

**THE EFFECTS OF HALL CURRENT ON THE FLOW DUE TO THE
OSCILLATING ROTATING POROUS DISK WITH A VISCOUS FLUID AT
INFINITY: GRAPHICAL SOLUTIONS USING MATLAB**

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ABSTRACT. The flow due to the oscillating rotating porous disk with a viscous fluid at infinity is studied under the influence of Hall current. Governing equations are implied with reasonable approximations and solved analytically to get the expressions for the velocity fields in closed form. Graphical results are presented for the velocity components for various values of parameters namely, the Hall, suction and blowing through MATLAB and a discussion is provided. It is important to note that presented results are valid for all values of the frequencies.

1. INTRODUCTION

In many MHD problems it is assumed that the electrical conductivity of the fluid is isotropic and as such a scalar quantity. However, this need not be so always in nature and the conductivity of the medium is an anisotropic if the medium is rarefied and /or if a string magnetic field is present. In the presence

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2020 *Mathematics Subject Classification.* 76B75.

Key words and phrases. Rotating porous Disk, Viscous fluid, Hall Current, MATLAB.

Submitted: 01.03.2021; *Accepted:* 22.03.2021; *Published:* 07.05.2021.

of a strong magnetic field, the charged particles are tied to the lines of force, and this prevents their motion transverse to the magnetic field. Then the tendency of the current flow in a direction normal both the electrical and magnetic field is known as Hall current. The Hall Effect is the production of a voltage difference (Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current. Thus the Hall Effect rotates the current vector away from the direction of the electric field generally reduces the level of the Lorentz force on the flow.

The Hall coefficient is defined as the ratio of the induced electric field to the product of the current density and applied magnetic field. It is a characteristic of the material from which the conductor is made, since its value depends on the type, number, and properties of the charge carriers that constitute the current. Hall problems are often used as magnetometers (i.e) to measure magnetic fields, or inspect materials (such as tubing or pipelines) using the principles of magnetic fluid leakage. Hall Effect devices produce a very low signal level and thus require amplification. While suitable for laboratory instruments, the vacuum tube amplifiers available in the first half of the 20th century were too expensive, power consuming, and unreliable for everyday applications. It was only with the development of the low cost integrated circuit that the Hall Effect sensor became suitable for mass application. Many devices now sold as Hall Effect sensors in fact contain both the sensor as described above plus a high gain integrated circuit (IC) amplifier in a single package. The flow of a viscous incompressible fluid due to non-coaxial rotations of a disk and the fluid at infinity has been studied by a number of researchers. An exact solution of this type of problem was obtained by Berker [17]. Coirier [16] studied the flow due to a disk and the fluid at infinity which are rotating non-coaxially at a slightly different angular velocity. The non-Newtonian flow due to a disk and the fluid at infinity which are rotating non-coaxially at a slightly different angular velocity was studied by Erdogan [15]. An exact solution of the three dimensional Navier-Stokes equations for the flow due to non coaxial rotation of a porous disk and the fluid at infinity was studied by Erdogan [13, 14]. Murthy and Ram [12] studied the magnetohydrodynamic flow and heat transfer due to eccentric rotations of a porous disk and the fluid at infinity. The unsteady flow due to non-coaxial rotations of a disk, oscillating in its own plane and the fluid at infinity was studied by Kasiviswanathan and Rao [11]. Chakraborti et al. [12]

studied the hydromagnetic flow due to non-coaxial rotations of a disk and the fluid at infinity with same angular velocity. The flow due to non-coaxial rotations of an oscillatory porous disk and the fluid at infinity about an axis passing through a fixed point parallel to the axis of rotation of the disk was investigated by Hayet et al. [10]. The flow due to non-coaxial rotations of an oscillating porous disk and the fluid at infinity which rotate about an axis passing through a fixed point parallel to the axis of rotation of the disk was studied by Guria et al. [5]. Hayet et al. [8] studied the unsteady MHD flow due to non-coaxial rotations of a porous disk and the fluid at infinity. Turkeyilmazoglu M. A class of exact solutions for the incompressible viscous magnetohydrodynamic flow over a porous rotating disk said by the three-dimensional hydromagnetic equations of motion are treated analytically to obtained exact solutions with the inclusion of suction and injection was obtained Turkeyilmazoglu [2020]. Hall effects on unsteady hydromagnetic flow induced by an eccentric–concentric rotation of a disk and a fluid at infinity. Rotating disk flows of conducting fluids have practical applications in many areas such as rotating machinery, lubrication, computer storage devices, viscometry and crystal growth process. In most cases the hall term was ignored in applying Ohm’s law as it has no marked effect for small and moderate values of the magnetic field. In recent years, considerable interest has been shown in mass addition to boundary layer flows, especially in connection with the cooling of the turbine blades and the sting of high speed aero vehicles. In the present paper, proposed to study the theoretical study of the Hall Effect in steady MHD flow over a rotating porous disk with a viscous fluid at infinity.

2. MATHEMATICAL FORMULATION

Consider an incompressible viscous fluid which fills the space $z > 0$ and is in contact with an infinite porous disk making oscillations in its own plane. Introduce a Cartesian coordinate system with the z - axis normal to the disk, which lies in the plane $z = 0$. The axis of rotation of both the disk and the fluid, are assumed to be in the plane $x = 0$, with the distance between the axes being l . The geometry of the problem is shown in Fig 1. Initially, the disk and the fluid at infinity are rotating with the same angular velocity Ω about the z^1 - axis and at time $t = 0$, the disk start to oscillate suddenly along the x - axis and to

rotate impulsively about the z -axis with Ω and the fluid continues to rotate with Ω about the z^1 -axis. A uniform magnetic field B_0 is applied in the positive z -direction

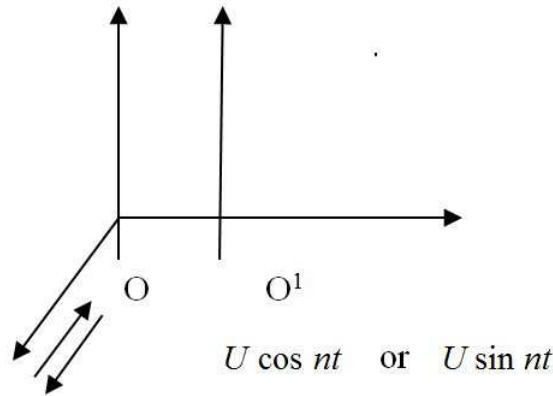


FIGURE 1.

