag{2}$$

where $\Theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$. An $MA(q)$ is always stationary, it is invertible when all the roots of $\Theta(B)$ are outside the unit circle.

Moving average autoregressive model $ARMA(p,q)$.

This process verifies the following equation:

$$\Phi(B)X_t = \Theta(B)\varepsilon_t \tag{3}$$

where $\begin{cases} \Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \\ \Theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q \end{cases}$

The conditions of stationarity of an $ARMA$ are determined by the roots of the polynomial associated to its component AR, while its invertibility depends on the roots of the polynomial associated to its component MA

Integrated moving average autoregressive model $ARIMA(p,d,q)$.

When the given series $(X_t)_{t\in\mathbb{Z}}$ is not stationary, it should be modeled using an $ARIMA(p,d,q)$ where d denotes the order of differentiation (or integration).

$(X_t)_{t\in\mathbb{Z}}$ is a stationary integrated mobile average autoregressive model noted $ARIMA(p,d,q)$ if it admits the following representation:

$$\Phi(B)(1-B)^d X_t = \Theta(B)\varepsilon_t \tag{4}$$

2.1.2. *The Box-Jenkins procedure.* The Box-Jenkins methodology allows to determine the appropriate $ARIMA(p, d, q)$ model for the modeling of a time series. This procedure suggests four steps:

1-Identification: based on functions of autocorrelation and partial autocorrelation study. This step is the most important. Its purpose is to find the p and q values of the ARMA processes. The rules which facilitate the search of them are: Deseasonalisation and stationary in terms of trend. If the study of simple correlograms and statistical tests, point to a series with a trend, it is appropriate to study the characteristics -according to the Dickey Fuller Augmented test (1979) - in order to detect the presence of a unit root. After stationary we can identify the values of the parameters p and q of the ARMA model.

2- Estimation of ARMA processes: When the p and q values of the ARMA process are identified, the next step is to estimate the coefficients associated with the autoregressive term and moving average. Methods of estimation are different depending on the type of process diagnosed. Generally, the maximum likelihood method, ordinary least squares method, Durbin Watson method and nonlinear least squares method are used.

3- Validation: By using, significance of the coefficients parameters tests (student test), the null hypothesis test of homoscedasticity (tests ARCH, white, Brensch-Pagan) and null hypothesis of autocorrelation, for the residuals (Box-Pierre tests, Ljunge-Box). This step is about testing whether, the residuals are white noise or not. If the residuals are white noise, it will be necessary that the residuals series be stationary (fluctuating around a constant zero mean). Moreover, it is after application of the Box-Pierce and ARCH tests that the alternative hypotheses are rejected.This step must be followed by a comparison of the models qualities, which are validated. The criteria for choosing the model to be used can be standard or information. The most used criteria are: the Akaike Information Criterion (AIC), and the Schwarz Criterion (SC), mean absolute error (MAE), root-mean-square error (RMSE), mean absolute percentage error (MAPE) and the F test, where, T is the number of observations of the series X_t studied and e_t are the estimated residuals.

Forecasting is the last step in the Box and Jenkins methodology. Knowing the forecast horizon (h), the forecast made in T for the date $T+h$ is given by $\hat{X}_{T+h}$:

$$\hat{X}_{T+h} = IE[X_{T+1}/I_t] = \hat{X}_{T+h/t-1}$$

This expression represents the best forecast of the series X_t conditionally to the set of information available at date t.The term $\hat{X}_{T+h/t-1}$ means that the value of X_t is predicted on the basis of past observations $X_t, X_t, \ldots$ using the estimated value of the coefficients, in the same way of [24]). This paper used the SARIMA models with methdology of the Box-Jenkins for modeling and forecast Number of Injured in Road Accidents in Northeast Algeria.

2.2. **Heteroscedastic modeling:** $ARCH - GARCH$ **models.** Despite the advantages of $ARMA(p,q)$ models, they suffer from the failure to take into account certain structural constraints linked to the phenomenon being modeled. These constraints can reflect the volatility of a certain variable, and sometimes involve the use of nonlinear models, likely to make the $ARMA$ specification inadequate.

