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FRACTIONAL KINETIC EQUATIONS INVOLVING GENERALIZED V-FUNCTION VIA LAPLACE TRANSFORM

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ABSTRACT. In this study, a new and further generalized form of the fractional kinetic equation involving the generalized V−function has been developed. We have discussed the manifold generality of the generalized V−function in terms of the solution of the fractional kinetic equation. Also, the graphical interpretation of the solutions by employing MATLAB is given. The results are very general in nature, and they can be used to generate a large number of known and novel results.

1. Introduction and Preliminaries

Fractional calculus (FC) is regarded as a beneficial tool for studying fractional order integrals and derivatives. Fractional calculus has been adopted and used in a variety of scientific and engineering fields. Fractional differential equations and their applications are very useful in many fields and have played a very important role in a wide range of applications in applied science, chemistry ,physics, engineering and biology. The kinetic equations are a collection of

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differential equations that explain the rate of change in a star's chemical composition for each order in terms of production and destruction reaction rates Fractional kinetic equations in various forms have been widely and successfully used to describe and solve a variety of important physics and astrophysics problems over the last several decades (see, for example, [1–8, 11, 19, 20] and the references therein).

The special functions and their applications can be found in the solutions of fractional integral and differential equations, as well as in a variety of other areas of mathematics and mathematical physics problems. The authors have developed a generalized form of the fractional kinetic equations as well as the V-function series in light of the usefulness and significance of the fractional kinetic equations in some astrophysical problems. The V-function series' broad generality will enable us to deduce several special cases of the main results.

So, we recall the differential fractional equation for the rate of reaction change $\mathfrak{M} = \mathfrak{M}(t)$,the destruction rate $d = d(\mathfrak{M})$ and the production rate $p = p(\mathfrak{M})$ that given by Haubold and Mathai [12] as the follows

$$\frac{d(\mathfrak{M})}{dt} = -d(\mathfrak{M}_{\mathrm{t}}) + p(\mathfrak{M}_{\mathrm{t}}), \tag{1.1}$$

where $\mathfrak{M}_{\mathrm{t}}$ is the function identified by

$$\mathfrak{M}_{\mathrm{t}}(t^*) = \mathfrak{M}(t - t^*), t^* > 0. \tag{1.2}$$

Neglecting the inhomogeneity in the quantity $\mathfrak{M}(t)$ that is the equation

$$\frac{d\mathfrak{M}}{dt} = -c_i \mathfrak{M}_i(t) \tag{1.3}$$

is part of the initial condition $\mathfrak{M}_{\mathrm{i}}(t = 0) = \mathfrak{M}_0$ is the number of density of index $\imath i \jmath$ at time $t = 0$.

The equation solution (1.3)is referred as

$$\mathfrak{M}_{\mathrm{i}}(t) = \mathfrak{M}_0 \; e^{-c_i t} \tag{1.4}$$

On the other hand, we can take

$$\mathfrak{M}(t) - \mathfrak{M}_0 = c_0 \; D_t^{-1} \mathfrak{M}(t), \tag{1.5}$$

where the ${}_0D_t^{-1}$ is the standard fractional integral operator.In addition, the fractional generalization for the standard kinetic equation(1.5) defined by Haubold and Mathai [12] as the form

$$\mathfrak{M}(t) - \mathfrak{M}_0 = c^{\gamma}\, {}_0D_t^{-\gamma}\, \mathfrak{M}(t), \tag{1.6}$$

where ${}_0D_t^{-\gamma}$ is the Riemann-Liouville fractional integral operator expressed as (see, Samko et al. [17])

$$ {}_0D_t^{-\gamma} f(t) = \frac{1}{\Gamma(\gamma)} \int_0^t (t-\tau)^{\gamma-1} f(\tau)d\tau,\ (t>0, \Re(\gamma)>0). \tag{1.7}$$

Haubold and Mathai [12] provide the equation solution (1.6) in the form:

$$\mathfrak{M}(t) = \mathfrak{M}_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(\gamma n+1)} (ct)^{\gamma n}. \tag{1.8}$$

Further, Saxena and Kalla [18] expressed the following fractional kinetic equation as the form

$$\mathfrak{M}(t) - \mathfrak{M}_0 f(t) = -c^{\gamma}\, ({}_0D_t^{-\gamma}\, \mathfrak{M})(t),\ (\Re(\gamma)>0, \tag{1.9}$$

where $\mathfrak{M}(t)$ denotes the density number of a given species at every time t, $\mathfrak{M}_0 = \mathfrak{M}(0)$ is a density number that species at time t = 0, c is a constant and $f \in L(0,\infty)$.

Now, applying laplace transform in Eq:(1.9), we get

$$L\{\mathfrak{M}(t);\tau\} = \mathfrak{M}_0 \frac{F(\tau)}{1+c^{\gamma}\tau^{-\gamma}} = \mathfrak{M}_0 \Big(\sum_{n=0}^{\infty} (-c^{\gamma})^n \tau^{-\gamma n}\Big) F(\tau), \qquad \Big(n \in \mathfrak{M}_0, \Big|\frac{c}{\tau}\Big| < 1\Big), \tag{1.10}$$

where the Laplace transform [21] is given by

$$F(\tau) = L\{\mathfrak{M}(t);\tau\} = \int_0^{\infty} e^{-\tau t} f(t)dt, \qquad (R(\tau)>0). \tag{1.11}$$

Recently, the $V-$function is defined by Kumar [14] as follows:

$$\begin{aligned} V(t) &= V_n^{a_\mu,h,b_v}(p,\xi,\zeta,\delta,q,K_\mu,A_v,B_w,\alpha,\beta,\rho;t) \\ &= \Im\left(\sum_{n=0}^{\infty} \frac{(-p)^n \prod_{\mu=1}^{j}\big[(a_\mu)_{n+K_\mu}\big](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\big[(b_v)_{n+A_v}\big]\prod_{w=1}^{u}\big[(h)_{\alpha n\rho+B_w}\big]}\right)\left(\frac{t}{2}\right)^{n\zeta+h\delta+q}, \end{aligned} \tag{1.12}$$

where

(i) $p,\zeta,\delta,q,\beta,\rho,k_\mu(\mu = 1,2,\ldots,j), A_v(v = 1,2,\ldots,i), B_w(w = 1,2,\ldots,u)$ are real numbers;
(ii) $i, j,$ and u are natural numbers;
(iii) $a_\mu, b_v \geq 1(\mu = 1,2,\ldots,j; v = 1,2,\ldots,i);$

(iv) $\alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t$ is a complex variable and $\Im$ is an arbitrary constant;

(v) the series on the RHS of (1.12) converges absolutely if $j < i$ or $j = i$ with $| p(t/2)^{\zeta} | \leq 1$.

