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DECOMPOSITION OF GENERALIZED FAN GRAPHS

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ABSTRACT. Let $G = (V, E)$ be a finite graph. The Generalized Fan Graph $F_{m,n}$ is defined as the graph join $\overline{K_m} + P_n$, where $\overline{K_m}$ is the empty graph on m vertices and P_n is the path graph on n vertices. Decomposition of Generalized Fan Graph denoted by $D(F_{m,n})$. A star with 3 edges is called a claw S_3. In this paper, we discuss the decomposition of Generalized Fan Graph into claws, cycles and paths.

1. INTRODUCTION

Graph theory is proved to be tremendously useful in modeling the essential features of system with finite components. Graphical models are used to represent telephone network, railway network, communication problems, traffic network etc. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and the relations by edges [5].

The Generalized Fan Graph $F_{m,n}$ is defined as the graph join $\overline{K_m} + P_n$, where $\overline{K_m}$ is the empty graph on m vertices and P_n is the path graph on n vertices [6]. The case $m = 1$ corresponds to the usual Fan Graphs, while $m = 2$ corresponds to the Double Fan Graphs, etc. Number of edges of the Generalized Fan Graph

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$F_{m,n}$ with $(m+n)$ vertices is $mn+(n-1)$. A star S_n is the complete bipartite graph $K_{1,n}$; A tree with one internal vertex and n edges. A vertex $v \in V(G)$ is said to be *complete vertex* if $deg(v) = n-1$.

All graphs considered here are finite and undirected without loops, unless otherwise noted. For the standard graph-theoretic terminology the reader is referred to [7] and to study about the decomposition of graphs into paths, stars and cycles is referred to [1–4].

As usual C_n denotes the cycle of length n, P_{n+1} denotes the path of length n, S_3 denotes the claw and $D(F_{m,n})$ denotes decomposition of Generalized Fan Graph.

2. Basic Definitions

In this section, we see some basic definitions of graph decomposition and Generalized Fan Graph.

Let $L = \{H_1, H_2, \ldots, H_r\}$ be a family of subgraphs of G. An $L-decomposition$ of G is an edge- disjoint decomposition of G into positive integer α_i copies of H_i, where $i \in \{1,2,\ldots,r\}$. Furthermore, if each H_i $(i \in \{1,2,\ldots,r\})$ is isomorphic to a graph H, we say that G has an $H - decomposition$. It is easily seen that $\sum_{i=1}^{r} \alpha_i \; e(H_i) = e(G)$ is one of the obvious necessary conditions for the existence of a $\{H_1, H_2, \ldots, H_r\} - decomposition$ of G. For convenience, we call the equation, $\sum_{i=1}^{r} \alpha_i \; e(H_i) = e(G)$, a *necessary sum condition* [10].

The Generalized Fan Graph $F_{m,n}$ is a graph with vertex set $V(F_{m,n}) = V(\overline{K_m}) \bigcup V(P_n)$, where $V(\overline{K_m}) = \{v_1, v_2, \ldots, v_m\}$ and $V(P_n) = \{u_1, u_2, \ldots, u_n\}$ and edge-set consisting of all edges of the form $e_{ij} = v_i u_j$ and $e_k = u_k u_{k+1}$ where $1 \le i \le m$, $1 \le j \le n$ and $1 \le k \le n-1$ [6].

Obviously Fan Graph $F_{1,n}$ has every vertex of degree 3 except the complete vertex v_1 and the vertices u_1 and u_n are of degree 2. Complete vertex has degree n.

3. Decomposition of Fan Graphs $F_{1,n}$

In this section, we characterize the theorem of decomposition of Fan Graph $F_{1,n}$ into claws, cycles and paths.