Robert F. Engle [12] introduced the ARCH model class in 1982, then Tim p. Bollerslev [7]generalized it in 1986.

2.2.1. *Linear (symmetric)* $ARCH$ *processes.* The fendamental assumption underlying linear $ARCH$ is the symmetry of the quadratic specifications of the conditional variance of the error.

$ARCH(q)$ process.

Is written in the following form:

$$Var(\varepsilon_t/I_{t-1}) = h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \tag{5}$$

where $IE(\varepsilon_t/I_{t-1}) = 0$, $\alpha_0 > 0$ and $\alpha_i \geq 0$. $I_{t-1} = \{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, t-q\}$

The condition of stationarity is: $\sum_{i=1}^{q} \alpha_i < 1$. $I_{t-1} = \{\varepsilon_{t-1},\ \varepsilon_{t-2}, \ldots, \varepsilon_{t-i}, \ldots\}$.

Generalized $ARCH$ models.

Noting $GARCH(p,q)$ is given in the following form

$$h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}^2, \tag{6}$$

$\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ et $\sum \alpha_i + \sum \beta_j < 1$ is the condition of stationarity.

In the case where $\sum \alpha_i + \sum \beta_j = 1,$ the équation (6) becomes

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^{\max(p,q)=r} (\alpha_i + \beta_i)\, \varepsilon_{t-i}^2 + \mu_t - \sum_{j=1}^{p} \beta_j \mu_{t-j}, \tag{7}$$

where $\mu_t = \varepsilon_t^2 - h_t^2.$

Then, a $GARCH(p,q)$ process becomes an $ARMA(r,p)$ process.

$ARCH$ models on average $(ARCH - M)$.

This model takes in cosideration the existing relation between the mean and the variance of the analyzed variable. A variation of the conditional variance will be accompanied by a conditional variation of the mean.

This model is written in the following form:

$$\left\{ \begin{array}{c} \Phi(B)X_t = \Theta(B)\varepsilon_t + \delta h_t^2 \\ h_t^2 = \alpha_0 + \sum\limits_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum\limits_{j=1}^{p} \beta_j h_{t-j}^2 \end{array} \right. , \tag{8}$$

where X_t is a stationary $\Phi(B)$ and $\Theta(B)$ are, respectively, the autoregressive lagging and moving average polynomials.

2.2.2. *Nonlinear $ARCH$ Processes (Asymmetric)*. Another empirical characteristic of series of returns is the asymmetric behavior, the "leverage effect" which designates a greater volatility following a negative shock (Bad Noise), than following a positive shock (Good Noise). In [13] Manamba Epaphra, reveals that exchange rate series exhibts the empirical regularities such as clustering volatility, nonstationarity, non-normality and serial eccrelation that justify the application of the ARCH methodology.

$EGARCH$ Models.

The exponential $EGARCH$ model -difficult to interpret- is a specification adapted to the $GARCH$ model where " α_i and β_j" are negative, thus removing the constraints of non-negativity imposed on the parameters. This type of model is expressed as follows:

$$\log(h_t^2) = \alpha_0 + \sum_{i=1}^{q} \left[\alpha_i \left| \frac{\varepsilon_{t-i}}{h_{t-i}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{h_{t-i}} \right] + \sum_{j=1}^{p} \beta_j \log(h_{t-j}^2). \tag{9}$$

Special case of an $EGARCH(1,1)$

$$\log(h_t^2) = \alpha_0 + \left[\alpha_1 \left|\frac{\varepsilon_{t-1}}{h_{t-1}}\right| + \gamma_1 \frac{\varepsilon_{t-1}}{h_{t-1}}\right] + \beta_1 \log(h_{t-1}^2). \tag{10}$$

$TGARCH$ Model.

Threshold $ARCH$ or $GARCH$ modeling consists in integrating the effect of asymmetry in the quadratic specifications of the conditional variance of the errors. In a threshold model, h_t^2 is a function defined in pieses which allows to obtain different functions of volatility according to the sign and the values of the shocks.