The relevant boundary and initial conditions are

$$(2.1) \quad \begin{aligned} u &= -\Omega y + U \cos nt \text{ or } u = -\Omega y + U \sin nt; v = \Omega x \text{ at } z = 0 \text{ for } t > 0, \\ u &= -\Omega(y - l), v = \Omega x \text{ at } z = 0 \text{ for } t > 0, \\ u &= -\Omega(y - l), v = \Omega x \text{ at } t = 0 \text{ for } z > 0, \end{aligned}$$

in which n being the frequency of the non-torsional oscillations. The equations governing the flow are (2.1)-(2.5), (2.11), (2.12) and the following generalized Ohm's law which includes the Hall current [1]

$$(2.2) \quad J + \frac{W_e \tau_e}{B_0} (J \times B) = \sigma \left[E + V \times B + \frac{1}{en_e} \Delta p_e \right],$$

where e is the electron charge, p_e is the electron pressure, n_e is the electron number density, ω_e is the cyclotron frequency and τ_e is the electron collision time. Note that the ion-slip and thermoelectric effects are not included in equation (2.2). In the absence of an external applied electric field and with negligible effects of polarization of the ionized gas, E is taken as zero ($E = 0$). The induced magnetic field is negligible which is a valid consideration is on the laboratory scale. Further, it is assumed that $\omega_e \tau_e \approx \theta(1)$ and $\omega_1 \tau_1 \ll 1$, where ω_1 and τ_1 are cyclotron frequency and collision time for ions respectively.

Proceeding with the Equations (2.1), (2.11), (2.12) in [1] and (2.2) then using the above assumptions, the flow with Hall effects is governed by the following scalar equations:

$$(2.3) \quad \frac{\partial f}{\partial t} - \Omega g - W_0 \frac{\partial f}{\partial z} = -\frac{1}{\rho} \frac{\partial \hat{p}}{\partial x} + v \frac{\partial^2 f}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1-im)}(f - \Omega y),$$

$$(2.4) \quad \frac{\partial g}{\partial t} + \Omega f - W_0 \frac{\partial g}{\partial z} = -\frac{1}{\rho} \frac{\partial \hat{p}}{\partial x} + v \frac{\partial^2 g}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1-im)}(g - \Omega x),$$

$$(2.5) \quad \frac{1}{\rho} \frac{\partial \hat{p}}{\partial x} = \frac{\sigma B_0^2}{\rho(1-im)} W_0.$$

Here $m = \omega_e \tau_e$ is the Hall parameter and the modified pressure \hat{p} is

$$(2.6) \quad \hat{p} = p_1 - \frac{\rho r^2 \Omega^2}{2}, \quad r^2 = x^2 + y^2.$$

From equations (2.11) in [1] and (2.1), one can write

$$(2.7) \quad \begin{aligned} f(0, t) &= U \cos nt \text{ or } f(0, t) = U \sin nt, \quad g(0, t) = 0 \text{ for } t > 0, \\ f(z, t) &= \Omega l; \quad g(z, t) = 0 \text{ as } z \rightarrow \infty \text{ for all } t, \\ f(z, 0) &= \Omega l; \quad g(z, 0) = 0 \text{ for } z > 0. \end{aligned}$$

Eliminating \hat{p} from equations (2.3) to (2.5), using boundary conditions (2.7) and then combining the resulting equations one can write the following problem:

$$(2.8) \quad v \frac{\partial^2 G}{\partial z^2} - \frac{\partial G}{\partial t} + W_0 \frac{\partial G}{\partial z} - (N + i\Omega) G = 0,$$

$$(2.9) \quad \begin{aligned} G(0, t) &= \frac{u}{\Omega l} \cos nt - 1 \text{ or } G(0, t) = \frac{u}{\Omega l} \sin nt - 1; \quad t > 0, \\ G(0, t) &= 0, \text{ as } z \rightarrow \infty, \text{ for all } t, \\ G(z, 0) &= 0 \text{ for } z > 0, \end{aligned}$$

in which

$$(2.10) \quad G = \frac{f}{\Omega l} + i \frac{g}{\Omega l} - 1,$$

and

$$N = \frac{\sigma B_0^2(1 + im)}{\rho(1 + m^2)}.$$

Introducing

$$(2.11) \quad H = Ge^{i\Omega t}.$$

The governing problem becomes

$$(2.12) \quad v \frac{\partial^2 H}{\partial z^2} - \frac{\partial H}{\partial t} + W_0 \frac{\partial H}{\partial z} - NH = 0,$$

$$(2.13) \quad \begin{aligned} H(0, t) &= -1 + \frac{U}{\Omega l} \cos nt \text{ or } H(0, t) = -1 + \frac{U}{\Omega l} \sin nt; \quad t > 0, \\ H(z, t) &= 0 \text{ as } z \rightarrow \infty \text{ for all } t; \quad H(z, 0) = 0. \end{aligned}$$

3. EXACT SOLUTION TO THE PROBLEM FOR NON-RESONANT FREQUENCIES

$$(K = N/\Omega \neq 1)$$

By means of the Laplace transform, we obtain the following solutions for the resulting transformed problems for $U \cos nt$

$$(3.1) \quad \bar{H}(z, s) = \left[-\frac{1}{s - i\Omega} + \frac{U}{2\Omega l} \left\{ \frac{1}{s + i(n - \Omega)} + \frac{1}{s - i(n + \Omega)} \right\} \right] e_1; \quad n > \Omega,$$

$$(3.2) \quad \bar{H}(z, s) = \left[-\frac{1}{s - i\Omega} + \frac{U}{2\Omega l} \left\{ \frac{1}{s - i(\Omega - n)} + \frac{1}{s - i(n - \Omega)} \right\} \right] e_1; \quad n < \Omega,$$

and for $U \sin nt$

$$(3.3) \quad \bar{H}(z, s) = \left[-\frac{1}{s - i\Omega} + i \frac{U}{2\Omega l} \left\{ \frac{1}{s + i(n - \Omega)} - \frac{1}{s - i(n + \Omega)} \right\} \right] e_1; \quad n > \Omega,$$

$$(3.4) \quad \bar{H}(z, s) = \left[-\frac{1}{s - i\Omega} + i \frac{U}{2\Omega l} \left\{ \frac{1}{s - i(\Omega - n)} - \frac{1}{s - i(n - \Omega)} \right\} \right] e_1; \quad n < \Omega,$$

in which

$$(3.5) \quad e_1 = e^{-\left[\frac{W_0}{2v} + \sqrt{\left(\frac{W_0}{2v} \right)^2 + \frac{N}{v}} \right] z}.$$

After the inversion for the Laplace transform, the equations (3.1) to (3.5) yield the following suction solutions for $U \cos nt$, $n > \Omega$:

$$(3.6) \quad \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{-\sqrt{2}W\xi} \left\{ \begin{array}{l} -\frac{1}{2} \left(\begin{array}{l} e^{(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_2+iy_2) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_2+iy_2) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ + \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(x_3+iy_3)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_3+iy_3) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_3+iy_3)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_3+iy_3) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ - \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(x_4+iy_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_4+iy_4) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_4+iy_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_4+iy_4) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \end{array} \right\},$$

and for $n < \Omega$,

$$(3.7) \quad \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{-\sqrt{2}W\xi} \left\{ \begin{array}{l} -\frac{1}{2} \left(\begin{array}{l} e^{(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_2+iy_2) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_2+iy_2) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ + \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(x_5+iy_5)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_5+iy_5) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_5+iy_5)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_5+iy_5) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ - \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(x_4+iy_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_4+iy_4) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_4+iy_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_4+iy_4) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \end{array} \right\}.$$

