ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **10** (2021), no.5, 2549–2559
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.5.19

COVID-19 IN KERALA: A COMPARATIVE STUDY OF COMPARTMENTAL MODELS AND SOCIAL NETWORK-BASED ANALYSIS AND THE CURRENT PERFORMANCE OF THE STATE

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ABSTRACT. Kerala, a southern state of India, has shown better performance in the initial months of the spread of the disease. But in the last few months, the spread of the disease in the state has grown breaking all controls and the control and management system has shown very poor performance. Mathematical modeling of the spread of the disease is effectively is being used in the prediction and control of the disease world over. In this paper, we make a comparative study of the research conducted on this subject based on the compartmental models and social network analysis based models giving special emphasis to Kerala state. We also point out the drawbacks of the current studies in comparison with the intensity of the actual spread of disease.

1. INTRODUCTION

COVID-19, the unique incident of the 21st century that affected almost all parts of the world influenced almost all conscious minds in the world more or less a similar way. It originated in one of the powerful countries in the world, who showed the least interest in containing the disease in their country limits.

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2020 *Mathematics Subject Classification.* 92D30, 92-08, 92-10.
Key words and phrases. COVID-19, compartmental models, SIR models, social network analysis, dynamics of the disease, mathematical epidemiology.
Submitted: 04.02.2021; *Accepted:* 18.02.2021; *Published:* 19.05.2021.

Even though there are mainly two theories about the origin of the disease, everybody would agree about the lack of interest in controlling the spread of the disease in the initial days of its appearance in the world. According to the available information and supporting data, they have shown their determination and strength in controlling the disease at the later stage. It is a fact that will be recorded in history forever. Naturally, there arises a question. Is it possible to control the spread of a disease like COVID-19 and eliminate it from the world? It is very easy when the spread is limited in a small geographical area. Kerala, a southern state in India effectively controlled the outbreak of a much dangerous outbreak of the Nipah Virus in 2018. The state has a well-developed network of hospitals and related facilities including government and private hospitals. But the standard of medical facilities available in government hospitals is far below the international level. This is compensated through the service of private hospitals where the cost of the medical service is above the reach of the common man.

Mathematical models are being used to represent the dynamics of the spread of disease. It gives representation reliability, clarity, analytical ability, and accuracy. The findings of the mathematical models can be used to evaluate the current status of the spread and plan the control strategies accordingly. This is the most important aspect of a mathematical study. The comparison of the predictions of the models with the actual data obtained from the direct observation of the disease situation helps us to modify and improve the models. This way models also get benefit from the analysis. Models also serve another important purpose. It can be effectively used to explain a problematic situation to others in a reliable fashion. Mathematical models in epidemiological studies serve all these three purposes.

31 December 2019 and the few days preceding it are the dark days in the history of mankind. It was on 31 December 2019 the first case of SARS-CoV-2 or 2019-nCoV (old name) and later renamed as COVID-19 by WHO officially reported in Wuhan, China [1, 2]. On 13 January 2020, the Ministry of Public Health (MoPH), Thailand reported a case of COVID-19 in Thailand [3]. It was the first case of infection by virus reported outside China, which was followed by a case reported in Japan on 15 January 2020 [4]. It can be elicited from the data available that China, Japan, and Thailand are among the countries that effectively controlled the disease. There is a small state in India called Kerala,

which claimed that it effectively controlled the disease and is a model for the rest of the world. In Kerala, the disease appeared on 30th January 2020 in Thrissur district in Kerala [7]. It was the first reported case in India, which gave a reference to Kerala in all the literature that was published on the spread of COVID-19 in India. This case was a student who had returned for vacation from Wuhan University in China.