Theorem 3.1. *Any Fan Graph $F_{1,n}, n \ge 2$ can be decomposed into P_n and S_n.*

Proof. Proof is immediate from the definition of the Fan Graph $F_{1,n}$. □

Theorem 3.2. *Any Fan Graph $F_{1,n}$ can be decomposed into following ways.*

$$D(F_{1,n}) = \begin{cases} (4d-1)S_3 \text{ and } P_3,\ d=1,2,3,\dots & \text{if } n=6d,\ d=1,2,3,\dots \\ (4d)S_3 \text{ and } P_2,\ d=1,2,3\dots & \text{if } n=6d+1,\ d=1,2,3,\dots \\ 4(d-1)S_3 \text{ and } C_3,\ d=1,2,3,\dots & \text{if } n=6d-4,\ d=1,2,3,\dots \\ (4d-3)S_3 \text{ and } P_3,\ d=1,2,3,\dots & \text{if } n=6d-3,\ d=1,2,3,\dots \\ (4d-2)S_3 \text{ and } P_2,\ d=1,2,3\dots & \text{if } n=6d-2,\ d=1,2,3,\dots \\ (4d-1)S_3,\ d=1,2,3,\dots & \text{if } n=6d-1,\ d=1,2,3,\dots \end{cases}.$$

Proof. Let $V(F_{1,n}) = \{v_1, u_1, u_2, \dots, u_n\}$ and

$$E(F_{1,n}) = \{e_{ij}/i=1,\ 1 \le j \le n\} \bigcup \{e_k/1 \le k \le n-1\}.$$

Case 1. $n = 6d,\ d = 1,2,3,\dots$.

To Prove: $F_{1,n}$ decomposed into $(4d-1)S_3$ and $P_3,\ d=1,2,3,\dots$.

Let $E_i = \{e_j, e_{1(j+1)}, e_{j+1}\}$ where $j = 1,3,5,\dots,n-3$, $E_j = \{e_{1j}, e_{1(j+2)}, e_{1(j+4)}\}$ where $j = 1,7,13,\dots,n-5$ and $E_k = \{e_{n-1}, e_{1n}\}$. The edge induced subgraph $< E_i >$ forms $(3d-1)$ copies of S_3, the edge induced subgraph $< E_j >$ forms d copies of S_3 and the edge induced subgraph $< E_k >$ forms a path P_3 of length 2. Hence $F_{1,n}$ decomposed into $3d-1+d = 4d-1$ copies of S_3 and a path P_3.

Case 2. $n = 6d+1,\ d = 1,2,3,\dots$.

To Prove: $F_{1,n}$ decomposed into $(4d)S_3$ and $P_2,\ d = 1,2,3\dots$.

Let $E_i = \{e_j, e_{1(j+1)}, e_{j+1}\}$ where $j = 1,3,5,\dots,n-2$, $E_j = \{e_{1j}, e_{1(j+2)}, e_{1(j+4)}\}$ where $j = 1,7,13,\dots,n-6$ and $E_k = \{e_{1n}\}$. The edge induced subgraph $< E_i >$ forms $3d$ copies of S_3, the edge induced subgraph $< E_j >$ forms d copies of S_3 and the edge induced subgraph $< E_k >$ forms a path P_2 of length 1. Hence $F_{1,n}$ decomposed into $3d+d = 4d$ copies of S_3 and a path P_2.

Case 3. $n = 6d-4,\ d = 1,2,3,\dots$.

To Prove: $F_{1,n}$ decomposed into $4(d-1)S_3$ and $C_3,\ d = 1,2,3,\dots$.

Let $E_i = \{e_j, e_{1(j+1)}, e_{j+1}\}$ where $j = 1,3,5,\dots,n-3$, $E_j = \{e_{1j}, e_{1(j+2)}, e_{1(j+4)}\}$ where $j = 1,7,13,\dots,n-7$ and $E_k = \{e_{1(n-1)}, e_{n-1}, e_{1n}\}$. The edge induced subgraph $< E_i >$ forms $(3d-3)$ copies of S_3, the edge induced subgraph $< E_j >$ forms $(d-1)$ copies of S_3 and the edge induced subgraph $< E_k >$ forms a cycle

C_3 of length 3. Hence $F_{1,n}$ decomposed into $3d-3+d-1=4d-4=4(d-1)$ copies of S_3 and a cycle C_3.