For $U \sin nt$, $n > \Omega$,

$$(3.8) \quad \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{-\sqrt{2}W\xi} \left\{ \begin{array}{l} -\frac{1}{2} \left(\begin{array}{l} e^{(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_2+iy_2) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_2+iy_2) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ + \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(x_5+iy_5)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_5+iy_5) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_5+iy_5)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_5+iy_5) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ - \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(x_4+iy_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_4+iy_4) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_4+iy_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_4+iy_4) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \end{array} \right\},$$

and for $n < \Omega$,

(3.9)

$$\frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{-\sqrt{2}W\xi} \left\{ \begin{array}{l} -\frac{1}{2} \left(\begin{array}{l} e^{(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_2 + iy_2) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_2 + iy_2) \sqrt{\frac{\tau}{2}} \right) \end{array} \sqrt{\frac{\tau}{2}} \right) \\ + i \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(x_5+iy_5)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_5 + iy_5) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_5+iy_5)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_5 + iy_5) \sqrt{\frac{\tau}{2}} \right) \end{array} \sqrt{\frac{\tau}{2}} \right) \\ - i \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(x_4+iy_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_4 + iy_4) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(x_4+iy_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_4 + iy_4) \sqrt{\frac{\tau}{2}} \right) \end{array} \sqrt{\frac{\tau}{2}} \right) \end{array} \right\}.$$

Here

$$\begin{aligned} x_2 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ x_3 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k-1 - \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ x_4 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k+1 + \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ x_5 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2} - k\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ y_2 &= \left[\sqrt{\left(W^2 - \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ y_3 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k-1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ y_4 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k+1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \end{aligned}$$

$$\begin{aligned}
y_5 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2} - k\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\
x_2 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\
x_3 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k - 1 - \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\
x_4 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k + 1 + \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\
x_5 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2} - k\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\
y_2 &= \left[\sqrt{\left(W^2 - \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\
y_3 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k - 1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\
y_4 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k + 1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\
y_5 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2} - k\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}},
\end{aligned}$$

and

$$(3.10) \quad \xi = \sqrt{\frac{\Omega}{2v}}z, \quad k = \frac{n}{\Omega}, \quad \tau = \Omega t, \quad N_1 = \frac{\sigma B_0^2}{\rho \Omega}, \quad W = \frac{W_0}{2\sqrt{v\Omega}}.$$

Note that in Equations (3.6) to (3.9), the Equations (2.10) and (2.11) have also been used. The solutions (3.6) to (3.9) are unsteady and valid for the suction case. For blowing, the unsteady solutions can be directly taken from the suction case i.e from equations (3.6) to (3.9) by replacing W by $-W_1$ ($W_1 > 0$). Further,

the steady state solutions in the respective case can be obtained by using the following asymptotic values of the complementary error function

$$(3.11) \quad \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\gamma}} \pm (x_j + iy_j) \sqrt{\frac{\tau}{2}} \right) \rightarrow (0, 2), \quad j = 1 \text{ to } 4.$$

4. EXACT SOLUTION FOR THE RESONANT CASE ($n/\Omega = 1$)

The unsteady suction solutions for $U \cos nt$ and $U \sin nt$ can be respectively written as

$$(4.1) \quad \left\{ \begin{aligned} & \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{-\sqrt{2}W\xi} \\ & -\frac{1}{2} \left(\begin{aligned} & e^{(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_2 + iy_2) \sqrt{\frac{\tau}{2}} \right) \\ & + e^{-(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_2 + iy_2) \sqrt{\frac{\tau}{2}} \right) \end{aligned} \right) \sqrt{\frac{\tau}{2}} \\ & \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{aligned} & e^{(x_6+iy_6)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_6 + iy_6) \sqrt{\frac{\tau}{2}} \right) \\ & + e^{-(x_6+iy_6)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_6 + iy_6) \sqrt{\frac{\tau}{2}} \right) \end{aligned} \right) \sqrt{\frac{\tau}{2}} \\ & + \frac{U}{2\Omega l} e^{ik\tau} \left(\begin{aligned} & e^{(x_7+iy_7)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_7 + iy_7) \sqrt{\frac{\tau}{2}} \right) \\ & + e^{-(x_7+iy_7)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_7 + iy_7) \sqrt{\frac{\tau}{2}} \right) \end{aligned} \right) \sqrt{\frac{\tau}{2}} \end{aligned} \right\}$$

$$(4.2) \quad \left\{ \begin{aligned} & \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{-\sqrt{2}W\xi} \\ & -\frac{1}{2} \left(\begin{aligned} & e^{(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_2 + iy_2) \sqrt{\frac{\tau}{2}} \right) \\ & + e^{-(x_2+iy_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_2 + iy_2) \sqrt{\frac{\tau}{2}} \right) \end{aligned} \right) \sqrt{\frac{\tau}{2}} \\ & + i \frac{U}{2\Omega l} e^{-k\tau} \left(\begin{aligned} & e^{(x_6+iy_6)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_6 + iy_6) \sqrt{\frac{\tau}{2}} \right) \\ & + e^{-(x_6+iy_6)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_6 + iy_6) \sqrt{\frac{\tau}{2}} \right) \end{aligned} \right) \sqrt{\frac{\tau}{2}} \\ & - i \frac{U}{2\Omega l} e^{-k\tau} \left(\begin{aligned} & e^{(x_7+iy_7)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (x_7 + iy_7) \sqrt{\frac{\tau}{2}} \right) \\ & + e^{-(x_7+iy_7)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (x_7 + iy_7) \sqrt{\frac{\tau}{2}} \right) \end{aligned} \right) \sqrt{\frac{\tau}{2}} \end{aligned} \right\}.$$

Here

$$x_6 = \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2} \right)^2 + \left(1 + \frac{N_1 m}{1+m^2} \right)^2} + \left(W^2 + \frac{N_1}{1+m^2} \right) \right]^{\frac{1}{2}},$$

$$x_7 = \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(2 + \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}},$$

$$y_6 = \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}},$$

$$y_7 = \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}.$$

For blowing $W = -W_1$ ($W_1 > 0$) and the respective unsteady solutions for $U \cos nt$ and $U \sin nt$ are

$$(4.3) \quad \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{-\sqrt{2}W\xi} \cdot \left\{ \begin{array}{l} -\frac{1}{2} \left(\begin{array}{l} e^{(\tilde{x}_2 + i\tilde{y}_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_2 + i\tilde{y}_2) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_2 + i\tilde{y}_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_2 + i\tilde{y}_2) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ + \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(\tilde{x}_3 + i\tilde{y}_3)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_3 + i\tilde{y}_3) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_3 + i\tilde{y}_3)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_3 + i\tilde{y}_3) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ \frac{U}{2\Omega l} e^{-ik\tau} \left(\begin{array}{l} e^{(\tilde{x}_4 + i\tilde{y}_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_4 + i\tilde{y}_4) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_4 + i\tilde{y}_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_4 + i\tilde{y}_4) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \end{array} \right\}$$

$$(4.4) \quad \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{-\sqrt{2}W\xi} \cdot \left\{ \begin{array}{l} -\frac{1}{2} \left(\begin{array}{l} e^{(\tilde{x}_2 + i\tilde{y}_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_2 + i\tilde{y}_2) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_2 + i\tilde{y}_2)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_2 + i\tilde{y}_2) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ + i \frac{U}{2\Omega l} e^{-k\tau} \left(\begin{array}{l} e^{(\tilde{x}_3 + i\tilde{y}_3)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_3 + i\tilde{y}_3) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_3 + i\tilde{y}_3)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_3 + i\tilde{y}_3) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \\ -i \frac{U}{2\Omega l} e^{-k\tau} \left(\begin{array}{l} e^{(\tilde{x}_4 + i\tilde{y}_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_4 + i\tilde{y}_4) \sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_4 + i\tilde{y}_4)\xi} \operatorname{erfc} \left(\frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_4 + i\tilde{y}_4) \sqrt{\frac{\tau}{2}} \right) \end{array} \right) \sqrt{\frac{\tau}{2}} \end{array} \right\}.$$

