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EXISTENCE OF PERIODIC SOLUTIONS OF NONAUTONOMOUS PREDATOR-PREY MODEL WITH BEDDINGTON-DEANGELIS FUNCTIONAL RESPONSE AND TIME DELAY

S. Mahalakshmi[1] and V. Piramanantham

ABSTRACT. In this paper we establish some easily verifiable sufficient conditions for the existence of periodic solutions of nonautonomous Predator-Prey Model with Beddington-DeAngelis Functional response and time delay using Mowhins Coincidence degree method.

1. INTRODUCTION

The study of qualitative analysis of the biological models is one of the central themes in Mathematical Biology, espcially the existence of positive periodic solution of those systems is inevitable because the effects of the periodically changing environment occur in the biological system. In recent years there have been lot of interest in analysing Interections between the predator and prey in ecological theory.

In the Predator-prey models, one of the most important component in the Predator-Prey relationship is the predator's rate of feeding upon prey, known as the Predator' functional response. The functional responses may depend on the

[1]*corresponding author*

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number of Predator and prey. Generally the functional response can be classified into two types: prey- dependent and predator-dependent.

Prey dependent means that the functional response is only a function of the prey's density, called Holling type functional responses, However the prey-dependent functional responses fail to model the interference among predators, and have been facing challenges from the biology communities. Predator-dependent means that the functional response is a function of both the prey's and the predator's densities which provide better description of predator feeding over a range of predator-prey abundances. This type of functional response is called the Beddington - DeAngelis type

The predator-prey system with the Beddington -DeAngelis functional response was originally proposed by Beddington [1] and DeAngelis et al. [2], independently. The dynamics of this model is described by differential equations in the form

$$\begin{aligned} \dot{x} &= rx(t)\left[1-\frac{x(t)}{K}-\frac{bx(t)y(t)}{1+nx(t)+my(t)}\right], \\ \dot{y} &= y(t)\left[-d+\frac{fbx(t)}{1+nx(t)+my(t)}\right]. \end{aligned} \tag{1.1}$$

Numerous results are found for various predator-prey models with/without delay on investigating the existence of periodic solutions and a diversity of solutions [3–9, 11].

In this paper we explore the existence of postiive periodic solutions of the following delayed predaotr-prey system with Beddington-DeAngelis functional response and

$$\begin{aligned} &x_1'(t) = \\ &\quad x_1(t)\left[r_1(t)-a_1(t)x_1(t-\tau_1(t))-\frac{b_1(t)x_2(t)}{m(t)+m_1(t)x_1(t)+m_2(t)x_2(t)}\right], \\ &x_2'(t) = \\ &\quad x_2(t)\left[-r_2(t)-a_2(t)x_2(t-\tau_2(t))+\frac{b_2(t)x_1(t-\tau_3(t))}{m(t)+m_1(t)x_1(t)+m_2(t)x_2(t)}\right], \end{aligned} \tag{1.2}$$

with the initial conditions

$$\begin{aligned} &x_i(\theta)=\phi_i(\theta_i), \theta\in[-\tau,0], \phi_i(0)>0, \\ &\phi_i\in C([\tau,0],\mathbb{R}), i=1,2. \end{aligned} \tag{1.3}$$

The coincidence degree method has been applied to establish the sufficient conditions for the existence of positive periodic solution of the system (1.2).

For the sake of convenience, we assume the following properties of the known functions involved in the system (1.2).

(A_1) the quantities $\tau_1(t), \tau_2(t)$ and $\tau_3(t)$ are negative periodic continuous function with a common period $\omega > 0$ and $\tau = \max\limits_{t\in[0,\omega]}\{\tau_i(t), i = 1, 2\}$.

(A_2) the functions r_i $a_i, b_i(i = 1, 2)$ and $m(t), m_1(t), m_2(t)$ are positive continuous ω periodic functions.

The main objective of this paper is to obtain sufficient conditions for the existence of positive periodic solutions of the system (1.2) by using the Mowhin's coincidence degree theory.