2. Solution of generalized fractional kinetic equations

In this section, we use the technique of Laplace transform to solve the fractional kinetic equation associated with the generalized $V-$function.

Remark 2.1. *In this section, solutions for fractional kinetic equations are obtained in terms of the generalized Mittag-Leffler Function $E_{\alpha,h}(z)$ (see [15], which is described as the form:*

$$E_{\alpha,h}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + h)}, \ \Re(\alpha) > 0, \ \Re(h) > 0. \tag{2.1}$$

Theorem 2.1. *let $\mathfrak{R}(\gamma) > 0, d > 0; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i, j, u \in \mathbb{N}, \ a_\mu$, $b_v \geq 1, \alpha > 0, \mathfrak{R}(\xi) > 0, \mathfrak{R}(h) > 0, t > 0, and \ \Im > 0$ is an arbitrary constant; $(\mu = 1, \ldots, j), (v = 1, \ldots, i), (w = 1, \ldots, u)$ then the following equation:*

$$\mathfrak{N}(t) - \mathfrak{N}_0 \left(V_n^{a_\mu, h, b_v}(p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w, \alpha, \beta, \rho; t) \right) = -d^\gamma \ {}_0D_t^{-\gamma} \mathfrak{N}(t) \tag{2.2}$$

has a solution given by

$$\begin{aligned} \mathfrak{N}(t) = \Im \, \mathfrak{N}_0 & \left(\sum_{n=0}^{\infty} \frac{(-p)^n \prod_{\mu=1}^{j} \left[(a_\mu)_{n+K_\mu} \right] (h + \alpha n + \beta)^{-\xi}}{\prod_{v=1}^{i} \left[(b_v)_{n+A_v} \right] \prod_{w=1}^{u} \left[(h)_{\alpha n \rho + B_w} \right]} \right) \left(\frac{t}{2} \right)^{n\zeta + h\delta + q} \\ & \times \Gamma(n\zeta + h\delta + q + 1) \ E_{\gamma, (n\zeta + h\delta + q + 1)} \left(-d^\gamma t^\gamma \right). \end{aligned} \tag{2.3}$$

Proof. The Laplace transform of Riemann- Liouville fractional integral operator is presented as

$$L \left\{ {}_0D_t^{-\gamma} f(t); \tau) \right\} = (\tau)^{-\gamma} F(\tau), \tag{2.4}$$

where $F(\tau)$ is define in (1.11).

Now, after we apply the Laplace transform to both sides of equation (2.2) and using (2.4) we have

$$L\Big(\mathfrak{N}(t);\tau\Big) = \mathfrak{N}_0\ L\Big(V_n^{a_\mu,h,b_v}(p,\xi,\zeta,\delta,q,K_\mu,A_v,B_w,\alpha,\beta,\rho;t)\Big) - d^\gamma\ L\Big({}_0D_t^{-\gamma}\mathfrak{N}(t);\tau\Big), \tag{2.5}$$

that is

$$\mathfrak{N}(\tau) = \Im\,\mathfrak{N}_0 \int_0^\infty e^{-\tau t}\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\big[(a_\mu)_{n+K_\mu}\big](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\big[(b_v)_{n+A_v}\big]\prod_{w=1}^{u}\big[(h)_{\alpha n\rho+B_w}\big]}\right)\left(\frac{t}{2}\right)^{n\zeta+h\delta+q} dt - d^\gamma(\tau)^{-\gamma}\mathfrak{N}(\tau). \tag{2.6}$$

By interchanging the order of integration and summation in the equation (2.6), we obtain

$$\mathfrak{N}(\tau)\big[1+d^\gamma(\tau)^{-\gamma}\big] = \Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\big[(a_\mu)_{n+K_\mu}\big](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\big[(b_v)_{n+A_v}\big]\prod_{w=1}^{u}\big[(h)_{\alpha n\rho+B_w}\big]}\right)\left(\frac{1}{2}\right)^{n\zeta+h\delta+q} \times \int_0^\infty e^{-\tau t}t^{n\zeta+h\delta+q}dt. \tag{2.7}$$

Equation (2.7) leads to

$$\mathfrak{N}(\tau)\big[1+d^\gamma(\tau)^{-\gamma}\big] = \Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\big[(a_\mu)_{n+K_\mu}\big](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\big[(b_v)_{n+A_v}\big]\prod_{w=1}^{u}\big[(h)_{\alpha n\rho+B_w}\big]}\right)\left(\frac{1}{2}\right)^{n\zeta+h\delta+q} \times \frac{\Gamma(n\zeta+h\delta+q+1)}{\tau^{n\zeta+h\delta+q+1}} \tag{2.8}$$

Equation (2.8) leads to

$$\mathfrak{N}(\tau) = \Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\big[(a_\mu)_{n+K_\mu}\big](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\big[(b_v)_{n+A_v}\big]\prod_{w=1}^{u}\big[(h)_{\alpha n\rho+B_w}\big]}\right)\left(\frac{1}{2}\right)^{n\zeta+h\delta+q} \times \Gamma(n\zeta+h\delta+q+1)\left\{\tau^{-(n\zeta+h\delta+q+1)}\sum_{s=0}^{\infty}\Big[-\Big(\frac{\tau}{d}\Big)^{-\gamma}\Big]^s\right\} \tag{2.9}$$