Here

$$\begin{aligned}\tilde{x}_3 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(\frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ \tilde{x}_4 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(2 + \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ \tilde{x}_4 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(2 + \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}, \\ \tilde{y}_4 &= \left[\sqrt{\left(W^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k - 1 - \frac{N_1 m}{1+m^2}\right)^2} + \left(W^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}.\end{aligned}$$

In order to determine the steady state solutions, equations (3.11) are used and get for $U \cos nt$

$$(4.5) \quad \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{\sqrt{2}w_1\xi} \left\{ \begin{array}{l} -e^{(\tilde{x}_2+\tilde{y}_2)\xi} + \frac{U}{\Omega l} e^{-i\tau} e^{(\tilde{x}_3+i\tilde{y}_3)\xi} + \\ \frac{u}{\Omega l} e^{i\tau} e^{(\tilde{x}_4+\tilde{y}_4)\xi} \end{array} \right\},$$

and for $U \sin nt$,

$$(4.6) \quad \frac{f}{\Omega l} + i \frac{g}{\Omega l} = 1 + e^{\sqrt{2}w_1\xi} \left\{ \begin{array}{l} -e^{(\tilde{x}_2+\tilde{y}_2)\xi} + i \frac{U}{\Omega l} e^{-i\tau} e^{(\tilde{x}_3+i\tilde{y}_3)\xi} - \\ i \frac{u}{\Omega l} e^{i\tau} e^{(\tilde{x}_4+\tilde{y}_4)\xi} \end{array} \right\}.$$

5. RESULTS AND DISCUSSION

In this chapter, the effects of Hall current on the flow due to non-coaxial rotation of an oscillating disk and fluid at infinity in the presence of suction and blowing is investigated. For small times, the analytic solutions have been obtained using Laplace transform method. The analytic solutions for large times have been computed through asymptotic behavior of the complementary error function. We also compared the present velocity profiles mathematically and graphically with those given in the reference. The results are found in well agreement. The mathematical problem contains altogether six dimensionless parameters (ξ, k, τ, W, N_1 and ϵ). Here the investigation is made for of the hall parameter for cosine and sine oscillations when the angular velocity is greater

than, smaller than or equal to the frequency of oscillations for both suction and blowing.

The illustration is for how the Hall Effect modifies the structure of flow, the profiles of velocity for both cosine and sine oscillations when $\tau = 0.3, \frac{U}{4\Omega l} = 1, N_1 = 4$. The effect of Hall parameter $m = 0.5, 1, 2$ on velocity profiles for cosine oscillations when $W = 0$ and $k = -4, 1, 4$ are shown in Figs.2 (i, ii), respectively. It is seen from these Figures that the magnitudes of $f/\Omega l$ increases and $g/\Omega l$ decreases with the increase of m . Moreover, it is observed that boundary layer thickness for $k = 1$ is smallest when compared with $k < 1$ and $k > 1$. Also, the boundary layer thickness in case of $k < 1$ is smaller than that of $k > 1$.

In order to see the variation of hall parameter $m = 0.5, 1, 2$ on the velocity profiles in presence of suction $W = 0.5$ and cosine oscillation, we display Figures.3 (i, ii) for $k = -4, 1, 4$. It appears that when the applied magnetic field is strong, both $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ depend strongly on the hall parameter m . The magnitude of $\frac{f}{\Omega l}$ increases while $\frac{g}{\Omega l}$ decreases for large m . Here, the boundary layer thickness is minimum and the velocity profiles are maximum when $k = 1$. Also, the velocity profiles for $k > 1$ are greater than for $k < 1$. Further, the comparison of Figures 2 (i, ii) and 3 (i, ii) indicate that velocity profiles and boundary layer thickness are smaller in case of suction. This is not surprising; it is known that suction causes reduction in the boundary layer thickness.

To demonstrate the effect of hall parameter $m = 0.5, 1, 2$ on velocity profiles in blowing $W = -0.5$ and cosine oscillations, the Figures 4 (i, ii) are prepared for $k = -4, 1, 4$. These Figures elucidate that the magnitude of velocity profiles is largest for $k = 1$ while the boundary layer thickness is smallest. It is also evident that velocity profiles $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ are larger for $k > 1$ when compared with that of $k < 1$.

Figures 5 (i, ii) to 7 (i, ii) illustrate the variation of Hall parameter $m = 0.5, 1, 2$ on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for $W = 0$. $W > 0$ and $W < 0$ for sine oscillations. It is obvious from Figure 2 (i, ii) to 4.7(i, ii) that boundary layer thickness in case of sine oscillations is smaller than that of cosine oscillations.

6. CONCLUSIONS

Effects of Hall current on the flow characteristics due to non-coaxial rotations of disk and a fluid has been investigated in this chapter. The following conclusions have been emerged:

- It is observed that in the presence of hall parameter, the asymptotic steady solution for blowing and resonance exists.
- When the external magnetic field is strong, the role of hall parameter becomes more significant.
- As the Hall parameter increases the magnitude of primary velocity $\frac{f}{\Omega l}$ increases while both secondary velocity $\frac{g}{\Omega l}$ and boundary layer thickness decreases.
- The boundary layer thicknesses in case of sine oscillations are smaller than for cosine oscillations

$$W = 0, N_1 = 5, \tau = 0.3, \frac{n}{\Omega} = 5.$$

The Figures 2, 3 and 4 show the effect of hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the absence of suction and blowing at $(\frac{u}{4\Omega l} = 1)$

