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NUMERICAL CALCULATION METHOD OF PIPELINE TRANSPORT OF LOW-COMPRESSIBLE FLUID

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ABSTRACT. It is known that similar quasi-one-dimensional nonlinear equations of conservation of momentum and mass are used when modeling the processes of pipeline transport of super- and low-compressible media. In the case of low-compressible media, the relationship between pressure and density of the medium is expressed as a linear relationship. Given this dependence, a numerical method is proposed in this paper for solving the problems of pipeline transport of a low-compressible fluid, where the hydrodynamic fluid rate and the propagation velocity of small disturbances in the fluid-pipe system are involved. In the framework of the study, the unknown powers in the terms of the momentum and mass conservation equations are reduced by introducing an auxiliary function in the form of natural logarithm of the reduced fluid density. Linear equations are compiled by the transition to the waves running along the flow and against the flow, relative to the new unknown quantities, where the convective terms and the terms of resistance force are nonlinear. A numerical method for solving equations using the iteration method and the implicit approximation scheme is developed taking into account the direction of disturbance propagation when the pressure is set at the inlet and the flow rate is changed over time at the outlet. The initial conditions of the problem are formulated based on a constant flow rate. The method was tested for the cases of spasmodic and sinusoidal changes in boundary conditions and satisfactory results were obtained from the point of view of the physics of the process.

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1. INTRODUCTION

It is known that a quasi-one-dimensional description of the state of a super- and low-compressible fluids leads to partial differential equations [1]. The terms of equations that express convection and resistance forces have the third degree of unknowns, and the remaining terms have the first and second degree of unknowns [2, 3].

In analytical solution of the problems, the convective component of the medium inertia is usually discarded, and the quadratic law of resistance is linearized [3, 4]. The introduction of mass flow rate makes it possible to construct separate equations for hydrostatic pressure and mass flow rate of fluid [5]. Depending on the considered force factors, complete or truncated telegraph-type equations are formed [6]. In particular, when inertia terms are ignored, a parabolic equation is formed, and when resistance force is ignored, a hyperbolic equation is formed [7]. These two methods of simplification are often used in solving problems of pipeline transport of media and are known as "long" and "short" pipelines approaches [8].

The success of the method of separation of variables (one of the most commonly used methods for the analytical solution of the telegraph-type equation) is the separation of the eigenfunctions of functional series by the frequency indicator [9]. High-frequency oscillations, characteristic of parabolic equations, are damped quickly, and low-frequency oscillations persist longer both in time and in distance.

An option of a quasi-resonant frequency formation is possible. In this case, the excitation amplitude is expressed as the product of an exponentially decreasing function and a linear time function [10]. At the initial period of time the amplitude increases linearly [7, 11]. But after reaching its maximum, it decreases exponentially. Therefore, we used the term "quasi-resonant" [7].

When using the method of separation of variables, the cases of a discontinuous solution with the formation of compression and rarefaction waves can be considered [12]. They can be fixed with the involvement of a large number of terms of the functional series. But this will be an approximate-analytical solution, as discrete components of the equations are discarded or linearized [10].

When solving problems numerically, these shortcomings can be eliminated [13]. With sufficiently small steps of numerical integration and applying the iteration, one can obtain solutions with the discontinuities of indicators. Moreover, the temperature regime can be studied [14].

Using numerical method, the difficulties associated with nonlinear boundary conditions in the form of a supercharger characteristic, fluid outflow can be overcome (the N.E. Zhukovsky formula for limiting the outflow rate [11], an account for a damper and other).

Below, a numerical method is proposed for solving a simple problem of pipeline transport of a low-compressible fluid when setting time changes in hydrostatic pressure at the inlet and in fluid rate - at the outlet [15]. Unlike other well-known models, it is assumed here that the fluid density linearly depends on pressure. Moreover, the linear dependence coefficient is related by Young's models of fluid and the pipe material, as well as by diameter and thickness of the pipe material [11].

An auxiliary function and traveling waves are introduced for the numerical solution to the problem. Linear equations have been compiled with respect to traveling waves, (Note that a similar approach was used in acoustics [16], in computational hydrodynamics [17].) However, the equations have nonlinear terms, which result in an iterative process.

The equations of traveling waves are approximated by an implicit scheme, involving the A. A. Samarsky "against the flow" scheme.