Naturally, the government health care machinery, acted against the spread, and with the support of a group of dedicated, efficient health workers and government officials the state successfully reduced the rate of spread within a controllable limit. The previous experience they have gained from the Nipah outbreak and subsequent dynamics have given them the confidence to perform well [10]. Some people having vested interest have started claiming that the state is performing extremely well and it is a model for the rest of the world. The state managed to control the situation some way up to August. But then things got changing from then onwards. The daily reported cases started increasing drastically. The daily death toll continues to increase gradually. Health care facilities in some districts are being reported insufficient. We analyze the situation from the perspective of the scientific study of the dynamics of the spread of the disease in the state and the applications of the modeling techniques for the improvement of the situation. We also make a comparison of the predictions made by the models and the actual spread of the disease indicated by the data published by government sources. The literature search is done in the database of Google scholar using the keywords COVID-19, Kerala, and Mathematical modeling on 30th September 2020. The search aims to collect all relevant information about the published articled in the form of journal papers (Both refereed and non-refereed), published books, book chapters, and other relevant materials. The search was limited by the period 2019 onwards (including 2019). Since the disease appeared only in December 2019 in the world, this period is justifiable. The search showed 435 items, which was analyzed to select the papers exclusively, deal with the mathematical modeling aspect of COVID-19 in Kerala. It is also examined whether any paper discusses the application of the findings of the mathematical model-based analysis to control the spread of the disease and the ways to control it. As per the available sources of information, it is not possible to cure the disease by using drugs or generate artificial immunity completely by applying vaccines [1]. But the countries in the world that have controlled the disease have followed the disease control strategies. In this paper, we also discuss the control strategies that are possible to apply in the state. In the following section, we discuss the mathematical models that are used so far to predict the disease in the state. In section 3, we make a comparison of the model predictions with the actual data available from government sources. Some important suggestions that can be considered while modifying the models in the subsequent stages are included. Discussion in Section 4 is on the strategic improvement to better the performance of disease control mechanisms in the state. It is then followed by the conclusion and directions for future research.

2. Mathematical models used to represent the disease spread in Kerala

The most popular mathematical model used in the study of COVID-19 is the Kermak and McKendrick compartmental model. These models divide the whole population into the compartments such as susceptible, infected, recovered, exposed, quarantined, symptomatic, asymptomatic, etc depending upon the nature and type of the disease studied. The transfer of an individual from one compartment to another is represented by a set of differential equations which are solved to obtain the size of each component at a particular time. Compartmental models are developed using the assumption that each susceptible in the population has an equal chance to meet with an infected. This is an oversimplified assumption that is true only when the infected do not have proper information about the infected individuals. This situation is unlikely to happen in a society that depends on mass media television and newspapers and social media such as WhatsApp, YouTube, Tweeter, Facebook, Telegram, etc. The behavior of the people in society also reflects on the spread of the disease and greatly affects the dynamics. Moreover, the social relations in the form of contacts (only physical contact is relevant in this case) of the members in a society, and geographical factors affecting the human to human contact relation play a significant role in determining the spread of the disease from one person to other. In the case of COVID-19, even coming close to an infected person can be treated as a contact leading to transmission. Next, we proceed to consider the studies on COVID-19, conducted by researchers using compartmental models and the conclusions made by them in the context of Kerala, a southern state of India. The actual data of the spread is used to compare the results of the compartmental models to evaluate the performance made by the models in predicting the disease state. Kerala is mentioned in almost all research articles published on the spread of COVID- 19 in India because the first case of infection in India was reported in Kerala. So in a search made about COVID-19 in Google scholar, with the keyword Kerala almost all papers related to COVID-19 in India are displayed. Out of the papers listed a detailed search with the application of mathematical models in the study of the spread of COVID-19 in Kerala shows only three papers directly deals with the mathematical modeling techniques in the study.

In the paper [8], Anoop et al. have used an advanced SEIR model to represent the disease situation and predictions are made according to the model. The advanced SEIR model divides the entire population into six compartments. Susceptible (S), Exposed (E), Infected (I), Quarantined (Q), Recovered (R) and Dead (D) are the six compartments. The sum of the number of members of these components is N (the total population). Susceptible is a group of individuals who can be infected by the virus. When an individual gets an infection, he becomes a member of the Exposed group, leaving the susceptible group. He then remains in the group for the period of incubation, before leaving to enter the Infectious group. Infectious is the group of people who can infect the people in the susceptible group by contact. A subgroup of the infectious people is kept under quarantine (Q). A fraction of the members of both I and Q die and the remaining recover. D represents the people who have died and R represents the population who have recovered. Homogenous mixing among the members of S and I is the basic assumption of the model. We know that the contact between two individuals can cause infection only if one belongs to I and the other belongs to S. Total number of such contacts is SI. A fraction of these contacts may lead to infection. Let β be the rate at which contact between an infective and a susceptible person leads to infection. The exposed individuals are removed at a rate ε from E to I. Also $\varepsilon = \frac{1}{\text{incubation period}}$. We assume that a fraction q of the infective is taken to quarantine. The remaining individuals in component I either recover at a rate $\gamma = \frac{1}{\text{infectious period}}$ or die at the rate e. The quarantined people either recover at the rate $\gamma' = \frac{1}{\text{period of quarantine}}$ or die at the rate e. Also, we have $beta = R_0\gamma$, were R_0 is the basic reproduction rate. Hence we have the following differential equations, which represent the dynamics of the disease.