Now, taking inverse Laplace transform on both sides of the equation (2.9), and using

$$L^{-1}\{\tau^{-\gamma};t\}=\frac{t^{\gamma-1}}{\Gamma(\gamma)},\ (\mathcal{R}(\gamma)>0) \tag{2.10}$$

we have

$$\begin{aligned} &L^{-1}\{\mathfrak{N}(\tau)\}\\ &=\Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\left[(a_\mu)_{n+K_\mu}\right](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{u}\left[(h)_{\alpha n\rho+B_w}\right]}\right)\left(\frac{1}{2}\right)^{n\zeta+h\delta+q}\\ &\times\Gamma(n\zeta+h\delta+q+1)\ L^{-1}\left(\sum_{s=0}^{\infty}(-1)^s(d)^{\gamma s}(\tau^{-(n\zeta+h\delta+q+\gamma s+1)}\right), \end{aligned} \tag{2.11}$$

that is

$$\begin{aligned} \mathfrak{N}(t)&=\Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\left[(a_\mu)_{n+K_\mu}\right](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{u}\left[(h)_{\alpha n\rho+B_w}\right]}\right)\left(\frac{1}{2}\right)^{n\zeta+h\delta+q}\\ &\times\Gamma(n\zeta+h\delta+q+1)\left(\sum_{s=0}^{\infty}(-1)^s(d)^{\gamma s}\frac{t^{(n\zeta+h\delta+q+\gamma s)}}{\Gamma(\gamma s+n\zeta+h\delta+q+1)}\right) \end{aligned} \tag{2.12}$$

$$\begin{aligned} \mathfrak{N}(t)&=\Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n prod_{\mu=1}^{j}\left[(a_\mu)_{n+K_\mu}\right](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{u}\left[(h)_{\alpha n\rho+B_w}\right]}\right)\left(\frac{t}{2}\right)^{n\zeta+h\delta+q}\\ &\times\Gamma(n\zeta+h\delta+m+1)\left(\sum_{s=0}^{\infty}(-1)^s\frac{(t^\gamma d^\gamma)^s}{\Gamma(\gamma s+n\zeta+h\delta+q+1)}\right). \end{aligned} \tag{2.13}$$

Now, we can Written equation (2.13) as

$$\begin{aligned} \mathfrak{N}(t)&=\Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\left[(a_\mu)_{n+K_\mu}\right](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{u}\left[(h)_{\alpha n\rho+B_w}\right]}\right)\left(\frac{t}{2}\right)^{n\zeta+h\delta+q}\\ &\times\Gamma(n\zeta+h\delta+q+1)\ E_{\gamma,(n\zeta+h\delta+q+1)}\ (-d^\gamma t^\gamma). \end{aligned} \tag{2.14}$$

□

Theorem 2.2. *Let* $\Re(\gamma)>0, d>0; p,\xi,\zeta,\delta,q,K_\mu,A_v,B_w\in\Re; b,i,j,u\in\mathbb{N}$*,* $a_\mu,b_v\geq 1,\alpha>0,\Re(\xi)>0,\Re(h)>0,t>0, and\ \Im>0$ *is an arbitrary constant;*

$(\mu=1,\ldots,j),(v=1,\ldots,i),(w=1,\ldots,u)$ *then the following equation:*

$$\mathfrak{N}(t)-\mathfrak{N}_0\left(V_n^{a_\mu,h,b_v}(p,\xi,\zeta,\delta,q,K_\mu,A_v,B_w,\alpha,\beta,\rho;d^\gamma t^\gamma)\right) = -\,d^\gamma\ {}_0D_t^{-\gamma}\mathfrak{N}(t) \tag{2.15}$$

has a solution given by

$$\mathfrak{N}(t)=\Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\left[(a_\mu)_{n+K_\mu}\right](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{u}\left[(h)_{\alpha n\rho+B_w}\right]}\right)\left(\frac{d^\gamma t^\gamma}{2}\right)^{n\zeta+h\delta+q} \times\Gamma(n\zeta\gamma+h\delta\gamma+\gamma q+1)\ E_{\gamma,(n\zeta+h\delta+q)\gamma+1}\left(-d^\gamma t^\gamma\right). \tag{2.16}$$

Theorem 2.3. *Let* $\Re(\gamma)>0,c>0,d>0,c\neq d;p,\xi,\zeta,\delta,q,K_\mu,A_v,B_w\in\Re;b,i,$ $j,u\in\mathbb{N},\ a_\mu,b_v\geq 1,\alpha>0,\Re(\xi)>0,\Re(h)>0,t>0, and\ \Im>0$ *is an arbitrary constant;* $(\mu=1,\ldots,j),(v=1,\ldots,i),(w=1,\ldots,u)$ *then the following equation:*

$$\mathfrak{N}(t)-\mathfrak{N}_0\left(V_n^{a_\mu,h,b_v}(p,\xi,\zeta,\delta,q,K_\mu,A_v,B_w,\alpha,\beta,\rho;d^\gamma t^\gamma)\right) = -\,c^\gamma\ {}_0D_t^{-\gamma}\mathfrak{N}(t) \tag{2.17}$$

has a solution given by

$$\mathfrak{N}(t)=\Im\,\mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-p)^n\prod_{\mu=1}^{j}\left[(a_\mu)_{n+K_\mu}\right](h+\alpha n+\beta)^{-\xi}}{\prod_{v=1}^{i}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{u}\left[(h)_{\alpha n\rho+B_w}\right]}\right)\left(\frac{d^\gamma t^\gamma}{2}\right)^{n\zeta+h\delta+q} \times\Gamma(n\zeta\gamma+h\delta\gamma+\gamma q+1)\ E_{\gamma,(n\zeta+h\delta+q)\gamma+1}\left(-c^\gamma t^\gamma\right). \tag{2.18}$$

Proof. Proof of Theorems 2.2 and 2.3 are similar to the proof of Theorem 2.1 so it is omitted here. □

3. Special cases

(i) If we choose $\mu=1,v=2,w=1,a_1=1,b_1=1,b_2=1,p=1,\xi=1,\zeta=2,\delta=1,q=0,k_1=0,A_1=0,A_2=0,B_1=0,\alpha=1,\beta=0,\rho=1\ and\ \Im=\frac{1}{\Gamma(h)}$, then the Theorems 1, 2 and 3 are reduces to the following form including the Bessel function $J_h(t)$ (see, eg, [10]).