$GARCH$ threshold models $(TGARCH)$ are written:

$$h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i^- \varepsilon_{t-i}^2 I_{\varepsilon_{t-i<0}} + \sum_{i=1}^{q} \alpha_i^+ \varepsilon_{t-i}^2 I_{\varepsilon_{t-i\geq 0}} + \sum_{j=1}^{p} \beta_j h_{t-j}^2, \tag{11}$$

where $I_{\varepsilon_{t-i}}$ indicates the indicator function such as:

$$\begin{cases} I_{\varepsilon_{t-i<0}} = 1, & \text{si } \varepsilon_{t-i} < 0 \\ I_{\varepsilon_{t-i\geq 0}} = 0, & \text{si} non. \end{cases}$$

The $TGARCH(1,1)$ process is given in the following form:

$$h_t^2 = \alpha_0 + \alpha_1^- \varepsilon_{t-1}^2 I_{\varepsilon_{t-1<0}} + \alpha_1^+ \varepsilon_{t-1}^2 I_{\varepsilon_{t-1\geq 0}} + \beta_1 h_{t-1}^2 \tag{12}$$

3. Data, Results and Discussion

Our objective is to forecast the price of oil using a univariate forecasting model, on a daily data sample of 3652 observations covering the period from January 1, 2010 to December 31, 2019.

3.1. **Statistical analysis of the oil price series.** Figure 1 reveals the existence of a growing trend for the "OIL" series, so this graph indicates non-stationarity in mean and in variance (especially in variance). There are also volatility groupings, which means this series is volatile. This volatility changes over time. This allows to say that a $GARCH$ type process could be adapted to the modelling of the oil price series. In order to reduce the variability of the oil price series "OIL", we need to transform the series "OIL" into a logarithm series "lnOIL"

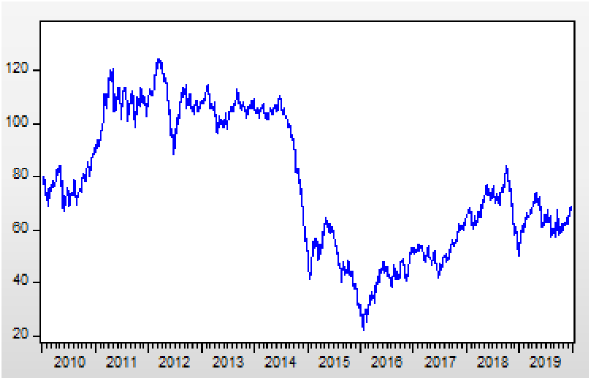


Figure 1 : OIL

According to the statistical indicators of the petroleum series, we notice that the Kurtosis coefficient (2.53) is lower than the value of the Kurtosis of the normal law which is equal to 3 (*i.e.* the distribution is platykurtic). The Skewness coefficient (0.92) is different from zero (ie the distribution is spread to the right). This asymmetry can be an indicator of non-linearity.

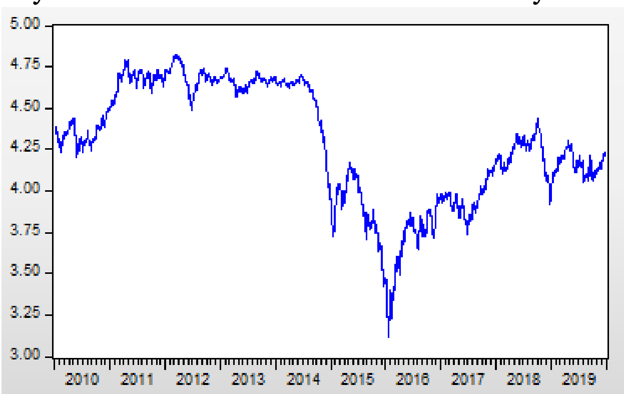


Figure 2 : lnoil

3.2. **Study of stationarity.** The graph of the "lnOIL" series of petroleum shows that there is a unit root which is confirmed by the Dikey-Fuller Increase test (DFA) at the 5% level.

The first difference is applied to station the "OIL" and "lnOIL" series. The following table summarizes the results of the ADF test.