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dE}{dt} = \beta IS - \varepsilon E$$
$$\frac{dI}{dt} = \varepsilon E - \gamma I - eI - qI$$
$$\frac{dQ}{dt} = qI - \gamma' Q - eQ$$
$$\frac{dR}{dt} = \gamma I + \gamma' Q$$
$$\frac{dD}{dt} = eI + eQ$$

Using the above model predictions are made about the spread in Kerala for a population of 1,80,000 individuals. The model shows a peak of the disease after about 150 days from the day the disease first appeared in Kerala. It shows a maximum 20, 000 infected cases and 150 deaths for a population having size 1,80,000. The total population of Kerala is now estimated as equal to 3.5 Crore. So multiplying the number of infected and the number of deaths by 190 ($\frac{35000000}{18000} = 190$ approximately), we can obtain an estimate of the total infected people as 38,00,000 and the total deaths as 28500. We can realize that these figures estimated are far from reality when we go through the actual data published by the government [13]. In the second paper [9] the authors have studied the trajectory of the disease during the first four months of the outbreak. They have applied the model to study the impact of the measures taken by the authorities to combat the spread of the disease using different models. In the model, the authors have used the generic class of SEIR model with modification by including two additional partitions of the population. The additional partitions are hospitalized cases and out-of-hospital cases. In addition to this, people traveling to the state are also taken into account. Different variations of the model are tested against the effect of reduced testing, no travel restrictions, no out of hospital measures, no in-hospital quarantine, and all measures removed. This study mainly focuses on the initial period of five months the predictions of the simulation almost matches with the actual data. But the real change in the spread of the disease in Kerala started after the period of the aforesaid study. So a revised analysis is needed using the models for the whole period of nine months for making some judgment about the performance of the model. In the third paper, Marbaniang [12] used a time series analysis model to make predictions on COVID-19 spread in Kerala. Further, a comparative study of the disease dynamics of the states of Maharashtra, Delhi, and Kerala, and India is made. Based on the COVID-19 data collected from the secondary sources from 16 March 2020 to 17 May 2020, the author predicted that over the next 20 days the confirmed cases of COVID-19 would increase to 2.45 lakhs in India, 93,709 in Maharashtra, 19,847 in Delhi, and 925 in Kerala. In the following section, we discuss the current scenario in the state concerning the data available from the sources of government and suggest ways to improve the performance of mathematical modeling using techniques of social network analysis.