The results obtained may be of interest for specialists in various fields.

2. MATERIALS

The system of quasi-one-dimensional equations of conservation of momentum and mass of a low-compressible fluid flowing through a relief pipeline has the form [11]:

$$\left\{ \begin{array}{l} -\frac{\partial p}{\partial x} = \rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \frac{\lambda}{2D}\rho\left|u\right|u + \rho g\, sin\alpha, \\ \\ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0. \end{array} \right. \tag{2.1}$$

Here $p(x,t)$, $\rho(x,t)$, $u(x,t)$ are the hydrostatic pressure, density and fluid rate in the distance x from the inlet to the section at a time t; D is the pipeline diameter; λ is the coefficient of friction resistance; g is the acceleration of gravity;

$z_1(x)$ is the leveling height of the pipeline axis, respectively,

$$\frac{dz_1(x)}{dx} = \sin\alpha(x).$$

The problem is solved for the case of low-compressible fluid transport through a pipeline. The low compressibility of a fluid is expressed in the fact that its density changes only at large pressure and momentum disturbances; in other cases the interval of change in fluid density is quite small.

The square of the velocity of small pressure disturbances in the pipe-fluid system is determined in [9] by the formula

$$c^2 = \left(\frac{\rho_0}{k} + \frac{D_0\rho_0}{E\delta}\right)^{-1},$$

where ρ_0 and D_0 are the fluid density and the pipeline diameter without pressure disturbances; k, E are the elastic moduli of transported fluid and pipe material; δ is the pipe thickness.

By the nature of the object, the square of the propagation velocity of small pressure disturbances represents the ratio of pressure increments and density [9]:

$$c^2 = \frac{\Delta p}{\Delta\rho}.$$

The equation of state of the fluid has the form:

$$p - p_* = c^2(\rho - \rho_*),$$

where for convenience we take $p_* = 0.1MPa$ and $\rho_* = 998.0\ kg/m^3$.

Then system (2.1) closes with the following dependence:

$$p(x,t) = c^2\rho(x,t) + p_* - c^2\rho_*. \tag{2.2}$$

In contrast to the equation of state of the gas [7,12], there is an addition term $p_* - c^2\rho_*$.

The initial velocity of undisturbed flow is taken equal to U_0: $u(x,0) = U_0$, the pressure at the inlet to the section is set as $p(0,0) = p_{00}$. Then from the first equation of system (2.1), at $t = 0$ follows

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\lambda}{2D}U_0^2 + g\frac{dz_1}{dx}$$

Considering

$$\rho\left(x,t\right)=\rho_{*}+\frac{p\left(x,t\right)-p_{*}}{c^{2}}=\frac{p\left(x,t\right)-p_{*}+\rho_{*}c}{c^{2}}.$$

The transformed equation can be written as:

$$-\frac{c^{2}}{p\left(x,t\right)-p_{*}+\rho_{*}c^{2}}\frac{\partial p}{\partial x}=\frac{\lambda}{2D}U_{0}^{2}+g\,\frac{dz_{1}}{dx}.$$

The left side of the dependence is presented as a derivative:

$$\frac{\partial}{\partial x}\ln\frac{p\left(x,t\right)-p_{*}+\rho_{*}c^{2}}{p_{*}}=-\frac{\lambda}{2Dc^{2}}U_{0}^{2}-\frac{g}{c^{2}}\,\frac{dz_{1}}{dx}.$$

Integrating from 0 to x, we obtain

$$\ln\frac{p\left(x,t\right)-p_{*}+\rho_{*}c^{2}}{p_{00}-p_{*}+\rho_{*}c^{2}}=-\frac{\lambda}{2Dc^{2}}U_{0}^{2}x-\frac{g}{c^{2}}\,\left(z_{1}\left(x\right)-z_{1}\left(0\right)\right).$$

Hence we find the initial pressure distribution

(2.3)
$$\begin{gathered}p\left(x,t\right)=p_{*}-\rho_{*}c^{2}+\\ +\left(p_{00}-p_{*}+\rho_{*}c^{2}\right)e^{-\frac{\lambda}{2Dc^{2}}U_{0}^{2}x-\frac{g}{c^{2}}\,\left(z_{1}\left(x\right)-z_{1}\left(0\right)\right)}.\end{gathered}$$

The boundary condition at the pipeline inlet is set in the form:

(2.4)
$$p(0,t)=p_{0}\left(t\right).$$

At the outlet from the section, the law of rate change is set as

(2.5)
$$u(l,t)=u_{l}\left(t\right).$$

In general, such a statement of nonlinear problem differs from the problems considered in [7, 12] in that it takes into account the initial and subsequent pressure distributions under the influence of pipeline slope with a variable coefficient $\sin\alpha\left(x\right)$.