3.3. $ARMA$ **modelling using Box Jenkins methodology.**

TABLE 1. ADFtest

Series	Calculated value	Critical value	P-Value
OIL	$-1,174$	$-2,862$	$0,687$
DOIL	$-49,840$	$-2,862$	$0,0001$
lnOIL	$-1,298$	$-2,862$	$0,632$
DlnOIL	$-31,848$	$-2,862$	$0,0001$

3.3.1. *Identification of the $ARMA(p,q)$ model.* The DlnOIL series is stationary, we will look for an $ARMA$ model (p,q). According to the correlogram of the DlnOIL series, there are four models to remember: $AR(1), MA(1), ARMA(1,1)$ and $ARMA(3,1)$. To conclude on the quality of estimation of the four models, we will use the criteria AIC and SC (the smallest).

3.3.2. *Estimation of the selected models.* In all four models, the calculated student statistic of the coefficients is greater in absolute value than 1.96. It is then observed that the coefficients are significantly different from zero. Which shows the following table:

TABLE 2. Model estimation

Model	Coefficients	S.E	T.S	P.V
$AR(1)$	$0,176$	$0,010$	$16,377$	$0,000$
$MA(1)$	$0,196$	$0,010$	$18,825$	$0,000$
$ARMA(1,1)$	$\phi_1 = -0,236$	$0,052$	$-4,544$	$0,000$
	$\theta_1 = 0,427$	$0,048$	$8,793$	$0,000$
$ARMA(3,1)$	$\phi_3 = 0,051$	$0,011$	$4,303$	$0,000$
	$\theta_1 = 0,195$	$0,010$	$18,915$	$0,000$

The model estimation step resulted in the conservation of the two models for review of the residue tests.

3.3.3. *Validation of the ARMA model.* The validation of the model involves the verification of tests for the absence of autocorrelation of the residuals. To do that, we must examine the correlograms of the residuals of each of the four models. For this we apply the $Lejung - Box(LB)$ test, for a maximum delay number 36 for the four models. The test results for each model are given in the table below:

TABLE 3. LB test

Model	$AR(1)$	$MA(1)$	$ARMA(1,1)$	$ARMA(3,1)$
$Q(36)$	$90,519$	$77,666$	$66,475$	$63,371$
$P-$Value	$0,0000$	$0,0000$	$0,0010$	$0,0020$
$\boxed{Q^2(36)}$	$1692,3$	$1700,9$	$1676,1$	$1680,6$
$P.Value$	$0,0000$	$0,0000$	$0,0000$	$0,0000$

The analysis of the residues resulting from the modelization ARMA, shows the existence of nonlinear relation. Indeed, reading Table 3 shows the existence of autocorrelation as indicated by the Ljungue-Box statistic of residuals and square residuals. then the null hypothesis of autocorrelation abscence is rejected.

The characteristics of the shape of the residuals of the four models are presented in the table below:

TABLE 4. Statistical indicators of the models

	$AR(1)$	$MA(1)$	$ARMA(1,1)$	$ARMA(3,1)$
Kurtosis	$10,519$	$10,388$	$10,268$	$10,332$
Skewness	$0,388$	$0,373$	$0,351$	$0,379$
Jarque.B	$8963,163$	$8382,721$	$8111,332$	$8265,802$

Table 4 also confirms the leptokurtic and asymmetric nature of the residues ($kurtosis > 3$ and $Skewness \neq 0$).

The form indicators and the analysis of the residuals indicate the presence of volatility (the presence of a non-stationary series in variance).

The choice of the most appropriate model among the estimated models is made on the basis of minimization of the criterion $Akaike$ (AIC) and $Schwarz$ (SIC) and maximization of log likelihood LL.

TABLE 5. Criterion for choosing the model

	$AR(1)$	$MA(1)$	$ARMA(1,1)$	$ARMA(3,1)$
AIC	$-5,882$	$-5,889$	$-5,891$	$-5,891$
SIC	$-5,882$	$-5,886$	$-5,885$	$-5,886$
LL	10747	10753	10757	10758

According to table 5, the model to be retained among the four validated models is the model $ARMA\,(3,1)$.