3. IMPORTANCE OF SOCIAL NETWORK ANALYSIS IN THE PREDICTION AND CONTROL STRATEGIES OF COVID-19

We know that our society is divided into heterogeneous subgroups that have their internal characteristics of social relationships and contacts. These characteristic variations have a great impact on the spread of the disease. Members of some communities are very close to each other and their contacts are much higher than some other communities. For example student community, police force, group of employees working in an office or organization, a mob of political parties protesting against some issues, fishermen community on the coastal area, etc are very good examples of communities having strong and intense patter of social contacts. Some of these communities have a structure that lasts for a long time. But there are many vulnerable groups of people that form itself for a relatively short period and can cause a fast spread of the disease in the society. Group of people who assemble at public places for some time and spread out after some time is an example. Beyond all these groups, the family is an important strongly knit group in our society. All these groups must be handled in different ways in connection with the disease control and prevention strategies, according to its social network characteristics. Table 1 given below gives us a picture of the district-wise confirmed cases, deaths, and population density (No of persons per square kilometer) as of 8^{th} October 2020 [13]. The districts having low density shows a very small size in the confirmed cases compared to the districts having high density. Thiruvananthapuram, Alappuzha, Kozhikode, and Malapuram are the densely populated districts in the decreasing order. An increased number of the COVID-19 confirmed cases in these districts indicates that a high density of population increases the chance of an increase in social contacts. Moreover, we can see that the number of cases has significantly reduced in the districts having a low density of population (Idukki, Wayanad, Palakkad, and Pathanamthitta). Moreover, Thiruvananthapuram, Eranakulam, and Kozhikode are three big cities in Kerala. In these cities, the control of social contacts is almost impossible. This also might have caused an increase in the number in the corresponding districts. From this, it is clear that a direct application of the above discussed mathematical models without considering the varying possibility of human to human contact will mislead us in the strategic planning for disease control. An illogical analysis of the figures in the table will lead us to conclusions such as the district authorities in the districts of Idukki, Wayanad, Pathanamthitta, etc are very efficient in planning and implementing the strategies of disease control and treatment. The fact is that the medical facilities available in these three districts are at its lowest quality, especially in the government sector compared to the three populous districts (Thiruvananthapuram, Eranakulam, and Kozhikkodu).

Next, we discuss the importance and the effect of network topology in the planning and control of the spread of disease. Different networks have different internal structures. In some networks, every node is a member of some cycle. Some networks do not contain any cycle. In some networks, every node may be connected to almost all remaining nodes. Some networks could be divided into two subsets such that all paths from a member of one subset to the other subset must pass through one node. Such properties determine the structure of a network. An example of a network is given to explain it further.

The network given in the Fig. 1 is the Karate club network [6] redrawn using Pajek software. The network is famous for clashes between two of its members, labeled 34 and 1. These actors are the most powerful in the network. All other members are divided into groups under their leadership and split forever. The importance of the actors in the network is determined by a simple measure called the degree. The degree of a node in the network is the number of connections that the node has in the network. A node having the highest degree is the most powerful. The whole network is divided into two groups. There are two big communities in the network. One is centered around the node 34 and

TABLE 1. District-wise confirmed cases, deaths as of 8^{th} October, 2020 and population density (2011 census)

Sl. No.	Name of District	Confirmed Cases	Deaths	Population density
1	Thiruvananthapuram	41939	290	1510.87
2	Kollam	19580	71	1059.08
3	Pathanamthitta	9961	6	450.81
4	Alappuzha	18459	54	1499.61
5	Kottayam	13807	28	897.27
6	Idukki	4628	5	254.24
7	Eranakulam	25219	80	1070.80
8	Thrissur	19144	59	1027.53
9	Palakkadu	15737	39	627.15
10	Malapuram	30126	90	1156.71
11	Kozhikode	26783	84	1317.50
12	Wayanadu	4519	5	383.36
13	Kannur	15390	59	852.96
14	Kazarkodu	13558	60	654.90

the other is centered around node 1. Some smaller communities also exist in the network. There exist communities with center at the nodes 2, 3, 8, 33, etc. The nodes 24, 25, 28, and 32 form a community. Nodes 6, 7, and 17 form a small community. The nodes 3, 9, 14, 20, 28, 29, 31, 32, 33, etc are particularly important, because they have a key role in the network as mediators at the time of clashes. In the spreading of diseases, such nodes become very crucial. An epidemic that is spreading in one community can pass on to the other community only through these mediators. First, suppose node 1 is infected. If the contacts in the community continue for a long time the disease will be transmitted to all nodes in the community. At this level, the nodes having common connections with both communities become very critical. The disease will reach the other community through theses nodes. Breaking these contacts is very important for the healthy existence of the whole network. If nodes 1 or 34 are infected then the disease can spread in the entire group within there steps. This situation can be avoided by quarantining the members who have the maximum number of contacts in the network in the first instant. Thus the information about the structure of the network is an important part of the planning and implementation of the control strategies. In a real-world network, the situation is more complicated due to its complex structure and relatively big size.