Considering (2.2) and dividing both sides of the equations (2.1) by ρ, we compose a system of equations

(2.6)
$$\begin{cases}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+c^{2}\frac{\partial\varphi}{\partial x}=-\frac{\lambda}{2D}\left|u\right|u-g\,sin\alpha,\\ \\ \frac{\partial\varphi}{\partial t}+\frac{\partial u}{\partial x}+u\frac{\partial\varphi}{\partial x}=0.\end{cases}$$

Here an auxiliary function [5] is introduced:

(2.7)
$$\varphi(x,t)=\ln\tfrac{\rho(x,t)}{\rho_{*}}.$$

The system of equations (2.6) in [5] was obtained for a super-compressible fluid, where the gas state deviation from the Mendeleev-Clapeyron law is taken into account. In our case, it is used for a low-compressible fluid when the density change is significant in the presence of pressure and momentum jumps.

Following [10], equations (2.6) are transformed.

Proceed to dimensionless quantities. The length of section $l,$ the wave travel time of section l/c , and the sound velocity c are taken as the scales of distance, time and flow rate. The density scale ρ_* was defined above, and $p_* = 0.1MPa$ is taken as the pressure scale. Then the system of equations (2.6) takes the form:

$$\begin{cases} \frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \varphi}{\partial \bar{x}} = -\frac{\lambda l}{2D}\left|\bar{u}\right|\bar{u} - \frac{lg}{c^2}\sin\alpha, \\ \frac{\partial \varphi}{\partial \bar{t}} + \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{u}\frac{\partial \varphi}{\partial \bar{x}} = 0. \end{cases} \tag{2.8}$$

We represent the system of equations (2.8) in a matrix form

$$\frac{\partial W}{\partial \bar{t}} + A\frac{\partial W}{\partial \bar{x}} = B. \tag{2.9}$$

Here

$$W = \begin{pmatrix} \bar{u} \\ \varphi \end{pmatrix},\ A = \begin{pmatrix} \bar{u} & 1 \\ 1 & \bar{u} \end{pmatrix},\ B = \begin{pmatrix} F \\ 0 \end{pmatrix}, \tag{2.10}$$

$$F = -\frac{\lambda l}{2D}\left|\bar{u}\right|\bar{u} - \frac{lg}{c^2}\sin\alpha, \tag{2.11}$$

We represent matrix A as a product

$$A = V^{-1}\Lambda V,$$

Where V is the fundamental matrix, similar to A, consisting of elements of the eigenvectors of the matrix A [15]; V^{-1} is the inverse matrix V; Λ is a diagonal matrix whose nonzero elements are eigenvalues λ_1 and λ_2 of the matrix A.

The matrix A has eigenvalues

$$\lambda_{1,2} = \bar{u} \pm 1 \tag{2.12}$$

and unnormalized eigenvectors (1; 1) and (1; -1). Accordingly, the diagonal matrix has the form

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

and the fundamental matrix is

$$V = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{2.13}$$

Multiplying the equation

$$\frac{\partial W}{\partial \bar{t}} + V^{-1}\Lambda V \frac{\partial W}{\partial \bar{x}} = B \tag{2.14}$$

by V in the left-hand side, we come to the equation:

$$V\frac{\partial W}{\partial \bar{t}} + \Lambda V \frac{\partial W}{\partial \bar{x}} = VB. \tag{2.15}$$

Identity $VV^{-1} = E$ is taken into account here.