Case 4. $n=6d-3,\ d=1,2,3,\ldots.$

To Prove: $F_{1,n}$ decomposed into $(4d-3)S_3$ and $P_3,\ d=1,2,3,\ldots.$

Let $E_i=\{e_j,e_{1(j+1)},e_{j+1}\}$ where $j=1,3,5,\ldots,n-2$, $E_j=\{e_{1j},e_{1(j+2)},e_{1(j+4)}\}$ where $j=1,7,13,\ldots,n-8$ and $E_k=\{e_{1(n-2)},e_{1n}\}$. The edge induced subgraph $<E_i>$ forms $(3d-2)$ copies of S_3, the edge induced subgraph $<E_j>$ forms $(d-1)$ copies of S_3 and the edge induced subgraph $<E_k>$ forms a path P_3 of length 2. Hence $F_{1,n}$ decomposed into $3d-2+d-1=4d-3$ copies of S_3 and a path P_3.

Case 5. $n=6d-2,\ d=1,2,3,\ldots.$

To Prove: $F_{1,n}$ decomposed into $(4d-2)S_3$ and $P_2,\ d=1,2,3,\ldots.$

Let $E_i=\{e_j,e_{1(j+1)},e_{j+1}\}$ where $j=1,3,5,\ldots,n-3$, $E_j=\{e_{1j},e_{1(j+2)},e_{1(j+4)}\}$ where $j=1,7,13,\ldots,n-9$, $E_k=\{e_{1(n-3)},e_{1(n-1)},e_{1n}\}$ and $E_l=\{e_{n-1}\}$. The edge induced subgraph $<E_i>$ forms $(3d-2)$ copies of S_3, the edge induced subgraph $<E_j>$ forms $(d-1)$ copies of S_3, the edge induced subgraph $<E_k>$ forms 1 copy of S_3 and the edge induced subgraph $<E_l>$ forms a path P_2 of length 1. Hence $F_{1,n}$ decomposed into $3d-2+d-1+1=4d-2$ copies of S_3 and a path P_2.

Case 6. $n=6d-1,\ d=1,2,3,\ldots.$

To Prove: W_n decomposed into $(4d-1)S_3,\ d=1,2,3,\ldots.$

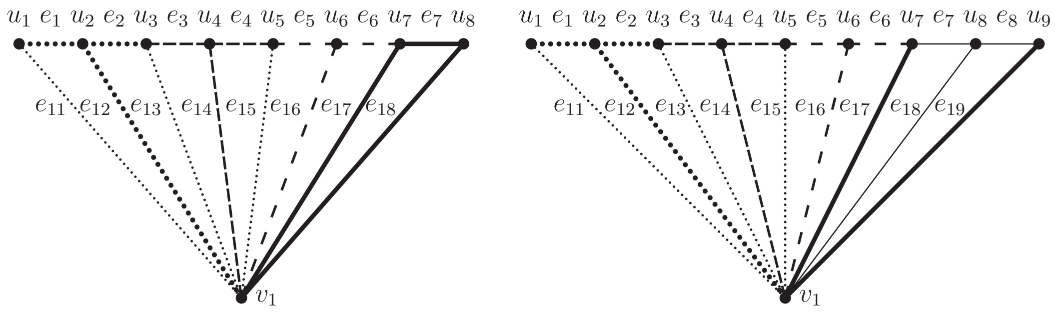
Let $E_i=\{e_j,e_{1(j+1)},e_{j+1}\}$ where $j=1,3,5,\ldots,n-2$ and $E_j=\{e_{1j},e_{1(j+2)},e_{1(j+4)}\}$ where $j=1,7,13,\ldots,n-4$. The edge induced subgraph $<E_i>$ forms $(3d-1)$ copies of S_3 and the edge induced subgraph $<E_j>$ forms d copies of S_3. Hence $F_{1,n}$ decomposed into $3d-1+d=4d-1$ copies of S_3. □

Illustration: Decomposition of Fan Graph $F_{1,n}$ on case 3 and case 4 explained through the following Figure 1.

