

A SEMI MARKOV PROCESS APPROACH IN EVALUATION OF SERVICE TIME OF BEACON MESSAGE TRANSMISSION IN THE DSRC MEDIUM ON VEHICULAR ENVIRONMENT

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ABSTRACT. On a highway each vehicle periodically tries to communicate with the RSU by transmitting beacon messages and other general messages like position, speed, destination etc. Beacon messages need to be given high priority and high-speed service since they are important for the broadcasting vehicle and also for the other vehicles within the stipulated radius. Instead of employing a Markov model, we employ a Semi Markov process and evaluate the service time transmission of the tagged state with a particular emphasis on the QOS of the beacon messages

1. INTRODUCTION

The exponential growth in the number of vehicles on the roads calls for a very high quality of service especially in terms of speed in broadcasting messages and in particular beacon messages. Each vehicle mounted with an OBU called the on the board unit, communicates with its neighboring vehicles in a particular radius through the road side units and also communicates with the driver giving him details of the other vehicles in its neighborhood. The main objective in

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this communication is the beacon messages which broadcast a request on any emergency for self or cautions the driver on the emergency of other vehicle. Hence a vehicular network is formed for a stipulated interval of time.

Thus, this well-organized and highly potential network, the Vehicular ad hoc network (VANET) broadcasts the position, speed, direction etc. of each vehicle in the given network range to and fro using the DSRC band via MAC at 5.9 GHz in every 300ms.

The three major challenges that the VANETs face are:

- (i) Reflections and signal scattering
- (ii) Vehicular mobility
- (iii) Strong ground reflections

Keeping quality of service primary and efficiency and low cost implementation without loss of data and delay in time, we employ a semi markov process in capturing the tagged states successful beacon message delivery. Using a semi markov process model with absorbing states, the service time distribution for the VANET is derived.

2. SEMI-MARKOV PROCESS

A semi-Markov process unlike a Markov process evaluates on every state continuously and just not only on the jumps. It is a stochastic process defined by Levy (1954) and Smith (1955) in 1950s, which spans over the complete time. Since every state has to be defined for a given time, we have the state S_t at any time, defined as $S_t = X_n$, for $t \in [T_n, T_{n+1})$. Hence the process $\{S_t\}_{t \geq 0}$ is a semi Markov process, with the times $0 = T_1 < T_2 < T_3 < \dots T_n < \dots$ as the jump times of $\{S_t\}_{t \geq 0}$, and $\tau_n = T_n - T_{n-1}$ are the sojourn times.

We see a detailed study on the MAC - level performance of the beacon message dissemination in the DSRC channel in saturated mode in [1], [2] and [3]. In [4] Bastani's model has a fixed time between the transmission of the already generated message and the generation of the next new message. This may delay the transmission or even contention of a beacon message generated. The standards [5], SAE J2735 DSRC Message Set defines the message types called as data frames and also specify that the users should comply to transmit messages in the 5.9 Ghz DSRC/WAVE medium.

3. THE MODEL DESCRIPTION

Beacon messages are generated periodically, of which most of them are updated and a few are not. In either case the newly generated messages are replaced by the older ones. We employ a semi Markov process model to capture the state of the messages and compute the probabilities of the service time. We define the service time as the time between the moments when the packet starts contending to catch the channel till the complete transmission of the packet. To capture the transmission of a tagged message, we employ the SMP with an absorbing state.

Notations

We define the following notation used in the computation of the service time probabilities.

$TA \rightarrow$ Packet transmission service time. That is the time taken by the tagged message to reach the absorbing state.

$F_{TA}(t) \rightarrow$ CDF of TA

$p_f \rightarrow$ Probability of the updated beacon message or the probability of a newly generated beacon message which replaces already generated message

$p_b \rightarrow$ Probability that the channel is busy.

$p_q \rightarrow$ Probability of the channel busy period during DIFS.

$r_q \rightarrow$ The probability that the channel is detected busy in DIFS time by the tagged vehicle after another neighbour of the tagged vehicle finishes transmission.

The beacon messages or the packets are generated periodically with a fixed interval of time denoted by τ . Also the replacement of the outdated messages occur only when the message generation time τ exceeds the service time, thus we have

$$(3.1) \quad p_f = P(TA > \tau) = 1 - P(TA \leq \tau) = 1 - F_{TA}(\tau).$$

We shall evaluate the probability of the updated message p_f , by deriving the cumulative distribution function of the service time using Laplace transform. We see from Figure 1, that two major cases arise on the transmission of the beacon message.