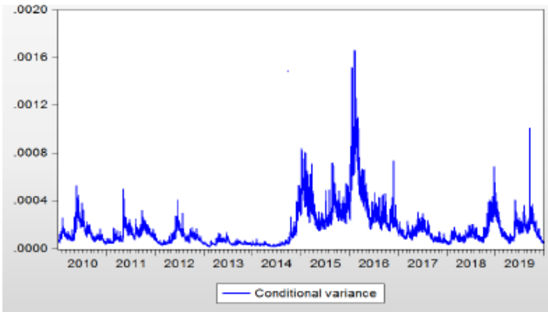


Figure 4: Test of Heteroscedasticity

We reject the assumption of normality of the residuals. This result is confirmed by the Jarque-Bera statistic $(8265,82 > \chi^2_{0,05}(2) = 5,99)$ and the associated probability is $0,00 < 5\%$.

3.3.4. *Heteroscedasticity test.* The number of delay "q" to be retained must first be determined. With regard to the correlogram of squared residuals of the $ARMA(3,1)$ model .

From this we therefore reject the null hypothesis of homoscedasticity in favor of the conditional heteroscedasticity, alternative for the model $ARMA(3,1)$.

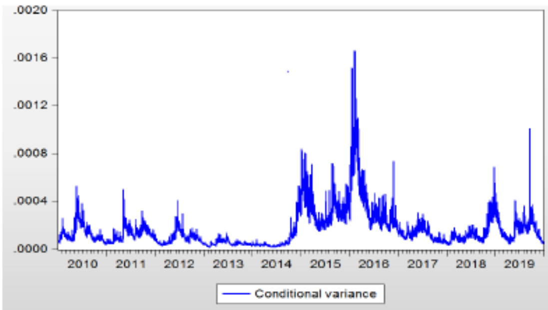


Figure 4: Test of Heteroscedasticity

Note that the above graph shows that the series of squares can follow a conditionally heteroscedastic $ARCH$ model, where there is a period of high volatility. We can now proceed to the LM test analogously to (2014) test [23].

TABLE 6. Choice of model

	$ARCH(1)$	$ARCH(2)$	$GARCH(3,1)$	$EGARCH(1,2)$	$TARCH(3)$
AIC	-5,9742	-5,9858	-6,2042	-6,2155	-6,0131
SIC	-5,9674	-5,9773	-6,1923	-6,2036	-6,0012
LL	10901,07	10923,14	11323,51	11344,10	10974,92
R^2	0,0360	0,0357	0,0366	0,0365	0,0357

F-statistic	53.38904	Prob. F(3,3644)	0.0000
Obs*R-squared	153.5920	Prob. Chi-Square(3)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 11/03/20 Time: 21:02
Sample (adjusted): 1/05/2010 12/31/2019
Included observations: 3648 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000113	8.93E-06	12.61393	0.0000
RESID^2(-1)	0.132008	0.016535	7.983454	0.0000
RESID^2(-2)	0.109665	0.016580	6.614425	0.0000
RESID^2(-3)	0.060955	0.016535	3.686437	0.0002

R-squared	0.042103	Mean dependent var	0.000162
Adjusted R-squared	0.041314	S.D. dependent var	0.000493
S.E. of regression	0.000483	Akaike info criterion	-12.43137
Sum squared resid	0.000851	Schwarz criterion	-12.42457
Log likelihood	22678.81	Hannan-Quinn criter.	-12.42894
F-statistic	53.38904	Durbin-Watson stat	2.007665
Prob(F-statistic)	0.000000		

3.4. **GARCH modelling.** The squared residuals correlogram of the $ARIMA$ model $(3,1,1)$ shows at least three autocorrelations (simple and partial) are significantly different from zero. Thus we estimate the following models: $ARCH(1)$, $ARCH(2)$, $GARCH(3,1)$, $EGARCH(1,2)$ and $TARCH(3)$.