Since the matrix V consists of constant elements, the property of transitivity of the differentiation operations and multiplication of the matrix can be applied to (2.15):

$$\frac{\partial (VW)}{\partial \bar{t}} + \Lambda \frac{\partial (VW)}{\partial \bar{x}} = VB. \tag{2.16}$$

Here

$$VW = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \bar{u} \\ \varphi \end{pmatrix} = \begin{pmatrix} \bar{u} + \varphi \\ \bar{u} - \varphi \end{pmatrix},$$

$$VB = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} F \\ 0 \end{pmatrix} = \begin{pmatrix} F \\ F \end{pmatrix}.$$

Given these dependencies, equation (2.16) takes the form:

$$\begin{cases} \dfrac{\partial (\bar{u} + \varphi)}{\partial \bar{t}} + (1 + \bar{u}) \dfrac{\partial (\bar{u} + \varphi)}{\partial \bar{x}} = F, \\ \dfrac{\partial (\bar{u} - \varphi)}{\partial \bar{t}} - (1 - \bar{u}) \dfrac{\partial (\bar{u} - \varphi)}{\partial \bar{x}} = F. \end{cases} \tag{2.17}$$

The resulting system of equations (2.17) states that the disturbances $(\bar{u} - \varphi)$ are displaced in the direction of increase $\bar{x}$ at velocity $1 + \bar{u}$, and the disturbances $(\bar{u} - \varphi)$ move with velocity $-(1 - \bar{u})$ against the direction of increase in coordinate $\bar{x}$. That is, these values represent analogues of traveling oncoming waves:

$$f_1(\bar{x}, \bar{t}) = \bar{u}(\bar{x}, \bar{t}) + \varphi(\bar{x}, \bar{t}), \tag{2.18}$$

$$f_2(\bar{x}, \bar{t}) = \bar{u}(\bar{x}, \bar{t}) - \varphi(\bar{x}, \bar{t}). \tag{2.19}$$

Regarding these functions, equations (2.17) are written in the form:

$$\begin{cases} \dfrac{\partial f_1}{\partial \bar{t}} + (1+\bar{u})\dfrac{\partial f_1}{\partial \bar{x}} = F, \\ \dfrac{\partial f_2}{\partial \bar{t}} - (1-\bar{u})\dfrac{\partial f_2}{\partial \bar{x}} = F. \end{cases} \tag{2.20}$$

The equations from (2.20) are linear with respect to the new unknown quantities, achieved by introducing matrices V, V^{-1} and Λ. At the same time, the convective terms and the right-hand sides of equations are nonlinear. With the known values of new unknowns, the values of flow rate $\bar{u}$, function φ, and fluid density $\bar{\rho}$ are easily calculated.

Consider the boundary conditions.

At the inlet, the law of pressure change (2.4) is set over time. A time change in density at the inlet to the section is found as

$$\rho(0,t) = \frac{p(0,t) - p_*}{c^2} + \rho_*$$

and function value as

$$\varphi(0,t) = \ln\left[1 + \frac{p(0,t) - p_*}{\rho_* c^2}\right].$$

This condition in dimensionless form is

$$\varphi(0,\bar{t}) = \ln\left[1 + p_*\frac{\bar{p}(0,\bar{t}) - 1}{\rho_* c^2}\right]. \tag{2.21}$$

Condition (2.5) in a dimensionless form is:

$$\bar{u}(1,\bar{t}) = u_l(t)/c. \tag{2.22}$$

The initial condition for velocity is

$$\bar{u}(\bar{x},0) = \frac{U_{00}}{c}.$$

The condition for density (2.3) at $\bar{t} = 0$ has a dimensionless form:

$$\varphi(\bar{x},0) = \ln\left[1 + p_*\frac{\bar{p}(\bar{x},0) - 1}{\rho_* c^2}\right]. \tag{2.23}$$

Thus, the mathematical model - the equations and single-valuedness conditions of their solution are built in dimensionless quantities. To solve the problem the data from [10] is used in the process of numerical solution.

A discrete coordinate $i \in [0, N]$with a constant step h and a discrete time $n = 0, 1, \ldots$ with a constant step τ are introduced. According to them, $\sin\alpha_i$ is calculated and the notation of grid functions $f_{1i}^{n,m}$, $f_{2i}^{n,m}$, $\bar{u}_i^{n,m}$, $\bar{\rho}_i^{n,m}$, $\varphi_i^{n,m}$ are introduced.

The initial conditions $\bar{u}(\bar{x}, 0)$ and $\varphi(\bar{x}, 0)$ are given. Passing to discrete coordinates, we compose f_{1i}^0, f_{2i}^0.

The sequence of calculations for the $n+1$-th time step is constructed as follows.