- a. The packet will start the service from the state CS_1 , if there is no replacement of the previously generated packet with probability $1 - p_f$.

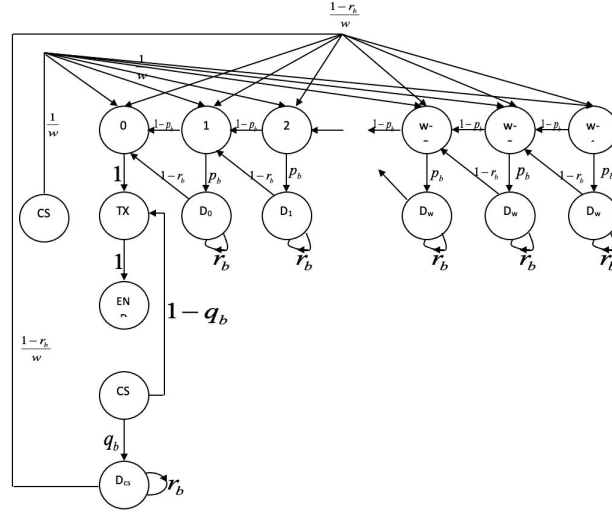


FIGURE 1.

- b. The packet will start the service from the state CS_2 , if the previously generated packet is replaced by the newly generated packet p_f .

Hence the transmission service can start either from CS_1 or CS_2 only. Let the time to reach the absorbing state from CS_1 and CS_2 be respectively TA_{CS_1} and TA_{CS_2} , and their corresponding probabilities be q_{CS_1} and q_{CS_2} .

Hence,

$$(3.2) \quad TA_j = \begin{cases} TA_{CS_1}, & \text{with probability } q_{CS_1} = 1 - p_f, \\ TA_{CS_2}, & \text{with probability } q_{CS_2} = p_f, \end{cases}$$

$$j \in \{CS_1, CS_2\}.$$

4. COMPUTATION OF SERVICE TIME

The packet transmission time, that is from the moment when the packet starts competing to access the channel till the completion of transmission, defines the service time and is given by,

$$(4.1) \quad TA = (1 - p_f) \cdot TA_{CS_1} + p_f \cdot TA_{CS_2}.$$

Since the sojourn time of the tagged message is deterministic in each state, we determine their Laplace - Stieltjes transform (LST):

$$(4.2) \quad L_{T_j}(s) = E(e^{-sT_j}) = \begin{cases} e^{-s \cdot A_1} & j = TX \\ e^{-s \cdot A_3} & j = CS_1, CS_2 \\ e^{-s \cdot A_4} & j = D_{CS} \\ e^{-s \cdot A_5} & j = D_0, D_1, \dots, D_{w-2} \\ 1 & j = 0 \\ e^{-s \cdot \sigma} & j = 1, 2, \dots, w - 1 \end{cases}$$

We shall evaluate TA_{CS_1} and TA_{CS_2} , Figure 1.

In accordance with the flow mechanism prescribed by the IEEE 802.11 DCF the model is designed in Figure 1. Also to have a better QoS and give priority to the beacon messages especially for the updated messages we adopt $D/G/1/1$ LIFO queue discipline, where D stands for the deterministic inter-arrival time of the messages, G stands for a general distribution for service time, the two 'ones' represent single server and single queue. Hence each vehicle has one $D/G/1/1$ LIFO interacting queue.

As seen above in (a) and (b), the beacon messages start transmission or contention from states CS_1 or CS_2 . If there is no replacement of the previously generated packet with probability $1 - p_f$, then the packet moves from state CS_1 to transmission state i.e., state TX with probability $1 - q_b$, else it moves to state D_{CS} with probability q_b and it stays in state D_{CS} with probability r_b and waits for the channel to be idle. Once the channel is sensed idle, after DIFS, the state moves from D_{CS} by choosing a back off counter from the contention window with probability $\frac{1-r_b}{w}$ and the countdown begins and once the countdown reaches zero the packet is transmitted. Each time the countdown is decremented by unity, and moves from state w_i to w_{i-1} with probability $1 - p_b$ and meanwhile, if the channel is sensed busy, it moves to the state $D_{w-(i-1)}$ with the probability p_b and then moves to state w_{i-1} with probability $1 - r_b$. Once the countdown counter reaches zero the packet is transmitted.

Similarly, if the previously generated packet is replaced by the newly generated packet then the transmission begins from the state CS_2 and chooses a counter value from the contention window for back-off with probability $\frac{1}{w}$ and the back-off counter is reduced by unity at a time. Once the state reaches zero, the packet is transmitted.