The models: $ARCH(1)$, $ARCH(2)$ and $GARCH(3,1)$ vanish the positivity condition of coefficients of variance and stationarity equation. The two processes $EGARCH$ $(1,2)$ and $TARCH(3)$ satisfy the condition of asymmetry.

The choice of the most suitable model is based on the selection criteria according to Table 7:

It therefore indicates that the only model retained for predicting volatility is the $EGARCH$ model $(1,2)$.

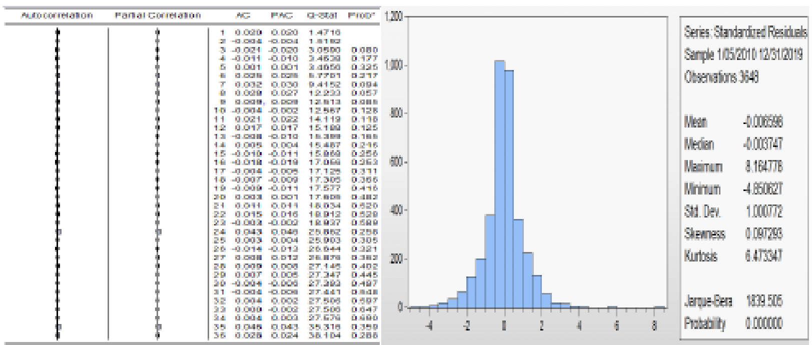
TABLE 7. Calculation of the standard criteria RMSE, MAE, MAPE and THEIL

	$ARCH(1)$	$ARCH(2)$	$GARCH(3,1)$	$EGARCH(1,2)$	$TARCH(3)$
$RMSE$	5,38E-05	8,42E-05	1,99E-05	1,51E-05	8,52E-05
MAE	1,50E-05	2,77E-05	5,49E-06	4,60E-06	2,76E-05
$MAPE$	726,9949	1033,348	50,42203	74,24842	870,1202
$THEIL$	0,057871	0,0852	0,022012	0,016558	0,085983

A very important analysis, remains to be undertaken, is a question of comparing the characteristics of predictive power of the various models. Hence, we present the calculation of the $RMSE$ (Root Mean Square Error), MAE (Mean Absolut Error), $MAPE$ (Mean Absolut Percentage Error) and $THEIL$ inequality coefficient (close to zero) for a forecast time ranging from 1 to 31 days (month of January 2020). We have noticed that the $EGARCH(1,2)$ model is the best model for the forecasts of the "OIL" series, Mohamed E. M. Abdelhafez (2018).

Table 9: Estimation of the EGARCH model(1,2)

Mean Equation			
	Coefficient	Z-Stastique	P-Value
AR(3)	0,061891	4,197647	0,0000
MA(1)	0,183594	10,52729	0,0000
Variance Equation			
C(3)	-0,086383	-10,54068	0,0000
C(4)	0,285618	14,24561	0,0000
C(5)	-0,208511	-10,32279	0,0000
C(6)	-0,046891	-13,84915	0,0000
C(7)	0,996243	1380,633	0,0000



Correlogram of residuals and statistic of the EGARCH model (1,2)

After examining the correlation of the residuals, we notice that the peaks are within the confidence interval $(P.Value > 0.05)$. Consequently, the null hypothesis of absence of autocorrelation of the residuals is accepted.

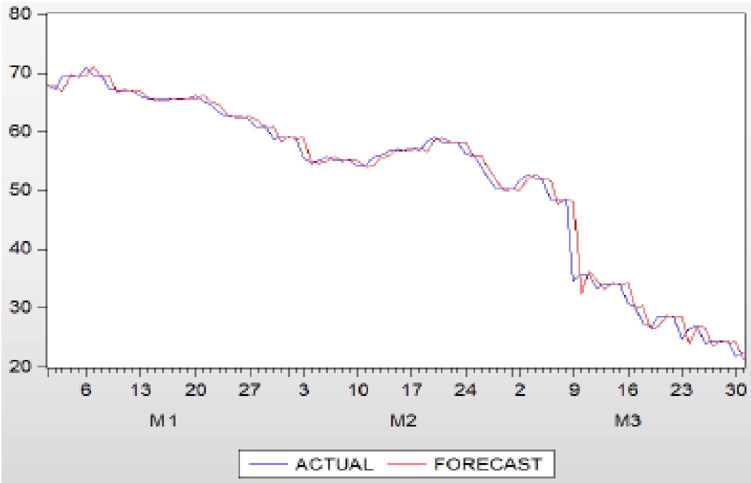
The conditional mean with conditional variance equation is given by