Corollary 3.1. *Let* $\Re(\gamma) > 0, d > 0; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i, j, u \in \mathbb{N},\ a_\mu, b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0, and\ \Im > 0$ *is an arbitrary constant;* $(\mu = 1, \ldots, j), (v = 1, \ldots, i), (w = 1, \ldots, u)$ *then the following equation:*

$$\mathfrak{N}(t) - \mathfrak{N}_0\left(V_n^{1,h,1,1}(1,1,2,1,0,0,0,0,0,1,0,1;t)\right) = -d^\gamma\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\begin{aligned}\mathfrak{N}(t) =\ & \mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\Gamma(n+h+1)}\right)\left(\frac{t}{2}\right)^{2n+h} \\ & \times \Gamma(2n+h+1)\ E_{\gamma,(2n+h+1)}\left(-d^\gamma t^\gamma\right)\end{aligned}$$

Corollary 3.2. *Let* $\Re(\gamma) > 0, d > 0; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i,\ j, u \in \mathbb{N},\ a_\mu, b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0, and\ \Im > 0$ *is an arbitrary constant;* $(\mu = 1, \ldots, j), (v = 1, \ldots, i), (w = 1, \ldots, u)$ *then the following equation:*

$$\mathfrak{N}(t) - \mathfrak{N}_0\left(V_n^{1,h,1,1}(1,1,2,1,0,0,0,0,0,1,0,1;d^\gamma t^\gamma)\right) = -d^\gamma\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\begin{aligned}\mathfrak{N}(t) =\ & \mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\Gamma(n+h+1)}\right)\left(\frac{d^\gamma t^\gamma}{2}\right)^{2n+h} \\ & \times \Gamma(2n\gamma+h\gamma+1)\ E_{\gamma,(2n+h)\gamma+1}\left(-d^\gamma t^\gamma\right).\end{aligned}$$

Corollary 3.3. *Let* $\Re(\gamma) > 0, c > 0, d > 0, c \neq d; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i, j, u \in \mathbb{N},\ a_\mu, b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0, and\ \Im > 0$ *is an arbitrary constant;* $(\mu = 1, \ldots, j), (v = 1, \ldots, i), (w = 1, \ldots, u)$ *then the following equation:*

$$\mathfrak{N}(t) - \mathfrak{N}_0\left(V_n^{1,h,1,1}(1,1,2,1,0,0,0,0,0,1,0,1;d^\gamma t^\gamma)\right) = -c^\gamma\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\begin{aligned}\mathfrak{N}(t) =\ & \mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\Gamma(n+h+1)}\right)\left(\frac{d^\gamma t^\gamma}{2}\right)^{2n+h} \\ & \times \Gamma(2n\gamma+h\gamma+1)\ E_{\gamma,(2n+h)\gamma+1}\left(-c^\gamma t^\gamma\right).\end{aligned}$$

(ii) If we choose $\mu = 1, v = 2, w = 1, a_1 = 1, b_1 = 3/2, b_2 = 1, p = 1, \xi = 1, \zeta = 2, \delta = 1, q = 1, k_1 = 0, A_1 = 0, A_2 = 0, B_1 = 1/2, \alpha = 1, \beta = 1/2, \rho = 1 \; and \; \Im = \frac{1}{\Gamma(h)\Gamma(3/2)}$, then the Theorems 1, 2 and 3 are reduces to the following form including the Struve function $H_h(t)$(see, e.g., [10]).

Corollary 3.4. *Let* $\Re(\gamma) > 0, d > 0; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i, j, u \in \mathbb{N},\ a_\mu$, $b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0, and\ \Im > 0$ *is an arbitrary constant;* $(\mu = 1, \ldots, j), (v = 1, \ldots, i), (w = 1, \ldots, u)$ *then the following equation:*

$$\mathfrak{N}(t) - \mathfrak{N}_0\left(V_n^{1,h,3/2,1}(1,1,2,1,1,0,0,0,1/2,1,1/2,1;t)\right) = -d^\gamma\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\mathfrak{N}(t) = \mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n+\frac{3}{2})\Gamma(n+h+\frac{3}{2})}\right)\left(\frac{t}{2}\right)^{2n+h+1} \times \Gamma(2n+h+2)\ E_{\gamma,(2n+h+2)}\ (-d^\gamma t^\gamma).$$

Corollary 3.5. *Let* $\Re(\gamma) > 0, d > 0; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i, j, u \in \mathbb{N},\ a_\mu$, $b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0, and\ \Im > 0$ *is an arbitrary constant;* $(\mu = 1, \ldots, j), (v = 1, \ldots, i), (w = 1, \ldots, u)$ *then the following equation:*

$$\mathfrak{N}(t) - \mathfrak{N}_0\left(V_n^{1,h,3/2,1}(1,1,2,1,1,0,0,0,1/2,1,1/2,1;d^\gamma t^\gamma)\right) = -d^\gamma\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\mathfrak{N}(t) = \mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n+\frac{3}{2})\Gamma(n+h+\frac{3}{2})}\right)\left(\frac{d^\gamma t^\gamma}{2}\right)^{2n+h+1} \times \Gamma(2n\gamma+h\gamma+\gamma+1)\ E_{\gamma,(2n+h+1)\gamma+1}\ (-d^\gamma t^\gamma).$$

Corollary 3.6. *Let* $\Re(\gamma) > 0, c > 0, d > 0, c \neq d; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i, j, u \in \mathbb{N},\ a_\mu, b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0, and\ \Im > 0$ *is an arbitrary constant;* $(\mu = 1, \ldots, j), (v = 1, \ldots, i), (w = 1, \ldots, u)$ *then the following equation:*

$$\mathfrak{N}(t) - \mathfrak{N}_0\left(V_n^{1,h,3/2,1}(1,1,2,1,1,0,0,0,1/2,1,1/2,1;d^\gamma t^\gamma)\right) = -c^\gamma\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\mathfrak{N}(t) = \mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n+\frac{3}{2})\Gamma(n+h+\frac{3}{2})}\right)\left(\frac{d^{\gamma}t^{\gamma}}{2}\right)^{2n+h+1} \times \Gamma(2n\gamma+h\gamma+\gamma+1)\; E_{\gamma,(2n+h+1)\gamma+1}\;(-c^{\gamma}t^{\gamma}).$$

(iii) If we choose $w = 1, h = 1, j = J, i = I, p = -2, \xi = 1, \zeta = 1, \delta = 0, q = 0, k_\mu = 0, A_v = 0, B_1 = -1, \alpha = 1, \beta = -1, \rho = 1$ *and* $\Im = 1$, then the Theorems 1, 2 and 3 are reduces to the following form including generalized hypergeometric function (see, e.g., [9]),

Corollary 3.7. *Let* $\Re(\gamma) > 0, d > 0; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i, j, u \in \mathbb{N},\ a_\mu,$ $b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0,$ *and* $\Im > 0$ *is an arbitrary constant;* $(\mu = 1,\ldots,j), (v = 1,\ldots,i), (w = 1,\ldots,u)$ *then the following equation:*

$$V_n^{a_\mu,1,b_v}(-2,1,1,0,0,0,0,-1,1,-1,1;t) = -d^{\gamma}\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\mathfrak{N}(t) = \mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(a_1)_n,\ldots,(a_j)_n}{(b_1)_n,\ldots,(b_i)_n}\right)\left(\frac{t}{n!}\right)^{n} \times \Gamma(n+1)\; E_{\gamma,(n+1)}\;(-d^{\gamma}t^{\gamma}).$$