4.1. The probability to reach the state 0 from state 2.

$$\begin{aligned}
&= e^{-s\sigma} (1 - p_b) e^{-s\sigma} (1 - p_b) + e^{-s\sigma} (1 - p_b) e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \\
&\quad \cdot \left[1 + \sum_{l=1}^{\infty} (r_b)^l (e^{-sA_5})^l \right] + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \left[1 + \sum_{l=1}^{\infty} (r_b)^l (e^{-sA_5})^l \right] \\
&\quad \cdot \left\{ e^{-s\sigma} (1 - p_b) + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \left[1 + \sum_{l=1}^{\infty} (r_b)^l (e^{-sA_5})^l \right] \right\} \\
&= (e^{-s\sigma})^2 (1 - p_b)^2 + (e^{-s\sigma})^2 (1 - p_b) p_b e^{-sA_5} (1 - r_b) \left[\sum_{l=0}^{\infty} (r_b e^{-sA_5})^l \right] \\
&\quad + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \left[\sum_{l=0}^{\infty} (r_b e^{-sA_5})^l \right] \\
&\quad \cdot \left\{ e^{-s\sigma} (1 - p_b) + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \left[\sum_{l=0}^{\infty} (r_b e^{-sA_5})^l \right] \right\} \\
&= (e^{-s\sigma})^2 (1 - p_b)^2 + (e^{-s\sigma})^2 (1 - p_b) p_b e^{-sA_5} (1 - r_b) \theta \\
&\quad + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \theta \left\{ e^{-s\sigma} (1 - p_b) + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \theta \right\} \\
&= (e^{-s\sigma})^2 (1 - p_b)^2 + (e^{-s\sigma})^2 (1 - p_b) p_b e^{-sA_5} (1 - r_b) \theta \\
&\quad + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \theta \left\{ e^{-s\sigma} (1 - p_b) + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \theta \right\} \\
&= (e^{-s\sigma})^2 (1 - p_b)^2 + (e^{-s\sigma})^2 (1 - p_b) p_b e^{-sA_5} (1 - r_b) \theta \\
&\quad + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \theta \left\{ e^{-s\sigma} (1 - p_b) + e^{-s\sigma} p_b e^{-sA_5} (1 - r_b) \theta \right\}
\end{aligned}$$

Here

$$\begin{aligned}
\theta &= \left[\sum_{l=0}^{\infty} (r_b e^{-sA_5})^l \right] \\
&= (e^{-s\sigma} (1 - p_b))^2 + e^{-s\sigma} (1 - p_b) e^{-s(\sigma+A_5)} p_b (1 - r_b) \theta \\
&\quad + e^{-s\sigma} (1 - p_b) e^{-s(\sigma+A_5)} p_b (1 - r_b) \theta + (e^{-s\sigma})^2 (e^{-sA_5})^2 (1 - r_b)^2 p_b^2 \theta^2 \\
&= (e^{-s\sigma} (1 - p_b))^2 + e^{-s\sigma} (1 - p_b) e^{-s(\sigma+A_5)} p_b (1 - r_b) \theta \\
&\quad + e^{-s\sigma} (1 - p_b) e^{-s(\sigma+A_5)} p_b (1 - r_b) \theta + (e^{-s\sigma})^2 (e^{-sA_5})^2 (1 - r_b)^2 p_b^2 \theta^2 \\
&= (e^{-s\sigma} (1 - p_b))^2 + 2 [e^{-s\sigma} (1 - p_b)] [e^{-s(\sigma+A_5)} p_b (1 - r_b) \theta] \\
&\quad + [e^{-s(\sigma+A_5)} p_b (1 - r_b) \theta]^2
\end{aligned}$$

$$\begin{aligned}
 (4.3) \quad &= \left[e^{-s\sigma} (1 - p_b) + e^{-s(\sigma+A_5)} p_b (1 - r_b) \theta \right]^2 \\
 &= \left[e^{-s\sigma} (1 - p_b) + e^{-s(\sigma+A_5)} p_b (1 - r_b) \theta \right]^2
 \end{aligned}$$