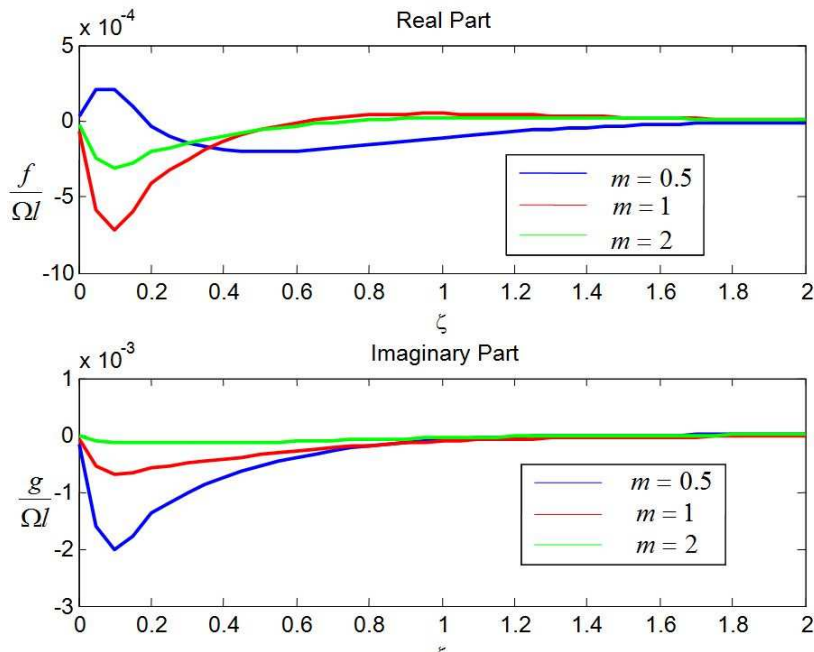
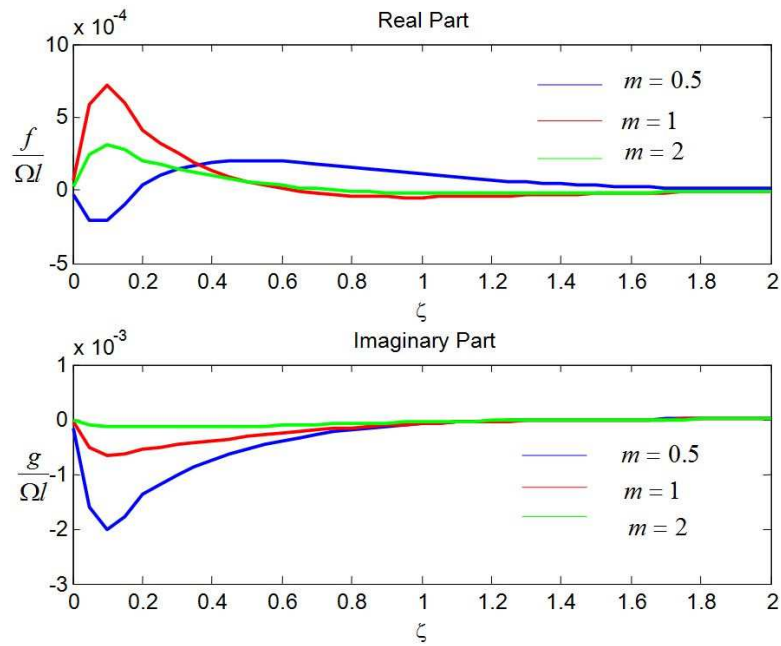
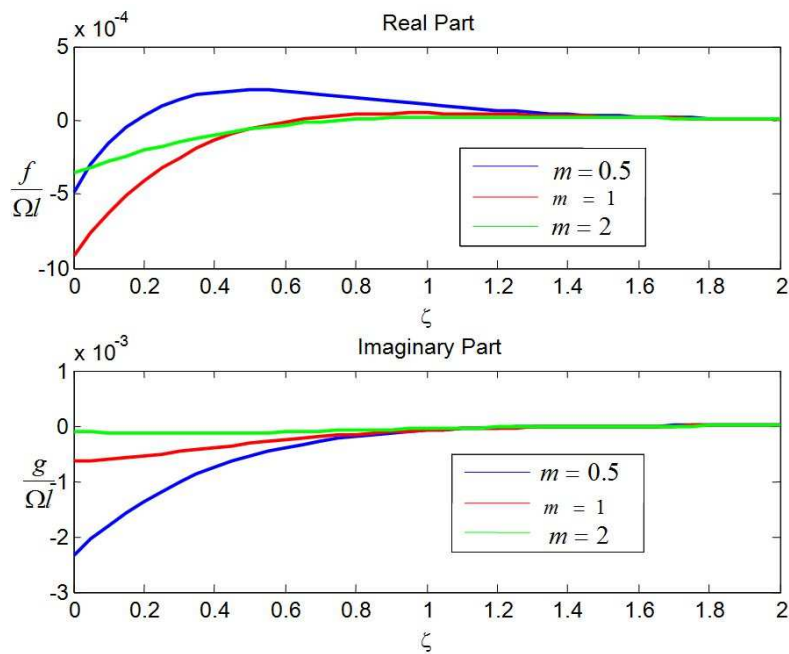


FIGURE 2. $W = 0, N_1 = 5, \tau = 0.3, \frac{n}{\Omega} = 5$


 FIGURE 3. $W = 0$ $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = -5$

 FIGURE 4. $W = 0$ $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 1$

The Figures 5, 6 and 7 show the effect of hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the absence of suction and blowing at $(\frac{u}{4\Omega l} = 1)$