Since the equations being solved are nonlinear ones, an iterative process is organized. The value of $\bar{u}_i^{n+1,m}$ in the coefficient of convective term and in the terms of resistance force is taken from the previous m-th approximation, the values $\bar{u}_i^n$ from the previous time layer are taken as zero approximation. Assuming $f_{2i}^{n,m}$ as the set values, we find the values of $f_{1i}^{n+1,m+1}$ at $i = 0..N$, the values of which are used to calculate $f_{2i}^{n+1,m+1}$ at $i = N..0$.

Let us dwell on calculations in the $m+1$-th approximation.

φ_0^{n+1} is calculated by the known value of the inlet pressure $\bar{p}_0^{n+1}$. Assuming the values of $f_{20}^{n+1,m+1}$ as known, we find $f_{10}^{n+1,m+1}$ and $\bar{u}_0^{n+1,m+1}$.

At $i = 1..N$ equation relative to f_1^{n+1} was approximated in an implicit form, taking into account the direction of disturbances propagation:

$$\begin{aligned} \frac{f_{1i}^{n+1,m+1} - f_{1i}^n}{\tau} + \left(1 + \bar{u}_i^{n+1,m}\right) \times \\ \times \frac{f_{1i}^{n+1,m+1} - f_{1i-1}^{n+1,m+1}}{h} = \Phi_{1i}^{n+1}, \end{aligned} \tag{2.24}$$

where

$$\Phi_{1i}^{n+1} = -\frac{\lambda l}{2D} \left|\bar{u}_i^{n+1,m}\right| \bar{u}_i^{n+1,m} - \frac{gl}{c^2} \sin\alpha_i.$$

From (2.24), when introducing notation $\sigma = \tau/h$, a recurrent dependence is compiled

$$f_{1i}^{n+1,m+1} = \frac{f_{1i}^n + \sigma\left(1 + \bar{u}_i^{n+1,m}\right) f_{1i-1}^{n+1,m+1} + \tau\Phi_{1i}^{n+1}}{1 + \sigma\left(1 + \bar{u}_i^{n+1,m}\right)}. \tag{2.25}$$

As seen from (2.25), $f_{1i-1}^{n+1,m+1}$ is taken from the results of the $m+1$-th approximation, i.e. the calculation is carried out using the over-relaxation method.

The last step is to calculate the value of $f_{1N}^{n+1,m+1}$ using formula (2.25). The values of $f_{2N}^{n+1,m+1}$ and $\varphi_N^{n+1,m+1}$ are calculated from the value of $\bar{u}_N^{n+1}$.

For decreasing values of $i = N - 1..0$, to approximate the second equation from (2.17), an implicit scheme was used, but, in contrast to [10], with over-relaxation for the convection term:

$$\begin{aligned} &\tfrac{f_{2i}^{n+1,m+1} - f_{2i}^{n}}{\tau} - (1 - \bar{u}_i^{n+1,})\times \\ &\times \tfrac{f_{2i+1}^{n+1,m+1} - f_{2i}^{n+1,m+1}}{h} = \Phi_{1i}^{n+1}. \end{aligned} \tag{2.26}$$

From here the recurrence dependence is compiled.

$$f_{2i}^{n+1,m+1} = \frac{f_{2i}^{n} + \sigma\left(1 - \bar{u}_i^{n+1,m}\right) f_{2i+1}^{n+1,m+1} + \tau \Phi_{1i}^{n+1}}{1 + \sigma\left(1 - \bar{u}_i^{n+1,m}\right)}. \tag{2.27}$$

At $i = N - 1..0$ the values of $\bar{u}_i^{n+1,m+1}$ and $\varphi_i^{n+1,m+1}$ are calculated from the values of $f_{1i}^{n+1,m+1}$ and $f_{2i}^{n+1,m+1}$. The procedure for the $m+1$-th approximation is completed by checking the fulfillment of conditions

$$\begin{aligned} &\max_{0\le i\le N}\left|\bar{u}_i^{n+1,m+1} - \bar{u}_i^{n+1,m}\right| < \varepsilon_u. \\ &\max_{0\le i\le N}\left|\varphi_i^{n+1,m+1} - \varphi_i^{n+1,m}\right| < \varepsilon_\varphi. \end{aligned} \tag{2.28}$$

The iterative process for the $n+1$-th time step is considered complete when both conditions are met. Then we can proceed to the next time step. Otherwise, calculations for the next $n+1$-th approximation for the $n+1$-th time layer are resumed.