$$\begin{aligned} D\ln OIL_t &= 0,061891 D\ln OIL_{t-3} - 0,183594\varepsilon_{t-1} + \varepsilon_t - 0,086383 \\ &+0,285619\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - 0,208511\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}}\right| - 0,046891\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \\ &+0,996243\log\left(\sigma_{t-1}^2\right) \end{aligned}$$

For the forecast of the first three months of the year 2020, we use the ARIMA (3,1,1) + EGARCH (1,2) model. The forecast values of the oil price are given in the table above.

Table 10: Forecast and Actual OIL price for the first three month of 2020

Day	January Forecast	January Actual	February Forecast	February Actual	March Forecast	March Actual
1	66,77	67.96	59,05	58.94	50,13	50.19
2	67,94	67.12	58,74	58.94	50,07	51.68
3	66,88	69.38	58,95	55.53	51,91	52.66
4	69,77	69.38	54,88	54.68	52,71	52.01
5	69,19	69.38	54,63	55.01	51,91	51.75
6	69,49	70.87	54,85	55.74	51,71	48.35
7	71,03	69.60	55,82	55.13	47,69	48.35
8	69,30	69.58	54,96	55.13	48,44	48.35
9	69,66	67.24	55,15	55.13	48,10	34.72
10	66,69	67.02	55,03	54.21	32,42	35.73
11	67,06	67.02	54,02	54.20	36,36	35.56
12	66,83	67.02	54,18	55.59	34,65	33.27
13	67,01	66.07	55,73	55.90	33,17	34.14
14	65,85	65.63	55,87	56.77	34,21	34.14
15	65,55	65.32	56,94	56.77	33,97	34.14
16	65,18	65.62	56,68	56.77	34,18	30.63
17	65,63	65.58	56,78	57.28	29,96	30.36
18	65,50	65.58	57,30	56.72	30,41	27.31
19	65,56	65.58	56,56	58.39	26,54	26.73
20	65,52	66.11	58,68	58.98	26,76	28.57
21	66,16	65.26	58,93	58.21	28,66	28.57
22	65,05	64.66	58,12	58.21	28,48	28.57
23	64,58	63.26	58,19	58.21	28,62	24.72
24	62,94	62.52	58,12	56.14	23,98	26.53
25	62,39	62.52	55,76	55.91	26,95	26.94
26	62,42	62.52	55,90	54.04	26,66	24.06
27	62,46	61.98	53,56	51.92	23,69	24.26
28	61,85	60.67	51,61	50.19	24,32	24.26
29	60,42	61.04	49,81	50.19	24,05	24.26
30	61,08	58.80			24,28	21.66
31	58,29	58.94			21,15	22.61



4. Conclusion

This paper has investigated daily oil price volatility. We have employed two univariate specifications of the generalized autoregressive conditional heteroscedastic (GARCH) model, including both symmetric and asymmetric models that capture most common stylized facts about oil price such as volatility clustering and leverage effect. We performed an ARCH test which rejected the null hypothesis of homoscedasticity. From this, we have deduced that a nonlinear $ARMA$ model of type $ARCH$ is adequate. Then we estimated five $ARMA$ models of the $ARCH$ type: $ARCH(1)$, $ARCH(2), GARCH(3,1)$, $EGARCH(1,2)$ and $TARCH(3)$. The criteria AIC, R^2 and LL lead us to choose the model $ARIMA(3,1,1) + EGARCH(1,2)$, as being the most adequate model for the forecast. From the outcome of our investigation it is possible to conclude that the $RMSE$, $MAPE$ and $U\ THEIL$ forecasting quality come up with a good forecasting model.

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