4.2. To reach END from CS_1 .

$$\begin{aligned}
 L_{TA_{CS_1}}(s) &= e^{-sA_3} (1 - q_b) e^{-sA_1} \cdot 1 + e^{-sA_3} q_b e^{-sA_4} \sum_{l=0}^{\infty} (e^{-sA_4} r_b)^l \left(\frac{1 - r_b}{w} \right) \\
 &\quad \sum_{i=0}^{w-1} \left[(1 - p_b) e^{-s\sigma} + e^{-s(\sigma+A_5)} p_b (1 - r_b) \sum_{l=0}^{\infty} (r_b e^{-sA_5})^l \right]^i e^{-sA_1} \cdot 1 \\
 &= (1 - q_b) e^{-s(A_1+A_3)} + q_b e^{-s(A_1+A_3)} e^{-sA_4} \sum_{l=0}^{\infty} (e^{-sA_4} r_b)^l \left(\frac{1 - r_b}{w} \right) \\
 &\quad \sum_{i=0}^{w-1} \left[(1 - p_b) e^{-s\sigma} + e^{-s(\sigma+A_5)} p_b (1 - r_b) \sum_{l=0}^{\infty} (r_b e^{-sA_5})^l \right]^i \\
 (4.4) \quad L_{TA_{CS_1}}(s) &= e^{-s(A_1+A_3)} \left\{ (1 - q_b) + q_b e^{-sA_4} \sum_{l=0}^{\infty} (e^{-sA_4} r_b)^l \left(\frac{1 - r_b}{w} \right) \right. \\
 &\quad \left. \sum_{i=0}^{w-1} \left[(1 - p_b) e^{-s\sigma} + p_b (1 - r_b) e^{-s(\sigma+A_5)} \sum_{l=0}^{\infty} (r_b e^{-sA_5})^l \right]^i \right\}
 \end{aligned}$$

4.3. To reach END from CS_2 .

$$\begin{aligned}
 (4.5) \quad &L_{TA_{CS_2}}(s) \\
 &= e^{-s(A_1+A_3)} \frac{1}{w} \sum_{i=0}^{w-1} \left[(1 - p_b) e^{-s\sigma} + p_b (1 - r_b) e^{-s(\sigma+A_5)} \sum_{l=0}^{\infty} (r_b e^{-sA_5})^l \right]^i
 \end{aligned}$$

From equation (4.1)

$$\begin{aligned}
 (4.6) \quad L_{TA}(s) &= \text{Laplace-stieltjes transform of } TA \\
 &= (1 - p_f) L_{TA_{CS_1}}(s) + p_f L_{TA_{CS_2}}(s)
 \end{aligned}$$

Hence, the Laplace transform of $F_{TA}(t)$ denoted as $F_{TA}^*(s)$

$$\begin{aligned}
F_{TA}(t) &= \text{CDF of TA} \\
F_{TA}^*(s) &= \text{Laplace transform of } F_{TA}(t) \\
&= \int_0^{\infty} e^{-st} F_{TA}(t) dt \\
&= \left[\frac{e^{-st}}{-s} F_{TA}(t) \right]_0^{\infty} - \int_0^{\infty} \left(\frac{e^{-st}}{-s} \right) dF_{TA}(t) \\
&= 0 + \frac{1}{s} \int_0^{\infty} e^{-st} dF_{TA}(t) = \frac{1}{s} \text{ Laplace-Stieltjes transform of CDF} \\
&= \frac{1}{s} \text{ Laplace-Stieltjes transform of } L_{TA} \\
&= \frac{1}{s} [L_{TA}(s)] \\
&= \frac{1}{s} [(1 - p_f) L_{TA_{CS_1}}(s) + p_f L_{TA_{CS_2}}(s)]
\end{aligned}$$

So, it follows that

$$(4.7) \quad F^*(s) = (1 - p_f) \frac{L_{TA_{CS_1}}(s)}{s} + p_f \frac{L_{TA_{CS_2}}(s)}{s}.$$

As per the model description, the generated message will be replaced by the next message if the service time exceeds τ that is the message generation interval. Hence we have,

$$(4.8) \quad F_{TA}(t) = \begin{cases} (1 - p_f) \cdot L^{-1} \left(\frac{L_{TA_{CS_1}}(s)}{s} \right) + p_f \cdot L^{-1} \left(\frac{L_{TA_{CS_2}}(s)}{s} \right) & t \leq \tau \\ 1 & t > \tau \end{cases}$$

Clearly we see that the sojourn time of the states is deterministic and so the service time is a discrete variable.

5. CONCLUSION

In this paper, we attempt to model an SMP with absorbing states to evaluate the service time of a beacon message dissemination in the DSRC. We elaborately derive the state probabilities of the SMP using balance equations. Since safety being the primary concern on the roads, the efficiency of the beacon messages

dissemination must be very high on the vehicular environment. Hence updating the beacon messages are done periodically even when the new messages are not generated. The SMP is especially employed so that every state is taken into account to have a better QoS.

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