2. Existence of positive periodic solutions

In this section, we prove the existence of solutions of periodic solution. For the reader's convenience, we provide some notations and definitions and also we first prepare the functional analytic settings.

Let X, Z be normed linear spaces, L : Dom $L \subset X \to Z$ be a linear transformation, and $N : X \to Z$ be a continuous function. The map L is knows as a Fredholm map of index zero if $\dim$ Ker L = codim Im $L < +\infty$ and Im L is closed in Z. If L is a Fredholm mapping of index zero there exist continuous projectors $P : X \to X$, and $Q : Y \to Y$ such that Im P = Ker L, Ker Q = Im L = Im $(I - Q)$. This implies that the restriction $L|_p$ of L to Dom $L \cap$ Ker P : $(I - P)X \to$ Im L is invertible. The inverse of L_P is denoted by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact. Since Im Q is isomorphic to Ker L, there exists an isomorphism J : Im $A \to$ Ker $L.$

We will make some notations and definitions which will be used in the proof of the main theorem

$$\bar{f} = \frac{1}{\omega}\int_0^\omega f(t)dt,$$

$$f^L = \sup_{t\in[0,\omega]} |f(t)|, \quad f^M = \inf_{t\in[0,\omega]} |f(t)|.$$

Lemma 2.1 (Ascoli-Arzela Theorem). *The set $\mathcal{F} \subset PC_\omega$ is relatively compact if, and only if,*

(i) *$\mathcal{F}$ is bounded, that is,* $\|u\| = \sup_{t\in[0,\omega]} \|u(t)\| \leq M$ *for each* $u \in \mathcal{F}$;
(ii) *$\mathcal{F}$ is quasi-equicontinuous.*

Our existence theorem for periodic solution of the equation (1.2) is proved with the help of the following theorem of Gaines and Mawhin [10].

Theorem 2.1. *Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Suppose that*

(i) *for each* $\lambda \in (0,1)$,*every solution x of* $Lx \neq \lambda Nx$ *is such that*$x \notin \Omega$*;*
(ii) *for each* $\lambda \in \partial\Omega \cap KerL, QNx \neq 0$;
(iii) $\deg\{JQN, \Omega \cap KerL, 0\} \neq 0$.

Then $Lx = Nx$ has atleast one solution lying in Dom $L \cap \overline{\Omega}$.

Theorem 2.2. *The system* (1.2) *has atleast one positive ω-periodic solution provided that*

$$\left(\frac{\bar{r}_2}{\bar{b}_2/m^L}\right) - 2\bar{r}_1\omega - \ln\left(\frac{\bar{a}_2 m_2^M}{\bar{b}_2 - \bar{r}_2 m_2^M}\right) - 4\omega\bar{r}_2 > 0$$

and the algebraic equations

$$\bar{r}_1 - \bar{a}_1\exp(u_1) - \frac{1}{\omega}\int_0^\omega \frac{\mu b_1\exp(u_2)}{m(t) + m_1(t)\exp(u_1) + m_2(t)\exp(u_2)}dt = 0,$$
$$-\bar{r}_2 - \bar{a}_2\exp(u_2) + \frac{1}{\omega}\int_0^\omega \frac{b_2\exp(u_1)}{m(t) + m_1(t)\exp(u_1) + m_2(t)\exp(u_2)}dt = 0,$$

has finite real-valued solution (u_1^*, u_2^*).

Proof. Let $x_1(t) = \exp(u_1(t)), x_2(t) = \exp(u_2(t))$. Then we obtain the following equivalent system:

$$\begin{aligned} u_1'(t) &= r_1(t) - a_1(t)\exp(u_1(t-\tau_1(t))) \\ &\quad - \frac{b_1(t)\exp(u_2(t))}{m(t) + m_1\exp(u_1(t)) + m_2(t)\exp(u_2(t))}, \\ u_2'(t) &= -r_2(t) - a_2(t)\exp(u_2(t-\tau_2(t)) \\ &\quad + \frac{b_2(t)\exp(u_1(t-\tau_3(t)))}{m(t) + m_1\exp(u_1(t)) + m_2(t)\exp(u_2(t))}. \end{aligned} \tag{2.1}$$

It is easy to see that if system (2.1) has one ω-Periodic solution $(u_1^*(t), u_2^*(t))$ then the corresponding $x^*(t) = (x_1^*(t), x_2^*(t))^T$ is a periodic solution of (1.2).