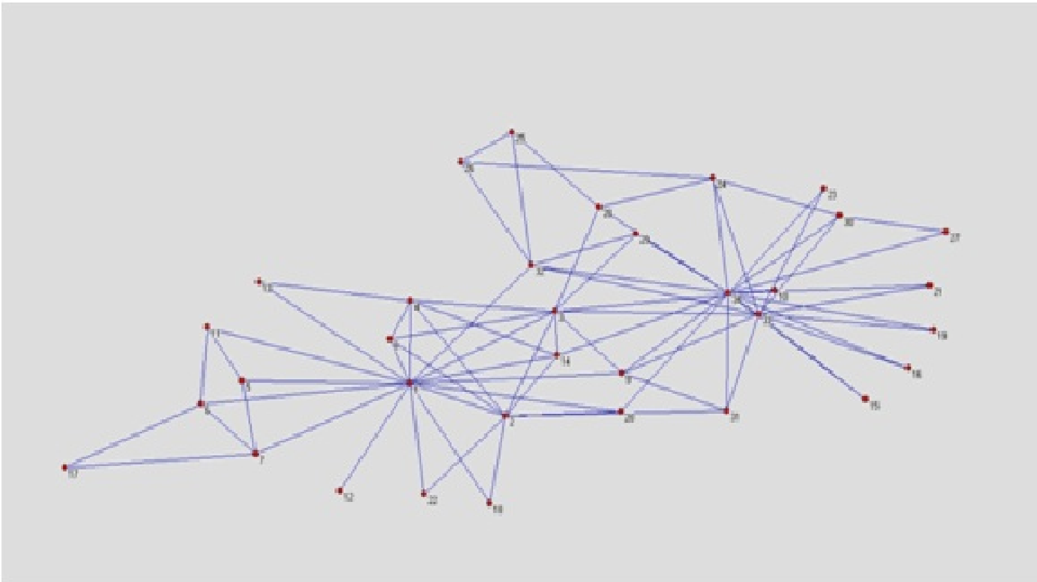

FIGURE 1. The Karate Club Network containing 34 nodes [6]

4. Conclusion

In this paper, we analyze the predictions made by the researchers based on the mathematical models used to study the spread of COVID-19 in Kerala, a southern state in India. The results obtained by the simulations of the models are compared to date representing the actual spread in the state. The reasons for the differences with the real data are discussed and possible ways to improve the performance of the modeling are given.

References

[1] https://www.who.int/csr/don/05-january-2020-pneumonia-of-unkown-cause-china/en/ accessed on 30th September 2020.

[2] https://www.who.int/csr/don/12-january-2020-novel-coronavirus-china/en/ accessed on 30th September 2020.

[3] https://www.who.int/csr/don/14-january-2020-novel-coronavirus-thailand/en/ accessed on 30th September 2020.

[4] https://www.who.int/csr/don/17-january-2020-novel-coronavirus-japan-ex-china/en/ accessed on 30th September 2020.

[5] https://en.wikipedia.org/wiki/List of districts of Kerala 30th September 2020.

[6] http://konect.uni-koblenz.de/networks/ucidata-zachary

[7] https://en.wikipedia.org/wiki/COVID-19/pandemic in Kerala

[8] P. ANOOP, AMBIKA, M. DIVAKARAN, N. MENON, U. DAS: *Comparison of some epidemiological models on Covid-19 spread in Kerala*, preprint available in ResearchGate, July 2020. accessed 30, September 2020.

[9] E. GOULT, S. SATHYENDRANATH, Åi. KOVAÄ■, A. ABDULAZIZ, N, MENON, G. GEORGE, C. EUNJIN KONG, T. PLATT: *COVID-19 in Kerala: analysis of measures and impacts*, Research Square, Accessed on 30-9-2020.

[10] R.J. KUMAR: *Nipah outbreak in Kerala-A network-based study*, Proceedings of the International conference, ICMMCMSE 2020, Alagappa University, Karaikudi, Tamil Nadu, 2020, to appear.

[11] R.J. KUMAR: *A Comparative Study of the SIR Prediction Models and Disease Control Strategies: A Case Study of the State of Kerala, India*, Computational Intelligence Methods in COVID-19: Surveillance, Prevention, Prediction and Diagnosis, Khalid Raza (Editor), Springer, 2021.

[12] S.P. MARBANIANG: *Forecasting the Prevalence of COVID-19 in Maharashtra, Delhi, Kerala, and India using an ARIMA model*, DOI: https://doi.org/10.21203/rs.3.rs-34555/v1

[13] https://dashboard.kerala.gov.in/

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