Corollary 3.8. *Let* $\Re(\gamma) > 0, d > 0; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i,\ j, u \in \mathbb{N},\ a_\mu,$ $b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0,$ *and* $\Im > 0$ *is an arbitrary constant;* $(\mu = 1,\ldots,j), (v = 1,\ldots,i), (w = 1,\ldots,u)$ *then the following equation:*

$$V_n^{a_\mu,1,b_v}(-2,1,1,0,0,0,0,-1,1,-1,1;d^{\gamma}t^{\gamma}) = -d^{\gamma}\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\mathfrak{N}(t) = \mathfrak{N}_0\left(\sum_{n=0}^{\infty}\frac{(a_1)_n,\ldots,(a_j)_n}{(b_1)_n,\ldots,(b_i)_n}\right)\left(\frac{d^{\gamma}t^{\gamma}}{n!}\right)^{n} \times\Gamma(\gamma n+1)\; E_{\gamma,(\gamma n+1)}\;(-d^{\gamma}t^{\gamma}).$$

Corollary 3.9. *Let* $\Re(\gamma) > 0, c > 0, d > 0, c \neq d; p, \xi, \zeta, \delta, q, K_\mu, A_v, B_w \in \Re; b, i,$ $j, u \in \mathbb{N},\ a_\mu, b_v \geq 1, \alpha > 0, \Re(\xi) > 0, \Re(h) > 0, t > 0,$ *and* $\Im > 0$ *is an arbitrary constant;* $(\mu = 1,\ldots,j), (v = 1,\ldots,i), (w = 1,\ldots,u)$ *then the following equation:*

$$V_n^{a_\mu,1,b_v}(-2,1,1,0,0,0,0,-1,1,-1,1;d^{\gamma}t^{\gamma}) = -c^{\gamma}\ {}_0D_t^{-\gamma}\mathfrak{N}(t)$$

has a solution given by

$$\mathfrak{N}(t) = \mathfrak{N}_0 \left(\sum_{n=0}^{\infty} \frac{(a_1)_n, \ldots, (a_j)_n}{(b_1)_n, \ldots, (b_i)_n} \right) \left(\frac{d^\gamma t^\gamma}{n!} \right)^n \times \Gamma(\gamma n + 1)\, E_{\gamma,(\gamma n+1)}\, (-c^\gamma t^\gamma).$$

(iv) If we choose $\mu = 1, v = 2, w = 1, a_1 = 1, b_1 = 1, b_2 = 1, p = 2, \xi = 1, \zeta = 1, \delta = 1, q = 0, k_1 = 0, A_1 = 0, A_2 = 0, B_1 = 0, \beta = 0, \rho = 1$ *and* $\Im = \frac{1}{\Gamma(h)}$, then the $V-$function (13) turns into the Wright generalized Bessel function (see, e.g., [10]),

$$V_n^{1,h,1,1}(2, 1, 1, 1, 0, 0, 0, 0, 0, \alpha, 0, 1; t) = J_h^\alpha(t).$$

(v) If we choose $\mu = 1, v = 1, w = 1, a_1 = 1, b_1 = 1, p = -2, \xi = 1, \zeta = 1, \delta = 0, q = 0, k_1 = 0, A_1 = 0, B_1 = -1, \beta = -1, \rho = 1$ *and* $\Im = \frac{1}{\Gamma(h)}$, then the $V-$function (13) turns into the Mittag-Leffler function (see, e.g., [13, 16]),

$$V_n^{1,h,1}(-2, 1, 1, 0, 0, 0, 0, -1, \alpha, -1, 1; t) = E_{\alpha,h}(t).$$

(vi) If we choose $\mu = 1, v = 2, w = 1, a_1 = 1, b_1 = (\tau + \epsilon + 3)/2, b_2 = (\tau - \epsilon + 3)/2, p = 1, \xi = 1, h = 1, \zeta = 2, \delta = \tau, q = 0, k_1 = 0, A_1 = 0, A_2 = 0, B_1 = -1, \alpha = 1, \beta = -1, \rho = 1$ *and* $\Im = 2^{\tau+1}/\{(\tau + \epsilon + 1)(\tau - \epsilon + 1)\}$,then the $V-$function (13) turns into the Lommel function (see, e.g., [10]),

$$V_n^{1,1,(\tau+\epsilon+3)/2,(\tau-\epsilon+3)/2}(1, 1, 2, \tau, 1, 0, 0, 0, -1, 1, -1, 1; t) = S_{\tau,\epsilon}(t).$$

4. Graphical interpretation

In this part, we plot the graphs of our kinetic equation solutions, which are established in Eqs. (2.3), (2.16) and (2.18). In each graph, we give four solutions of the results on the basis of assigning different values to the parameters, where values of the parameters are given as $\mu = w = a_1 = b_1 = b_2 = p = \xi = \delta = \alpha = d = c = \rho = 1, q = k_1 = A_1 = A_2 = B_1 = \beta = 0,\ v = \zeta = 2, \Im = \frac{1}{\Gamma(h)}, \gamma = 1.25, 1.5, 1.75, 2$ for solution of the Eq. (2.3) we plot four graphs of the Eq. (2.3) in figures 1234. Similarly, we plot the graph of the solution given in Eq. (2.16), which are in figure 5 and also give graph of the solution of Eq. (2.18) in figure 6.

Where values of the parameters are given as $\mu = w = a_1 = b_2 = p = \xi = \delta = q = d = c = \rho = h = \alpha = 1, k_1 = A_1 = A_2 = 0, B_1 = \beta = 1/2, v = \zeta = 2, b_1 = 3/2$, and $v\ \Im = \frac{1}{\Gamma(3/2)}$, for solutions of the Eqs. (2.3),(2.16) and (2.18) plotted in Figs. 789 respectively.