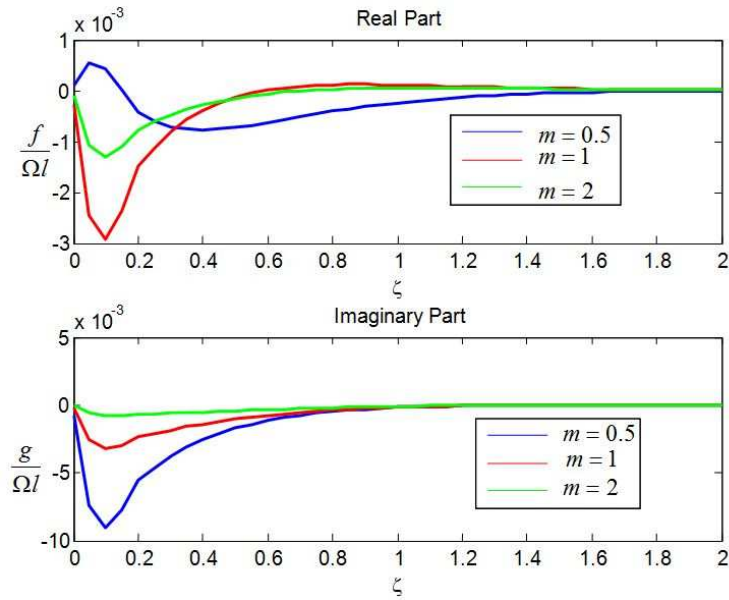


FIGURE 5. $W = 0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 5$

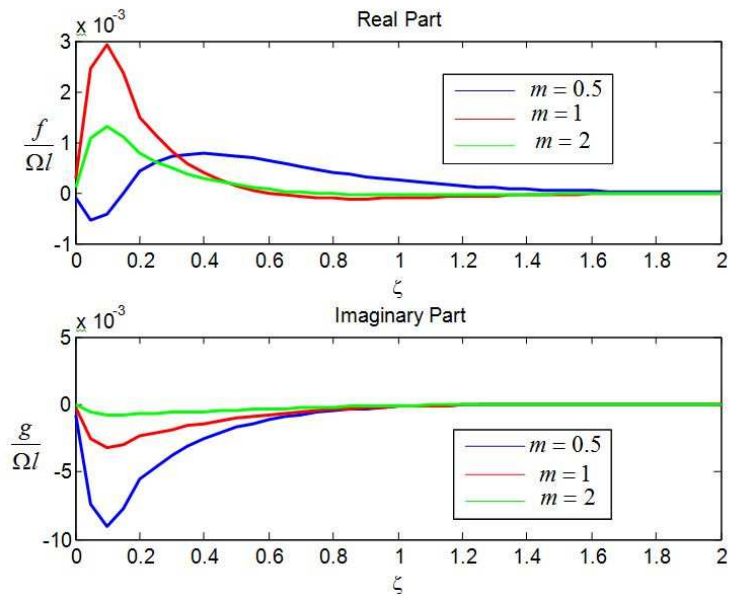
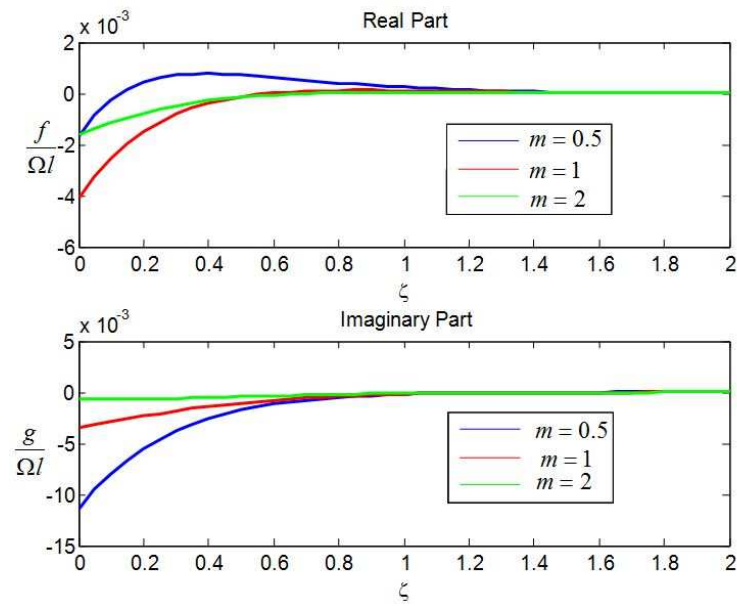
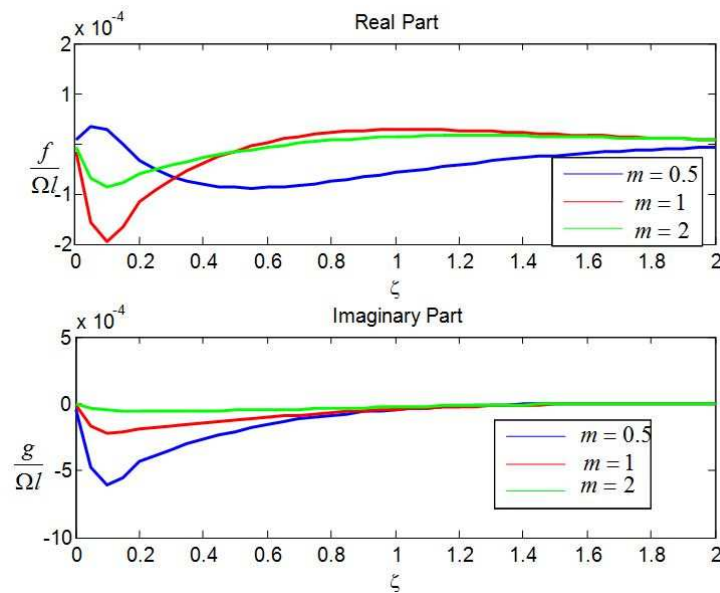
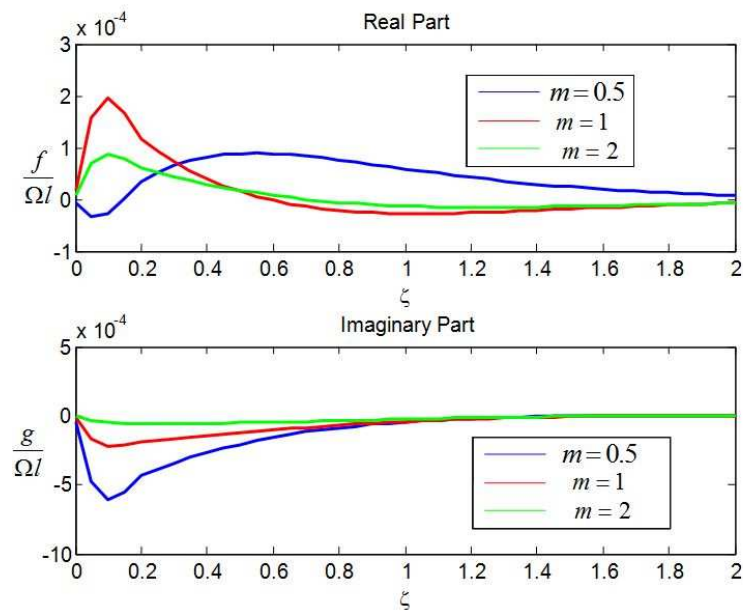
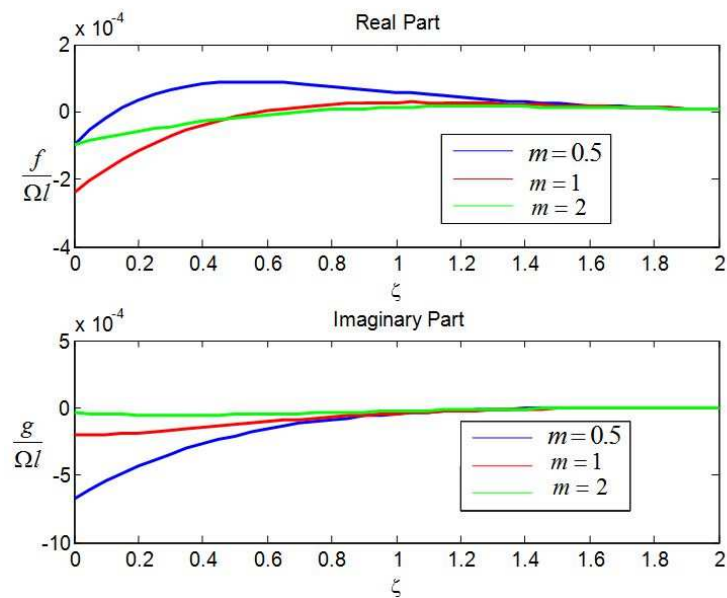


FIGURE 6. $W = 0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = -5$


 FIGURE 7. $W = 0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 1$

 FIGURE 8. $W = -0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 5$

FIGURE 9. $W = -0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = -5$ FIGURE 10. $W = -0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 1$

The Figures 8, 9 and 10 show the effect of hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the presence of suction ($\frac{u}{4\Omega l} = 1$)