The two figures represents decomposition of $F_{1,8}$ into 4 copies of S_3 and a cycle C_3 and decomposition of $F_{1,9}$ into 5 copies of S_3 and a path P_3 respectively.

All edges of the claws, cycle and path differentiated in the Figure 1.

Note 3.1. *In the above theorem Case 6 guarantees that there is a claw decomposition for Fan Graph $F_{1,n}$.*



$(n = 6d - 4,\ d = 2)\ F_{1,8}$ $(n = 6d - 3,\ d = 2)\ F_{1,9}$

FIGURE 1.

4. DECOMPOSITION OF DOUBLE FAN GRAPH $F_{2,n}$

In this section, we characterize the theorem of decomposition of Double Fan Graph $F_{2,n}$ into claws and paths.

Obviously Double Fan Graph $F_{2,n}$ has every vertex of degree 4 except the vertices v_1 and v_2 are of degree n and the vertices u_1 and u_n are of degree 3 respectively.

Theorem 4.1. *The Double Fan Graph $F_{2,n}$ can be decomposed into $(n-1)$ copies of S_3 and P_3.*

Proof. Let $V(F_{2,n}) = \{v_1, v_2, u_1, u_2, \ldots, u_n\}$ and

$$E(F_{2,n}) = \{e_{ij}/i = 1, 2,\ 1 \leq j \leq n\} \bigcup \{e_k/1 \leq k \leq n-1\}.$$

To Prove: $F_{2,n}$ decomposed into $(n-1)$ copies of S_3 and P_3.

Let $E_i = \{e_{1j}, e_j, e_{2j}\}$ where $j = 1, 2, 3, \ldots, n-1$ and $E_j = \{e_{1n}, e_{2n}\}$. The edge induced subgraph $< E_i >$ forms $(n-1)$ copies of S_3 and the edge induced subgraph $< E_j >$ forms a path P_3 of length 2. Hence $F_{2,n}$ can be decomposed into $(n-1)$ copies of S_3 and P_3. □

Illustration: Decomposition of Double Fan Graph $F_{2,n}$ explained through the following Figure 2.

The two figures represents decomposition of $F_{2,6}$ into 5 copies of S_3 and a path P_3 and decomposition of $F_{2,7}$ into 6 copies of S_3 and a path P_3 respectively.

All edges of the claws and path differentiated in the Figure 2.

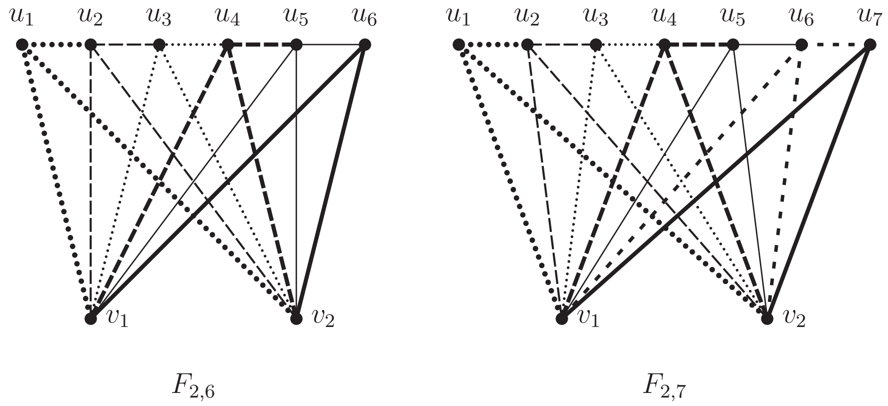


FIGURE 2.

Note 4.1. *In the above theorem guarantees that there is no claw decomposition for Double Fan Graph* $F_{2,n}$.

5. DECOMPOSITION OF TRIPLE FAN GRAPH $F_{3,n}$

In this section, we characterize the theorem of decomposition of Triple Fan Graph $F_{3,n}$ into claws and paths.

Obviously Triple Fan Graph $F_{3,n}$ has every vertex of degree 5 except the vertices v_1, v_2 and v_3 are of degree n and the vertices u_1 and u_n are of degree 4 respectively.