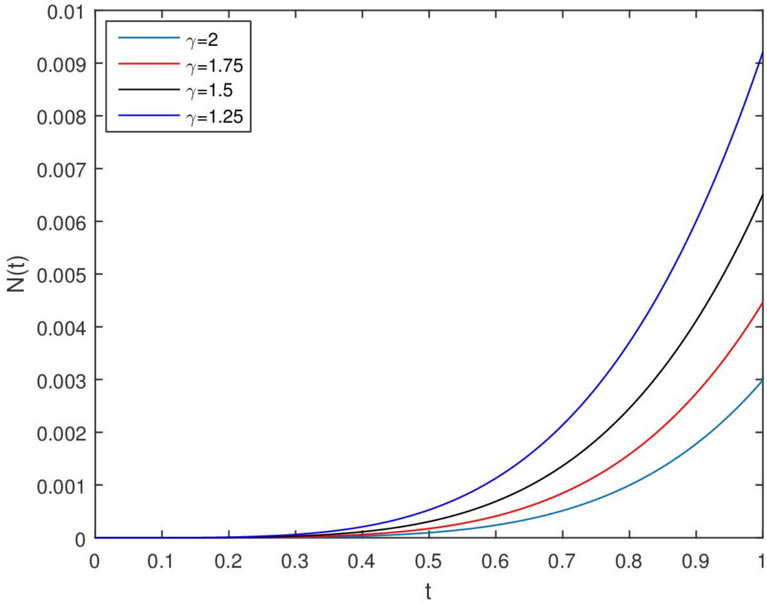

FIGURE 1. The Solution of (2.3) for $\mathfrak{N}(t)$ and $h = 1, \gamma = 1.25, 1.5, 1.75, 2$

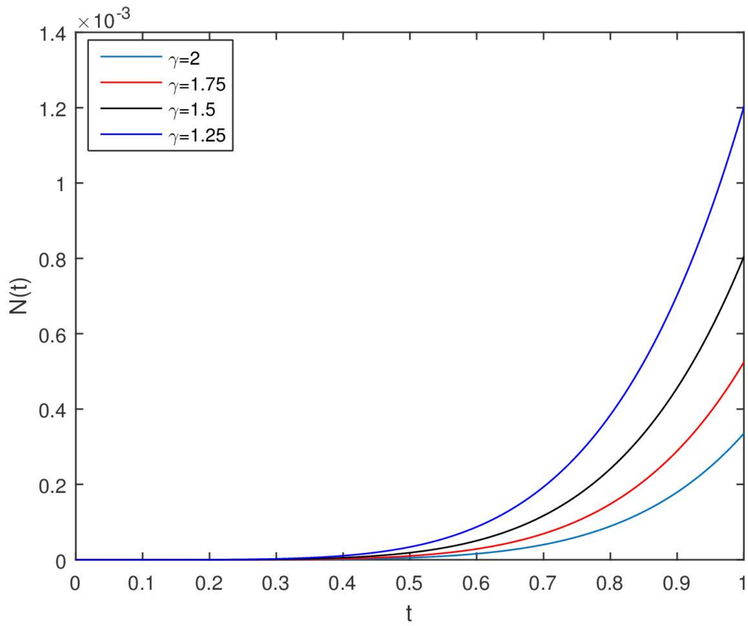

FIGURE 2. The Solution of (2.3) for $\mathfrak{N}(t)$ and $h = 2, \gamma = 1.25, 1.5, 1.75, 2$

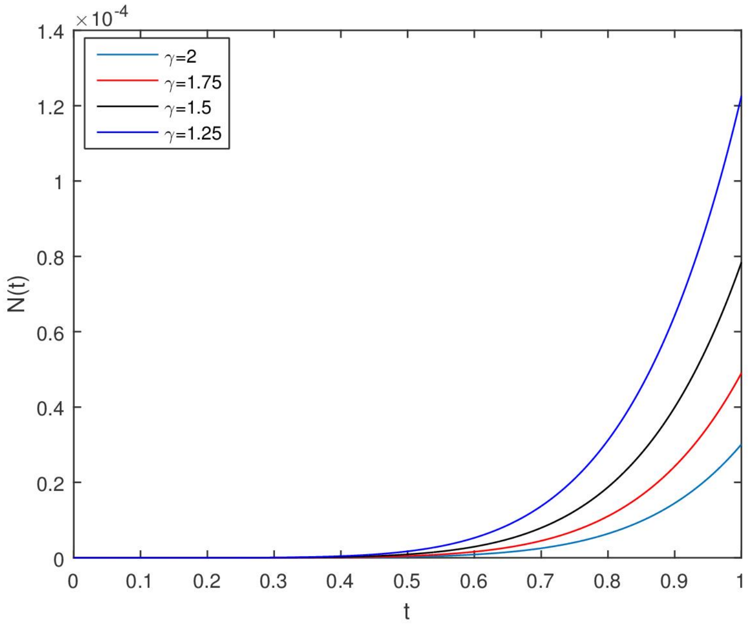

FIGURE 3. The Solution of (2.3) for $\mathfrak{N}(t)$ and $h = 3, \gamma = 1.25, 1.5, 1.75, 2$

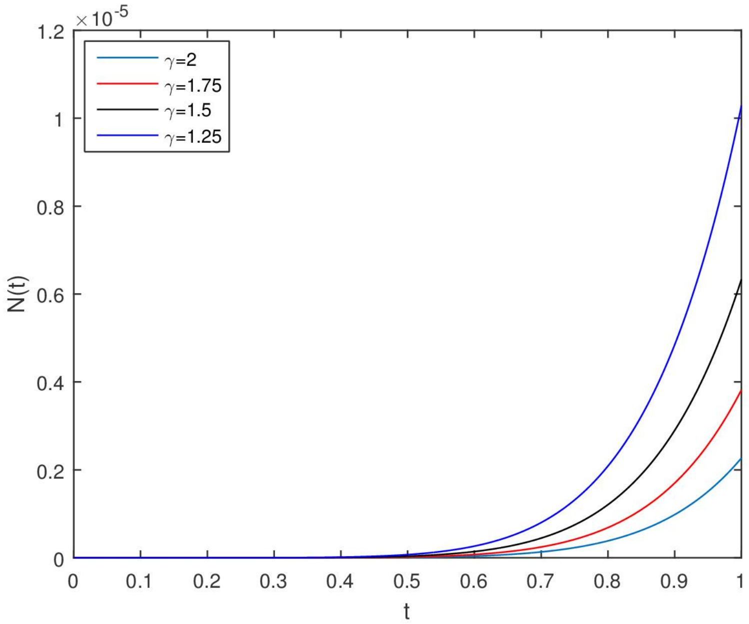

FIGURE 4. The Solution of (2.3) for $\mathfrak{N}(t)$ and $h = 4, \gamma = 1.25, 1.5, 1.75, 2$