3. Results

The calculations were carried out according to the data from [12]: $l = 1000.0$ m, $D = 0.200$ m $\lambda = 0.010$, $U_0 = 5.0$ m/s, $\sin\alpha = 0.1$. The speed of sound in an infinite space of water is taken equal to 1461.0 m/s, the density of water at rest - 998.0 kg/m^3, the pipe wall thickness - 0.005 m, Young's modulus for steel - 210 GPa. The propagation velocity of small pressure disturbances - c=1232.237 m/s. The dimensionless length step was 0.001 (N=1000), and the time step - $0.0002l/c$. These steps practically met the approximation conditions at $\varepsilon_u = \varepsilon_\varphi = 0.00001$ in the first approximation for a fixed time.

The calculations were performed for sinusoidal and discontinuous functions for the inlet pressure and outlet rate. We restrict ourselves to presenting some results of discontinuous nature. Figure 1 shows the pressure curves for various

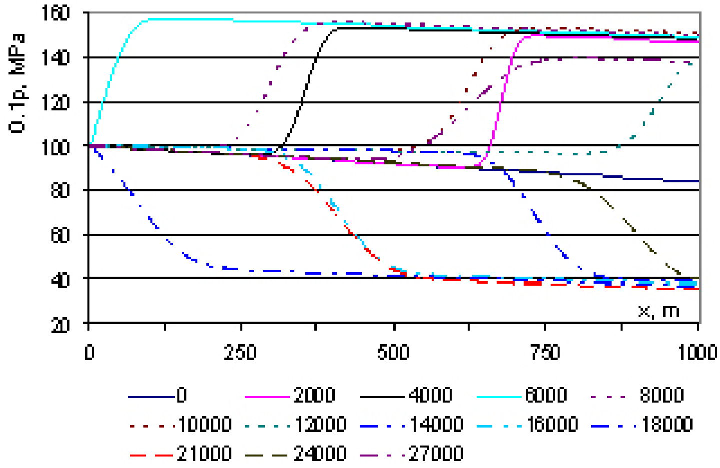

FIGURE 1. Pressure curves for various time steps. $p_{00} = 10.0\ MPa, U_0 = 5.0$ m/s The data are given in the text.

time sections after fluid deceleration at the end of the section. Before deceleration, the flow rate was $U_0 = 5.0$ m/s, and the pressure along the length practically dropped linearly.

The fluid deceleration at the end of the section forms a jump increase in pressure. The jump is displaced against the flow. Pressure graphs consist of three parts. Increased pressure is on the right side, initial pressure is on the left side. They are smeared by a pressure jump - by a compression wave.

After reaching the inlet section, the pressure disturbance returns back as a rarefaction wave. In the process, the pressure increment decreases, and reaching the end of the section the wave overturns, i.e. a negative pressure increment is formed. It goes to the beginning of the section and the return back. When returning back, it forms a new envelope.

The corresponding velocity graphs have four envelopes in the presented time interval (Figure 2). The upper envelope corresponds to the initial velocity distribution. Below is the envelope that exists in the interval of time $(2l/c,\ 2.5l/c)$. The third envelope from the top corresponds to the state of rest of fluid (0 m/s). The lower envelope is for a period of time $(l/c,\ 2l/c)$.

If the revolution of pressure jumps occurs at even coefficients of time-scale l/c, then the revolution of velocity jumps occurs at odd coefficients l/c.

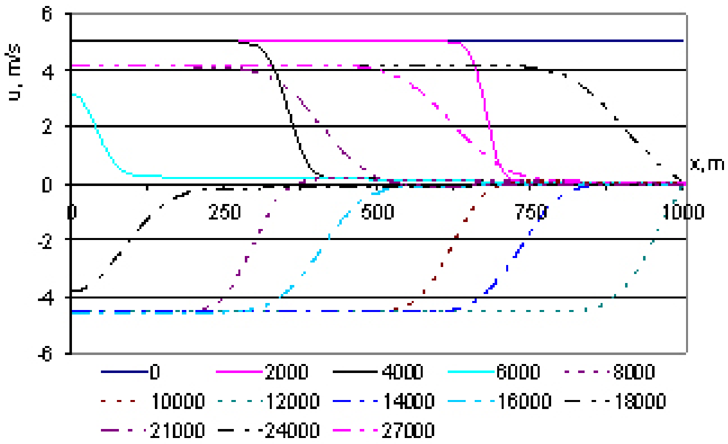

FIGURE 2. Velocity curves for various time steps.