Theorem 5.1. *Any Triple Fan Graph* $F_{3,n}$ *can be decomposed into following ways.*

$$D(F_{3,n}) = \begin{cases} (4d-1)S_3 \textit{ and } P_3,\ d = 1,2,3,\dots & \textit{if } n = 3d,\ d = 1,2,3,\dots \\ (4d+1)S_3,\ d = 1,2,3\dots & \textit{if } n = 3d+1,\ d = 1,2,3,\dots. \\ (4d-2)S_3 \textit{ and } P_2,\ d = 1,2,3\dots & \textit{if } n = 3d-1,\ d = 1,2,3,\dots \end{cases}$$

Proof. Let $V(F_{3,n}) = \{v_1, v_2, v_3, u_1, u_2, \dots, u_n\}$ and

$$E(F_{3,n}) = \{e_{ij}/i = 1,2,3,\ 1 \le j \le n\} \bigcup \{e_k/1 \le k \le n-1\}.$$

Case 1. $n = 3d,\ d = 1,2,3,\dots.$

To Prove: $F_{3,n}$ decomposed into $(4d-1)S_3$ and $P_3,\ d = 1,2,3,\dots.$

Let $E_i = \{e_{1(j+1)}, e_j, e_{2(j+1)}\}$ where $j = 1,2,3,\dots,n-1$, $E_j = \{e_{11}, e_{21}, e_{31}\} \bigcup \{e_{3j}, e_{3(j+1)}, e_{3(j+2)}\}$ where $j = 2,5,8,\dots,n-4$ and $E_k = \{e_{3(n-1)}, e_{3n}\}$. The edge induced subgraph $< E_i >$ forms $(3d-1)$ copies of S_3, the edge induced subgraph $< E_j >$ forms d copies of S_3 and the edge induced subgraph $< E_k >$ forms a path P_3 of length 2. Hence $F_{3,n}$ decomposed into $3d-1+d = 4d-1$ copies of S_3 and a path P_3.

Case 2. $n = 3d+1,\ d = 1, 2, 3, \ldots$.

To Prove: $F_{3,n}$ decomposed into $(4d+1)S_3,\ d = 1, 2, 3\ldots$.

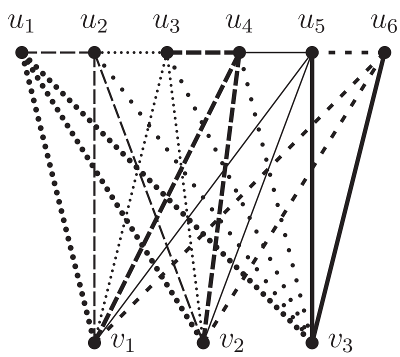
Let $E_i = \{e_{1(j+1)}, e_j, e_{2(j+1)}\}$ where $j = 1, 2, 3, \ldots, n-1$ and $E_j = \{e_{11}, e_{21}, e_{31}\} \bigcup \{e_{3j}, e_{3(j+1)}, e_{3(j+2)}\}$ where $j = 2, 5, 8, \ldots, n-2$. The edge induced subgraph $< E_i >$ forms $3d$ copies of S_3 and the edge induced subgraph $< E_j >$ forms $(d+1)$ copies of S_3. Hence $F_{3,n}$ decomposed into $3d+d+1 = 4d+1$ copies of S_3.

Case 3. $n = 3d-1,\ d = 1, 2, 3, \ldots$.

To Prove: $F_{3,n}$ decomposed into $(4d-2)S_3$ and $P_2,\ d = 1, 2, 3, \ldots$.

Let $E_i = \{e_{1(j+1)}, e_j, e_{2(j+1)}\}$ where $j = 1, 2, 3, \ldots, n-1$, $E_j = \{e_{11}, e_{21}, e_{31}\} \bigcup \{e_{3j}, e_{3(j+1)}, e_{3(j+2)}\}$ where $j = 2, 5, 8, \ldots, n-3$ and $E_k = \{e_{3n}\}$. The edge induced subgraph $< E_i >$ forms $(3d-2)$ copies of S_3, the edge induced subgraph $< E_j >$ forms d copies of S_3 and the edge induced subgraph $< E_k >$ forms a path P_2 of length 1. Hence $F_{3,n}$ decomposed into $3d-2+d = 4d-2$ copies of S_3 and a path P_2. □

Illustration: Decomposition of Triple Fan Graph $F_{3,n}$ explained through the following Figure 3.