Therefore, to complete proof, it suffices to show that the system (2.1) has atleast one ω periodic solution. Let

$$X = \{u = (u_1, u_2)^T \in C_\omega([0,\omega], \mathbb{R}^2) : u_i(t+\omega) = u_i(t), i = 1, 2\}, \quad Z = X.$$

Let us define for any $u \in Z$,

$$\|u\| = \max_{t\in[0,\omega]} |u_1(t)| + \max_{t\in[0,\omega]} |u_2(t)|.$$

Then X and Z are Banach spaces. Set L :Dom $L \cap X \to Z$,

$$L(u) = u'(t),$$

where

$$\text{Dom } L = \{u = (u_1, u_2)^T \in PC_\omega(\mathbb{R}, \mathbb{R}^2) : u_i \in PC_\omega, i = 1, 2\},$$

and $N : X \to Z$,

$$N\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} =$$
$$\begin{pmatrix} r_1(t) - a_1(t)\exp(u_1(t-\tau_1(t)) - \dfrac{b_1(t)\exp(u_2(t))}{m(t) + m_1\exp(u_1(t)) + m_2(t)\exp(u_2(t))} \\ -r_2(t) - a_2(t)\exp(u_2(t-\tau_2(t)) + \dfrac{b_2(t)\exp(u_1(t-\tau_3(t))}{m(t) + m_1\exp(u_1(t)) + m_2(t)\exp(u_2(t))} \end{pmatrix}.$$

$P : X \to X,\ P((u_1, u_2)^T) = (\overline{u_1}, \overline{u_2})^T$ and $Q : Z \to Z$,

$$Q\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\omega}\int_0^\omega \mathrm{u}_1(t)\,\mathrm{d}t \\ \frac{1}{\omega}\int_0^\omega \mathrm{u}_2(t)\,\mathrm{d}t \end{pmatrix}.$$

It is easy to see that

$$\text{Ker}L = \left\{u \in X : \exists\, c \in \mathbb{R}^2,\ (u_1(t), u_2(t)) = c,\ \text{for } t \in \mathbb{R}\right\}$$

and

$$\text{Im}L = \left\{u \in Z : \exists\, u \in \text{Dom}L,\ \int_0^\omega u(s)\mathrm{d}s = 0\right\}.$$

Since Im L is closed in Y and dimker L = codimIm $L = 2, L$ is a Fredholm mapping of index zero. Moreover, the generalized inverse (to L) K_p : Im $L \to$ Ker $P \cap$ Dom L is

$$K_P(u)=\int_0^t \mathrm{u}(s)\,\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^t \mathrm{u}(s)\,\mathrm{d}s\ \mathrm{d}t.$$

Then direct computation gives us

$$QN\begin{pmatrix}u_1\\u_2\end{pmatrix}=\begin{pmatrix}\frac{1}{\omega}\int_0^{\omega}\Big[r_1(t)-a_1(t)\exp(u_1(t-\tau_1(t)))-\dfrac{b_2(t)\exp(u_2(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\Big]\mathrm{d}t\\ \dfrac{1}{\omega}\int_0^{\omega}\Big[-r_2(t)-a_2(t)\exp(u_1(t-\tau_2(t)))+\dfrac{b_2(t)\exp(u_1(t-\tau_3(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\Big]\mathrm{d}t\end{pmatrix}$$