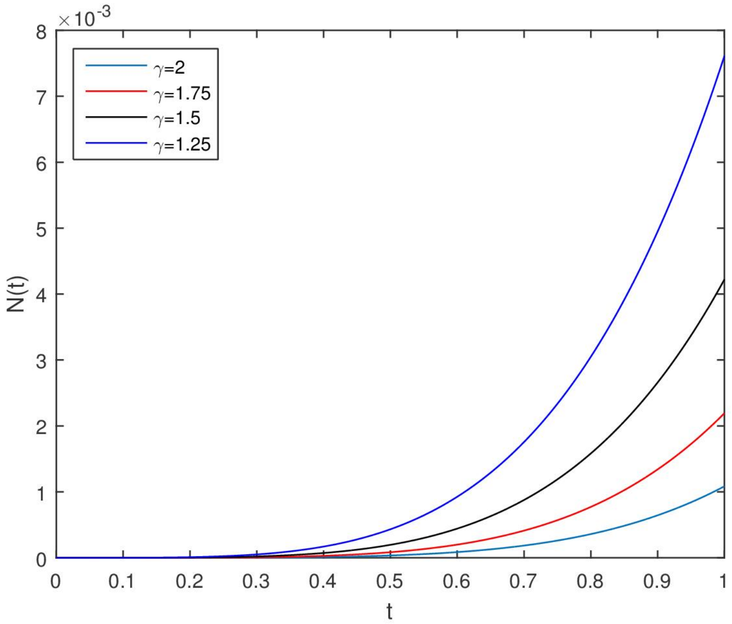

FIGURE 5. The Solution of (2.16) for $\mathfrak{N}(t)$ and $h = 1, \gamma = 1.25, 1.5, 1.75, 2$

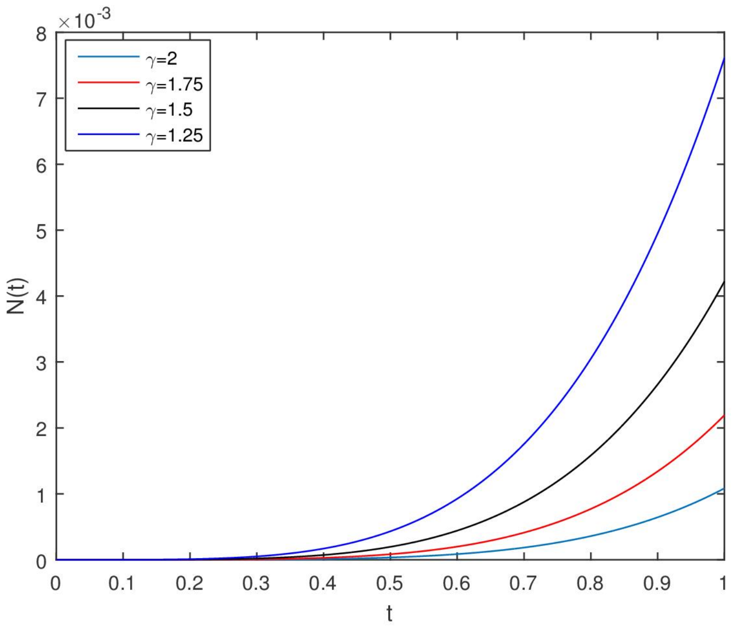

FIGURE 6. The Solution of (2.18) for $\mathfrak{N}(t)$ and $h = 1, \gamma = 1.25, 1.5, 1.75, 2$

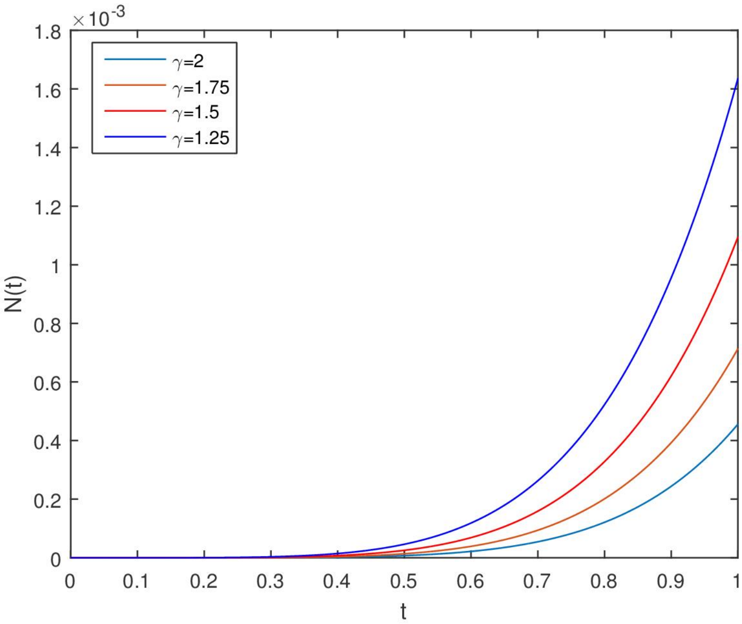

FIGURE 7. The Solution of (2.3) for $\mathfrak{N}(t)$ and $h = 1, \gamma = 1.25, 1.5, 1.75, 2$

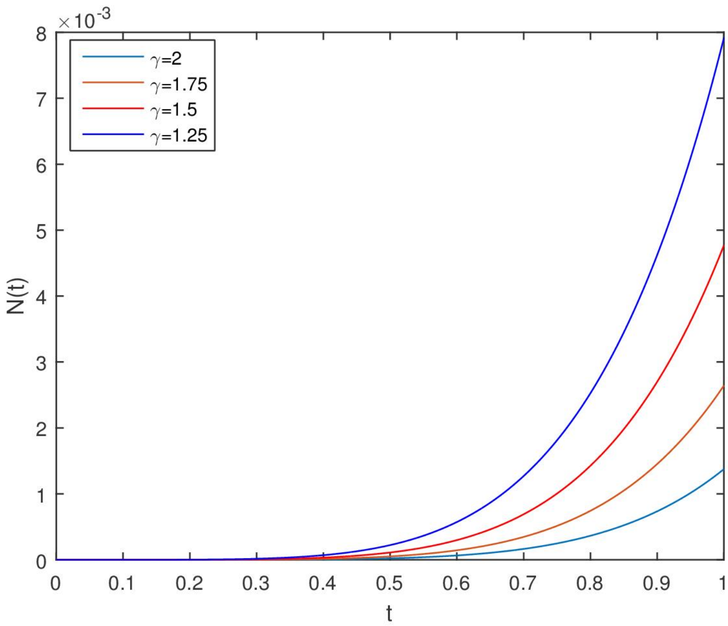

FIGURE 8. The Solution of (2.16) for $\mathfrak{N}(t)$ and $h = 1, \gamma = 1.25, 1.5, 1.75, 2$