$(n = 3d,\ d = 2)\ F_{3,6}$

FIGURE 3.

The above figures represents decomposition of $F_{3,6}$ into 7 copies of S_3 and a path P_3.

All edges of the claws and path differentiated in the above Figure 3.

Note 5.1. *In the above theorem Case 2 guarantees that there is a claw decomposition for Triple Fan Graph $F_{3,n}$.*

6. Decomposition of Quadruple Fan Graph $F_{4,n}$

In this section, we characterize the theorem of decomposition of Quadruple Fan Graph $F_{4,n}$ into claws and paths.

Obviously Quadruple Fan Graph $F_{4,n}$ has every vertex of degree 6 except the vertices v_1, v_2, v_3 and v_4 are of degree n and the vertices u_1 and u_n are of degree 5 respectively.

Theorem 6.1. *Any Quadruple Fan Graph $F_{4,n}$ can be decomposed into following ways.*

$$D(F_{4,n}) = \begin{cases} (10d-1)S_3 \text{ and } P_3,\ d=1,2,3,\dots & \text{if } n=6d,\ d=1,2,3,\dots \\ (10d+1)S_3 \text{ and } P_2,\ d=1,2,3\dots & \text{if } n=6d+1,\ d=1,2,3,\dots \\ (10d-8)S_3 \text{ and } P_4,\ d=1,2,3,\dots & \text{if } n=6d-4,\ d=1,2,3,\dots \\ (10d-6)S_3 \text{ and } P_3,\ d=1,2,3,\dots & \text{if } n=6d-3,\ d=1,2,3,\dots \\ (10d-4)S_3 \text{ and } P_2,\ d=1,2,3\dots & \text{if } n=6d-2,\ d=1,2,3,\dots \\ (10d-2)S_3,\ d=1,2,3,\dots & \text{if } n=6d-1,\ d=1,2,3,\dots \end{cases}.$$

Proof. Let $V(F_{4,n}) = \{v_1, v_2, v_3, v_4, u_1, u_2, \dots, u_n\}$ and

$$E(F_{4,n}) = \{e_{ij}/i = 1,2,3,4,\ 1 \leq j \leq n\} \bigcup \{e_k/1 \leq k \leq n-1\}.$$

Case 1. $n = 6d,\ d = 1,2,3,\dots$.

To Prove: $F_{4,n}$ decomposed into $(10d-1)S_3$ and $P_3,\ d = 1,2,3,\dots$.

Let $E_i = \{e_{1j}, e_{j-1}, e_{2j}\}$ where $j = 2,4,\dots,n$, $E_j = \{e_{3j}, e_j, e_{4j}\}$ where $j = 2,4,6,\dots,n-2$, $E_k = \{e_{1j}, e_{2j}, e_{3j}\}$ where $j = 1,3,5,\dots,n-1$, $E_l = \{e_{4j}, e_{4(j+2)}, e_{4(j+4)}\}$ where $j = 1,7,\dots,n-5$ and $E_m = \{e_{3n}, e_{4n}\}$. The edge induced subgraph $< E_i >$ forms $3d$ copies of S_3, the edge induced subgraph $< E_j >$ forms $(3d-1)$ copies of S_3, the edge induced subgraph $< E_k >$ forms $3d$ copies of S_3, the edge induced subgraph $< E_l >$ forms d copies of S_3 and the edge induced subgraph $< E_m >$ forms a path P_3 of length 2. Hence $F_{4,n}$ decomposed into $3d + 3d - 1 + 3d + d = 10d - 1$ copies of S_3 and a path P_3.

Case 2. $n = 6d + 1,\ d = 1, 2, 3, \ldots$.