and

$$\begin{aligned}&K_p(I-Q)N\begin{pmatrix}u_1\\u_2\end{pmatrix}\\&=\begin{pmatrix}\int_0^t\Big[r_1(s)-a_1(s)\exp(u_1(s-\tau_1(s))-\dfrac{b_1(s)\exp(u_2(s))}{m(s)+m_1\exp(u_1(s))+m_2(s)\exp(u_2(s))}\Big]\mathrm{d}s\\ \int_0^t\Big[-r_2(s)-a_2\exp(u_2(s-\tau_2(s))+\dfrac{b_2(s)\exp(u_1(s-\tau_3(s))}{m(s)+m_1\exp(u_1(s))+m_2(s)\exp(u_2(s))}\Big]\mathrm{d}s\end{pmatrix}\\&-\begin{pmatrix}\dfrac{1}{\omega}\int_0^{\omega}\int_0^t\Big[r_1(s)-a_1(s)\exp(u_1(s-\tau_1(s))-\dfrac{b_1(s)\exp(u_2(s))}{m(s)+m_1\exp(u_1(s))+m_2(s)\exp(u_2(s))}\Big]\mathrm{d}s\mathrm{d}t\\ \dfrac{1}{\omega}\int_0^{\omega}\int_0^t\Big[-r_2(s)-a_2\exp(u_2(s-\tau_2(s))+\dfrac{b_2(s)\exp(u_1(s-\tau_3(s))}{m(s)+m_1\exp(u_1(s))+m_2(s)\exp(u_2(s))}\Big]\mathrm{d}s\mathrm{d}t\end{pmatrix}\\&-\begin{pmatrix}(\dfrac{t}{\omega}-\dfrac{1}{2})\int_0^{\omega}\Big[r_1(t)-a_1(t)\exp(u_1(t-\tau_1(t))-\dfrac{b_1(t)\exp(u_2(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\Big]\mathrm{d}t\\ (\dfrac{t}{\omega}-\dfrac{1}{2})\int_0^{\omega}\Big[-r_2(t)-a_2\exp(u_2(t-\tau_2(t))+\dfrac{b_2(t)\exp u_1(t-\tau_3(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\Big]\mathrm{d}t\end{pmatrix}.\end{aligned}$$

Clearly, QN and $K_p(I-Q)N$ are continuous. Furthermore, it follows from Lemma 2.1 that $QN(\overline{\Omega})$ and $K_p(I-Q)N(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega\subset X$. Therefore, N is L- compact on $\overline{\Omega}$ for any open bounded set $\Omega\subset X$.

In the following, we consider the operator equation $Lu=\lambda Nu,\ \lambda\in(0,1)$, that is,

$$\begin{aligned}u_1'(t)&=\lambda\Big[r_1(t)-a_1(t)\exp(u_1(t-\tau_1(t))\\&\quad-\frac{b_2(t)\exp(u_2(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\Big],\\u_2'(t)&=\lambda\Big[-r_2(t)-a_2(t)\exp(u_2(t-\tau_2(t))\\&\quad+\frac{b_2(t)\exp(u_1(t-\tau_2(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\Big].\end{aligned}\tag{2.2}$$

Integration of both sides of the system (2.2) from 0 to ω gives

$$\int_0^\omega \left[a_1 \exp(u_1(t-\tau_1(t)) + \frac{b_1(t)\exp(u_2(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\right] \mathrm{d}t \tag{2.3}$$
$$= \int_0^\omega r_1(t)dt = \omega \overline{r}_1$$

(2.4)
$$\int_0^\omega \left[\frac{b_2(t)\exp(u_1(t-\tau_3(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))} - a_2(t)\exp(u_2(t-\tau_2(t))\right] \mathrm{d}t$$
$$= \int_0^\omega r_2(t)dt = \omega \overline{r}_2.$$

It follows from (2.2) that

$$\begin{aligned} \int_0^\omega |u_1'(t)|dt < \int_0^\omega &\Big[r_1(t) + a_1(t)\exp(u_1(t-\tau_1(t)) \\ &+ \frac{b_1(t)\exp(u_2(t)}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\Big] dt = 2\omega\overline{r}_1 \end{aligned} \tag{2.5}$$

$$\begin{aligned} \int_0^\omega |u_2'(t)|dt < \int_0^\omega &\Big[r_2(t) + a_2(t)\exp(u_2(t-\tau_2(t)) \\ &+ \frac{b_2(t)\exp(u_1(t-\tau_3(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\Big] dt = 2\omega\overline{r}_2. \end{aligned} \tag{2.6}$$