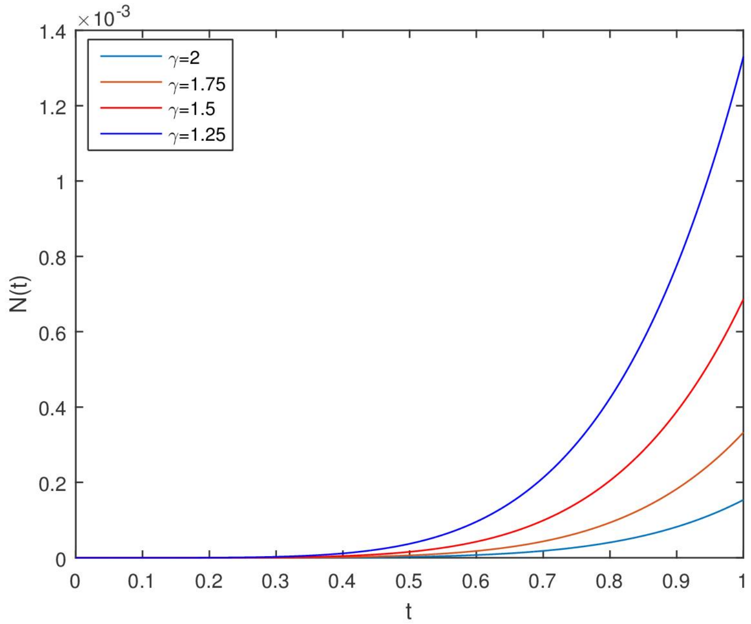

FIGURE 9. The Solution of (2.18) for $\mathfrak{N}(t)$ and $h = 1, \gamma = 1.25, 1.5, 1.75, 2$

5. CONCLUSION

We present a new fractional generalization of the standard kinetic equation as well as derive a solution for it in this paper. We can easily create various new and known fractional kinetic equations using the close relationship of the generalized $V-$function with several special functions. We also deduced from the graphical interpretation that the solutions to all three Eqs. (2.3), (2.16) and (2.18) for all positive values of the parameters $N(t)$ are Non-negative and $N(t) > 0$

REFERENCES

[1] P. AGARWAL, S.K. NTOUYAS, S. JAIN, M. CHAND, G .SINGH: *Fractional kinetic equations involving generalized k-bessel function via sumudu transform*, Alexandria Eng. J., 2017.

[2] W.F.S. AHMED, D.D. PAWAR: *Application of Sumudu Transform on Fractional Kinetic Equation Pertaining to the Generalized k-Wright Function*, Adv. Math., Sci. J. **9**(10) (2020), 8091- 8103.

[3] M. CHAND, R. KUMAR, S. BIR SINGH: *Certain Fractional Kinetic Equations Involving Product of Generalized k-Wright function*, Bulletin of the Marathwada Mathematical Society, **20**(1) (2019), 22-32.

[4] M. CHAND, ET AL: *Certain fractional integrals and solutions of fractional kinetic equations involving the product of S-function*, Mathematical Methods in Engineering. Springer, Cham, (2019), 213-244.

[5] V. B. L. CHAURASIA, D. KUMAR: *On the Solutions of Generalized Fractional Kinetic Equations*, Adv. Studies Theor. Phys., **4**(16) (2010), 773-780.

[6] V. B. L. CHAURASIA, S. C. PANDEY: *On the new computable solution of the generalized fractional kinetic equations involving the generalized function for the fractional calculus and related functions*, Astrophys. Space Sci. **317**(3) (2008), 213-219.

[7] G. A. DORREGO, D. KUMAR: *A Generalization of the Kinetic Eguation using the Prabhakar - type operators*, Honam Mathematical J. **39**(3) (2017), 401-416.

[8] B.K. DUTTA, L.K. ARORA, J. BORAH: *On the Solution of Fractional Kinetic Equation*, Gen. Math. Notes, **6**(1) (2011), 40-48.

[9] A. ERDELYI, W. MAGNUS, F. OBERHETTINGER, F. G. TRICOMI: *Higher Transcendental Functions, Vol. I. McGraw-Hill*, New York, 1953.

[10] A. ERDELYI, W. MAGNUS, F. OBERHETTINGER, F. G. TRICOMI: *Higher Transcendental Functions, Vol. II. McGraw-Hill*, New York, 1953.

[11] A. GUPTA, C. L. PARIHAR: *On solutions of generalized kinetic equations of fractional order*, Bol. Soc. Paran. Mat., **32**(1) (2014), 181-189.

[12] H.J. HAUBOLD, A.M. MATHAI: *The fractional kinetic equation and thermonuclear functions*, Astrophys. Space Sci. **273** (2000), 53-63.

[13] P. HUMBERT, R. P. AGARWAL: *Sur la fonction de Mittag-Leffler et quelques unesdeses generalisations*, Bull. Sci. Math. **77**(2) (1953), 180–185.

[14] V. KUMAR: *A general class of functions and N-fractional calculus*, J. Rajasthan Acad. Phys. Sci. **11**(3) (2012), 223-230.

[15] G. M. MITTAG-LEFFLER: *Sur la representation analytiqie d'une fonction monogene cinquieme note*, Acta Math., **29** (1905), 101-181.

[16] G. M. MITTAG-LEFFLER: *Sur la nouvelle fonction* $E_a(x)$, C.R. Acad. Sci. Paris **137** (1903), 554–558.

[17] S. G. SAMKO, A. A. KILBAS, O. I. MARICHEV: *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach, New York, 1993.

[18] R.K. SAXENA, S.L. KALLA: *On the solutions of certain fractional kinetic equations*, Applied Mathematics and Computation **199** (2008), 504-511.

[19] R. K. SAXENA, A. M. MATHAI, H. J. HAUBOLD: *Solutions of certain fractional kinetic equations and a fractional diffusion equation*, Journal Of Mathematical Physics **51** (2010), art.id. 103506.

[20] R.K. SAXENA, A.M. MATHAI, H.J. HAUBOLD: *On generalized fractional kinetic equations*, Physica A **344** (2004), 657-664.

[21] M. R. SPIEGEL: *STheory and Problems of Laplace Transforms*, Schaums Outline Series, McGraw-Hill, New York, 1965.

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