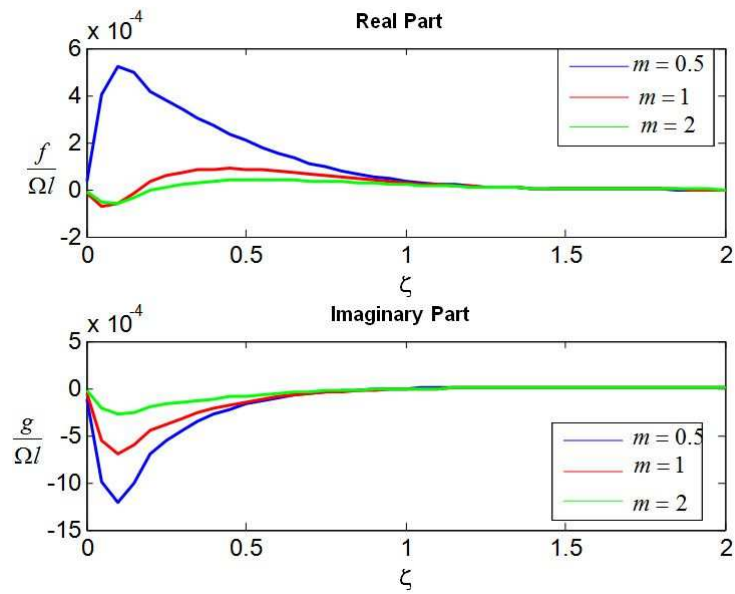


FIGURE 11. $W = 0$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 5$

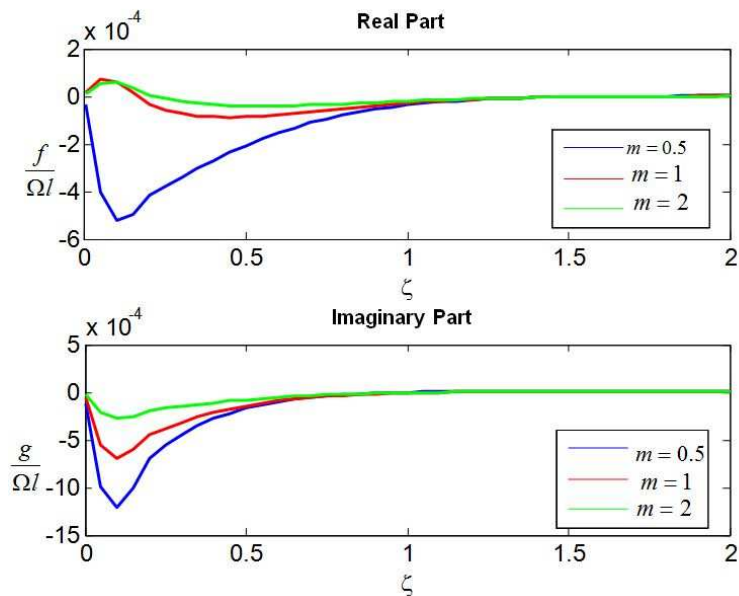
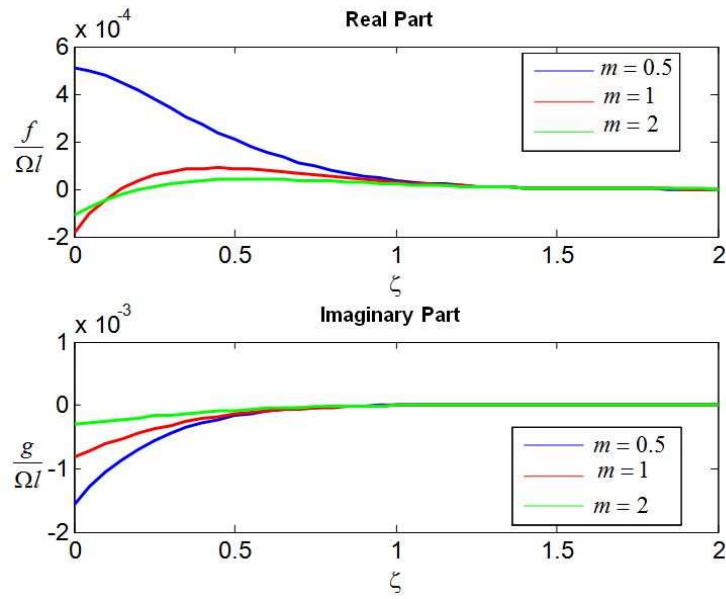
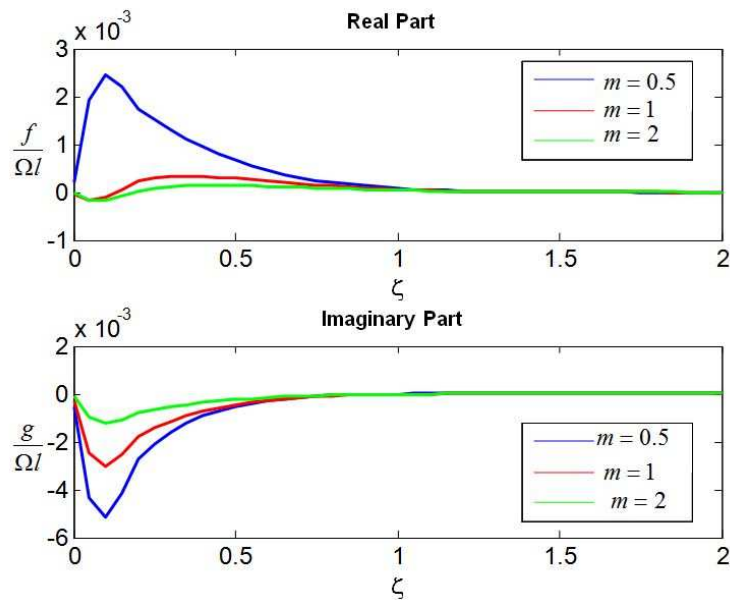
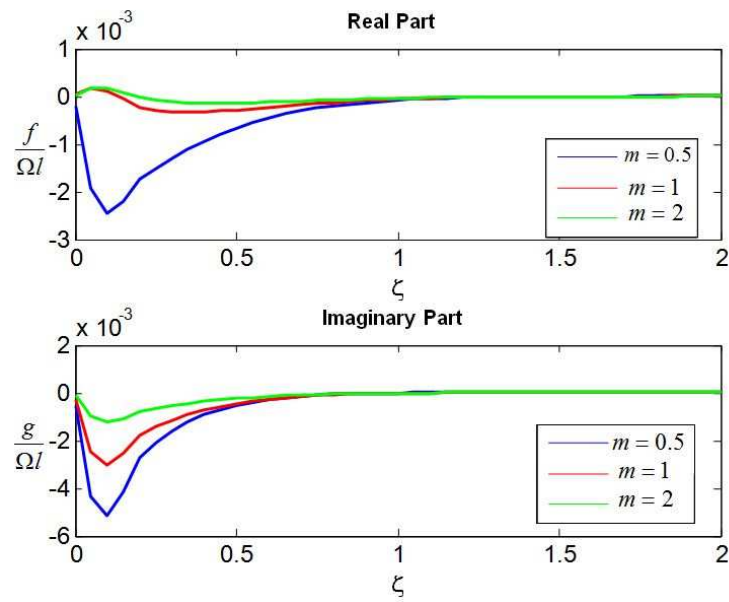
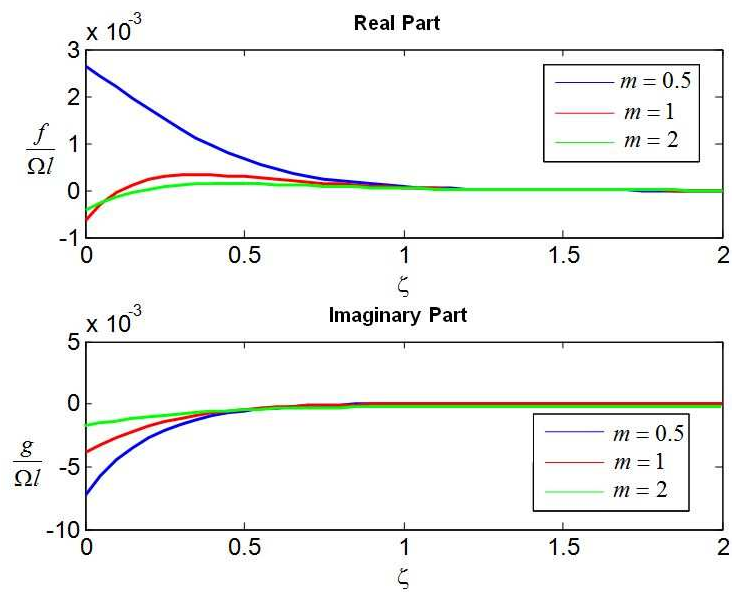