To Prove: $F_{4,n}$ decomposed into $(10d + 1)S_3$ and $P_2,\ d = 1, 2, 3, \ldots$.

Let $E_i = \{e_{1j}, e_{j-1}, e_{2j}\}$ where $j = 2, 4, \ldots, n - 1$, $E_j = \{e_{3j}, e_j, e_{4j}\}$ where $j = 2, 4, 6, \ldots, n-1$, $E_k = \{e_{1j}, e_{2j}, e_{3j}\}$ where $j = 1, 3, 5, \ldots, n$, $E_l = \{e_{4j}, e_{4(j+2)}, e_{4(j+4)}\}$ where $j = 1, 7, \ldots, n - 6$ and $E_m = \{e_{4n}\}$. The edge induced subgraph $< E_i >$ forms $3d$ copies of claws, the edge induced subgraph $< E_j >$ forms $3d$ copies of S_3, the edge induced subgraph $< E_k >$ forms $(3d + 1)$ copies of S_3, the edge induced subgraph $< E_l >$ forms d copies of S_3 and the edge induced subgraph $< E_m >$ forms a path P_2 of length 1. Hence $F_{4,n}$ decomposed into $3d + 3d + 3d + 1 + d = 10d + 1$ copies of S_3 and a path P_2.

Case 3. $n = 6d - 4,\ d = 1, 2, 3, \ldots$.

To Prove: $F_{4,n}$ decomposed into $(10d - 8)S_3$ and $P_4,\ d = 1, 2, 3, \ldots$.

Let $E_i = \{e_{1j}, e_{j-1}, e_{2j}\}$ where $j = 2, 4, \ldots, n$, $E_j = \{e_{3j}, e_j, e_{4j}\}$ where $j = 2, 4, 6, \ldots, n-2$, $E_k = \{e_{1j}, e_{2j}, e_{3j}\}$ where $j = 1, 3, 5, \ldots, n-1$, $E_l = \{e_{4j}, e_{4(j+2)}, e_{4(j+4)}\}$ where $j = 1, 7, \ldots, n - 7$ and $E_m = \{e_{3n}, e_{4n}, e_{4(n-1)}\}$. The edge induced subgraph $< E_i >$ forms $(3d-2)$ copies of S_3, the edge induced subgraph $< E_j >$ forms $(3d - 3)$ copies of S_3, the edge induced subgraph $< E_k >$ forms $(3d - 2)$ copies of S_3, the edge induced subgraph $< E_l >$ forms $(d - 1)$ copies of S_3 and the edge induced subgraph $< E_m >$ forms a path P_4 of length 3. Hence $F_{4,n}$ decomposed into $3d - 2 + 3d - 3 + 3d - 2 + d - 1 = 10d - 8$ copies of S_3 and a path P_4.

Case 4. $n = 6d - 3,\ d = 1, 2, 3, \ldots$.

To Prove: $F_{4,n}$ decomposed into $(10d - 6)S_3$ and $P_3,\ d = 1, 2, 3, \ldots$.

Let $E_i = \{e_{1j}, e_{j-1}, e_{2j}\}$ where $j = 2, 4, \ldots, n - 1$, $E_j = \{e_{3j}, e_j, e_{4j}\}$ where $j = 2, 4, 6, \ldots, n-1$, $E_k = \{e_{1j}, e_{2j}, e_{3j}\}$ where $j = 1, 3, 5, \ldots, n$, $E_l = \{e_{4j}, e_{4(j+2)}, e_{4(j+4)}\}$ where $j = 1, 7, \ldots, n - 8$ and $E_m = \{e_{4(n-2)}, e_{4n}\}$. The edge induced subgraph $< E_i >$ forms $(3d-2)$ copies of S_3, the edge induced subgraph $< E_j >$ forms $(3d - 2)$ copies of S_3, the edge induced subgraph $< E_k >$ forms $(3d - 1)$ copies of S_3, the edge induced subgraph $< E_l >$ forms $(d - 1)$ copies of S_3 and the edge induced subgraph $< E_m >$ forms a path P_3 of length 2. Hence $F_{4,n}$ decomposed into $3d - 2 + 3d - 2 + 3d - 1 + d - 1 = 10d - 6$ copies of S_3 and a path P_3.