In the future, the section tends to a state of rest, since the pressure remains constant at the inlet, and the velocity is zero at the outlet.

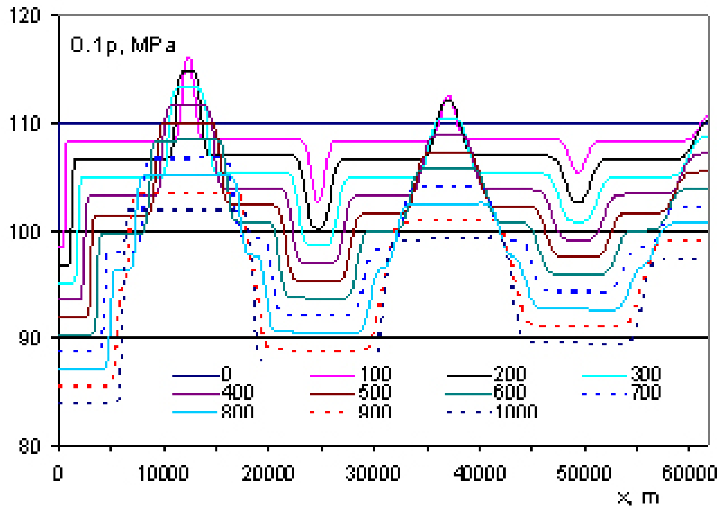

FIGURE 3. Time changes in pressure in various sections at jump increase in the inlet pressure from 10 MPa to 11 MPa.

The following example demonstrates the jump change in pressure at the inlet to the section at a constant flow rate at the outlet of the section. The graphs in Figures 3 and 4 represent time changes in pressure and flow rate in sections $0,\ 100,\ 200,\ ...,\ 1000\ m$.

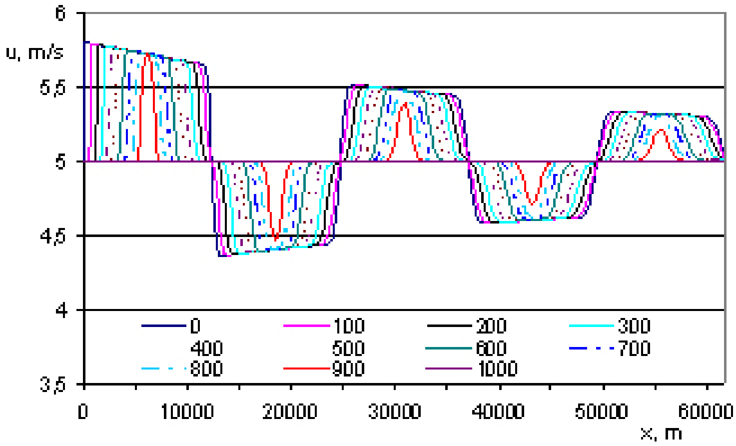

FIGURE 4. Time changes in velocity in various sections at jump increase in the inlet pressure from 10 MPa to 11 MPa.

4. CONCLUSION

Thus, the results of the qualitative analysis showed that the solutions of the system of functional differential equations with a delay argument, describing the functioning of liver cells and hepatitis B viruses, have the characteristics of existence, continuity, non-negativity, uniqueness and boundedness.

The equations of pipeline transport of a low-compressible fluid are reduced to the equations of analogues of traveling waves. The problem was considered over time when the pressure was set at the inlet to the section, and the fluid rate was set at the outlet.

A numerical method for solving the problem with approximation of equations of traveling waves with the first order accuracy in steps has been developed. An attempt to increase the approximation accuracy led to the static instability of computational process.

Numerical method was tested for a sinusoidal and discontinuous change in the inlet pressure and outlet flow rate over time.

The results obtained describe the aperiodic nature of the change in pressure and flow rate with the formation of new envelopes. At constant values of boundary conditions, a gradual transition to a state of rest of fluid in the section occurs.

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