FIGURE 12. $W = 0$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = -5$

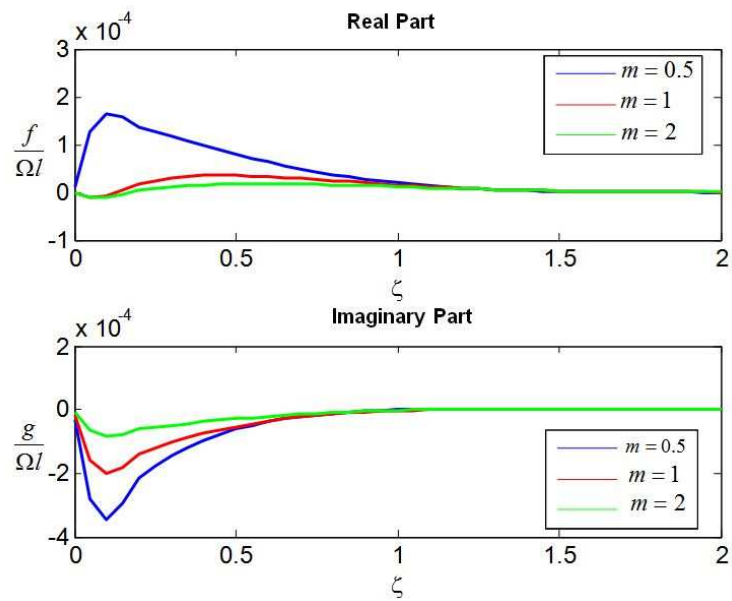
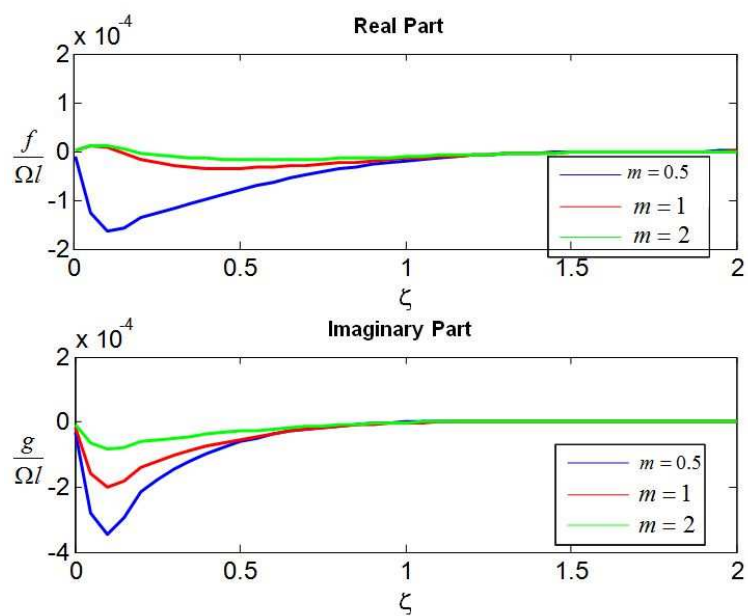
FIGURE 13. $W = 0$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 1$

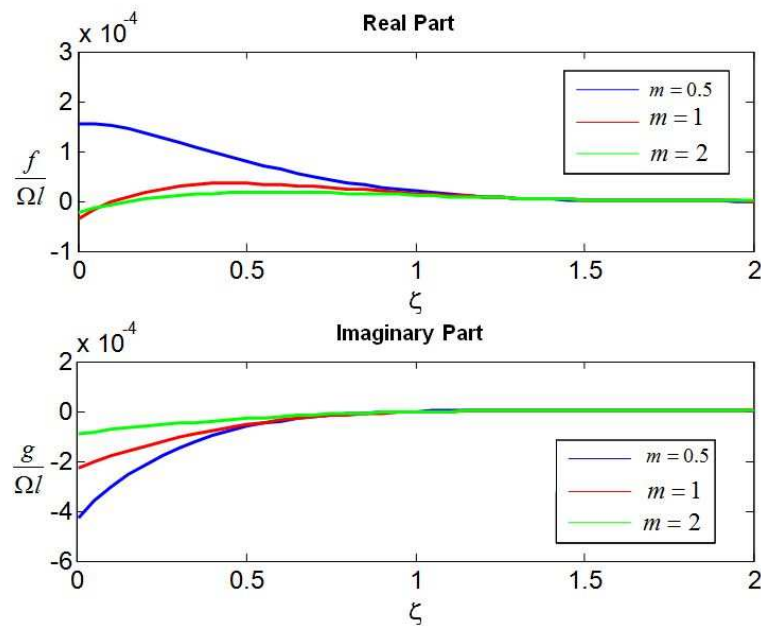
The Figures 11, 12 and 13 show the effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for sine oscillation in the absence of suction and blowing at $(\frac{u}{4\Omega l} = 1)$

FIGURE 14. $W = 0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 5$

FIGURE 15. $W = 0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = -5$ FIGURE 16. $W = 0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 1$

The Figures 14, 15 and 16 show the effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for sine oscillation in the presence of suction at $(\frac{u}{4\Omega l} = 1)$

FIGURE 17. $W = -0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 5$ FIGURE 18. $W = -0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = -5$

FIGURE 19. $W = -0.5$, $N_1 = 5$, $\tau = 0.3$, $\frac{n}{\Omega} = 1$

The Figures 17, 18 and 19 show the effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for sine oscillation in the presence of blowing at $(\frac{u}{4\Omega l} = 1)$

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