Case 5. $n = 6d - 2,\ d = 1, 2, 3, \ldots$.

To Prove: $F_{4,n}$ decomposed into $(10d - 4)S_3$ and $P_2,\ d = 1, 2, 3, \ldots$.

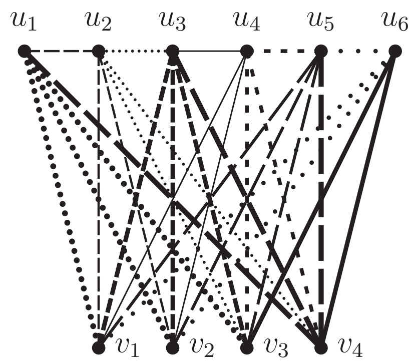
Let $E_i = \{e_{1j}, e_{j-1}, e_{2j}\}$ where $j = 2, 4, \ldots, n$, $E_j = \{e_{3j}, e_j, e_{4j}\}$ where $j = 2, 4, 6, \ldots, n-2$, $E_k = \{e_{1j}, e_{2j}, e_{3j}\}$ where $j = 1, 3, 5, \ldots, n-1$, $E_l = \{e_{4j}, e_{4(j+2)}, e_{4(j+4)}\} \bigcup \{e_{4(n-3)}, e_{4(n-1)}, e_{4n}\}$ where $j = 1, 7, \ldots, n-9$ and $E_m = \{e_{3n}\}$. The edge induced subgraph $< E_i >$ forms $(3d - 1)$ copies of S_3, the edge induced subgraph $< E_j >$ forms $(3d-2)$ copies of S_3, the edge induced subgraph $< E_k >$ forms $(3d-1)$ copies of S_3, the edge induced subgraph $< E_l >$ forms d copies of S_3 and the edge induced subgraph $< E_m >$ forms a path P_2 of length 1. Hence $F_{4,n}$ decomposed into $3d - 1 + 3d - 2 + 3d - 1 + d = 10d - 4$ copies of S_3 and a path P_2.

Case 6. $n = 6d - 1,\ d = 1, 2, 3, \ldots$.

To Prove: $F_{4,n}$ decomposed into $(10d - 2)S_3,\ d = 1, 2, 3, \ldots$.

Let $E_i = \{e_{1j}, e_{j-1}, e_{2j}\}$ where $j = 2, 4, \ldots, n-1$, $E_j = \{e_{3j}, e_j, e_{4j}\}$ where $j = 2, 4, 6, \ldots, n-1$, $E_k = \{e_{1j}, e_{2j}, e_{3j}\}$ where $j = 1, 3, 5, \ldots, n$ and $E_l = \{e_{4j}, e_{4(j+2)}, e_{4(j+4)}\}$ where $j = 1, 7, \ldots, n-4$. The edge induced subgraph $< E_i >$ forms $(3d - 1)$ copies of S_3, the edge induced subgraph $< E_j >$ forms $(3d - 1)$ copies of S_3, the edge induced subgraph $< E_k >$ forms $3d$ copies of S_3 and the edge induced subgraph $< E_l >$ forms d copies of S_3. Hence $F_{4,n}$ decomposed into $3d - 1 + 3d - 1 + 3d + d = 10d - 2$ copies of S_3. □

Illustration: Decomposition of Quadruple Fan Graph $F_{4,n}$ explained through the following Figure 4.



$(n = 6d,\ d = 1)\ F_{4,6}$

FIGURE 4.

The above figures represents decomposition of $F_{4,6}$ into 9 copies of S_3 and a path P_3.

All edges of the claws and path differentiated in the above Figure 4.

Note 6.1. *In the above theorem Case 6 guarantees that there is a claw decomposition for Quadruple Fan graph* $F_{4,n}$.

7. CONCLUSION

In this paper, we discussed decomposition of Fan Graphs, Double Fan Graphs, Triple Fan Graphs and Quadruple Fan Graphs into claws, cycles and paths.

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