Since $(u_1, u_2)^T \in X$, there exists $\xi_i, \eta_i \in [0, \omega], i = 1, 2$ such that

$$\begin{aligned} u_1(\xi_1) &= \min_{t\in[0,\omega]} u_1(t), \qquad & u_1(\eta_1) &= \max_{t\in[0,\omega]} u_1(t), \\ u_2(\xi_2) &= \min_{t\in[0,\omega]} u_2(t), \qquad & u_2(\eta_2) &= \max_{t\in[0,\omega]} u_2(t). \end{aligned} \tag{2.7}$$

It follow from (2.3) and (2.7) that

$$\int_0^\omega a_1(t)\exp(u_1(t-\tau_1(t))dt \quad < \quad \overline{r}_1\omega, \tag{2.8}$$

which gives

$$u_1(\xi_1) < \ln\frac{\overline{r}_1}{\overline{a}_1} := L_1. \tag{2.9}$$

Thus (2.5) together with (2.7) yields that

$$u_1(t) \le u_1(\xi_1) + \int_0^\omega |u_1'(t)|dt < \ln \frac{\bar{r}_1}{\bar{a}_1} + 2(\bar{r}_1\omega) := H_1. \tag{2.10}$$

It follows from the equation of (2.4) and (2.7) that

$$\begin{aligned} \int_0^\omega a_2(t)\exp(u_2(t-\tau_2(t))dt &\le \int_0^\omega \frac{b_2(t)\exp(u_2(t-\tau_3(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))} \\ &\le \int_0^\omega \frac{b_2(t)}{m_1(t)}dt \le \frac{\bar{b}_2}{m_1^L} \\ \exp(u_2(\xi_2)) &\le \frac{\overline{b_2}}{\bar{a}_2/m_1^L} \end{aligned}$$

$$u_2(\xi_2) \le \ln\left[\frac{\overline{b_2}}{\bar{a}_2/m_1^L}\right] = L_2, \tag{2.11}$$

which implies that

$$u_2(t) \le u_2(\epsilon_2) + \int_0^\omega |u_2'(t)|dt < \ln\left[\frac{\overline{b_2}}{\bar{a}_2/m_1^L}\right] + 2\omega\bar{r}_1 := K_1. \tag{2.12}$$

On the other hand, (2.3) yields that

$$\begin{aligned} \bar{r}_2\omega &\le \int_0^\omega \frac{b_2(t)\exp(u_1(t-\tau_3(t))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}dt \\ &\le \int_0^\omega \frac{b_2(t)}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}dt\exp(u_1(\eta_1)) \\ &\le (\bar{b}_2/m^L)\omega\exp(u_1(\eta_1)), \end{aligned} \tag{2.13}$$

implying that

$$u_1(\eta_1) \ge \ln\left[\frac{\bar{r}_2}{\bar{b}_2/m^L}\right] := l_1. \tag{2.14}$$

We derive from (2.13) and (2.14) that

$$u_1(t) \ge u_1(\eta_1) - \int_0^\omega |u_1'(t)|dt \ge \ln\left[\frac{\bar{r}_2}{\bar{b}_2/m^L}\right] - 2\bar{r}_1\omega := H_2. \tag{2.15}$$

The inequalities (2.13) and (2.15) lead that

$$\max_{t\in[0,\omega]} |u_1(t)| < \max\{|H_1|, |H_2|\} := H. \tag{2.16}$$

Once again from the second equation of (2.2)

$$\begin{aligned}
&\int_0^\omega a_2(t)\exp(u_2(t))dt\\
&= \int_0^\omega \frac{b_2(t)\exp(u_1(t-\tau_3(t)))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}dt - \int_0^\omega r_2(t)dt\\
&= \int_0^\omega \frac{b_2(t)\exp(u_2(t-\tau_3(t)))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}\\
&\quad \cdot \exp(u_1(t-\tau_3(t)) - \exp(u_2(t-\tau_3(t))))dt - \int_0^\omega r_2(t)dt\\
&\geq \{\exp(u_1(\xi_1)-u_2(\eta_2))\}\int_0^\omega \frac{b_2(t)\exp(u_2(t-\tau_3(t)))}{m(t)+m_1\exp(u_1(t))+m_2(t)\exp(u_2(t))}dt\\
&\quad - \int_0^\omega r_2(t)dt\\
&\geq \{\exp(u_1(\xi_1)-u_2(\eta_2))\}\int_0^\omega \frac{b_2(t)}{m_2(t)}dt - \int_0^\omega r_2(t)dt\\
&\geq \{\exp(u_1(\xi_1)-u_2(\eta_2))\}\frac{1}{m_2^M}\int_0^\omega b_2(t)dt - \int_0^\omega r_2(t)dt.
\end{aligned}$$

Therefore

$$\begin{aligned}
\exp(2u_2(\eta_2)) &\geq \exp(u_1(\xi_1))\left[\frac{\bar{b}_2-\bar{r}_2 m_2^M}{\bar{a}_2 m_2^M}\right]\\
u_2(\eta_2) &\geq \frac{1}{2}\left[u_1(\xi_1) - \ln\frac{\bar{a}_2 m_2^M}{\bar{b}_2-\bar{r}_2 m_2^M}\right].
\end{aligned} \tag{2.17}$$

From (2.15)

$$u_2(\eta_2) \geq \frac{1}{2}\left[\ln\left(\frac{\bar{r}_2}{\bar{b}_2/m^L}\right) - 2\bar{r}_1\omega - \ln\frac{\bar{a}_2 m_2^M}{\bar{b}_2-\bar{r}_2 m_2^M}\right] := l_2. \tag{2.18}$$

Hence, by (2.18) we obtain the inequality

$$\begin{aligned}
u_2(t) &\geq u_2(\eta_2) - \int_0^\omega |u_2'(t)|dt\\
&> \frac{1}{2}\left[\left(\frac{\bar{r}_2}{\bar{b}_2/m^L}\right) - 2\bar{r}_1\omega - \ln\left(\frac{\bar{a}_2 m_2^M}{\bar{b}_2-\bar{r}_2 m_2^M}\right) - 4\omega\bar{r}_2\right] := K_2.
\end{aligned} \tag{2.19}$$

So,

$$\max_{t\in[0,\omega]}\Big|u_2(t)\Big| < \max\left(|K_1|,|K_2|\right) := K. \tag{2.20}$$

Trivially H, K do not depend on λ. Take $B = H + K + L$ where$L$ is chosen sufficiently large enough such that each root (u_1^*, u_2^*) of the following algebraic equations

$$
\begin{aligned}
&\bar{r}_1 - \bar{a}_1 \exp(u_1) - \frac{1}{\omega}\int_0^\omega \frac{\mu b_1 \exp(u_2)}{m(t) + m_1(t)\exp(u_1) + m_2(t)\exp(u_2)}dt = 0,\\
&-\bar{r}_2 - \bar{a}_2 \exp(u_2) + \frac{1}{\omega}\int_0^\omega \frac{b_2 \exp(u_1)}{m(t) + m_1(t)\exp(u_1) + m_2(t)\exp(u_2)}dt = 0,
\end{aligned} \tag{2.21}
$$

where $\mu \in [0, 1]$ is any arbitrary parameter, satiesfies

$$
||(u_1^*, u_2^*)^T|| \;=\; |u_1^*| + |u_2^*| < B, \tag{2.22}
$$

if it exits Similar arguments as above lead that each soltion $(u_1^*, u_2^*)^T$ of (2.21) satisfies $H_1 < u_1^* < H_2$ and $K_1 < u_1^* < K_2$. Now we define $\Omega = \{u = (u_1, u_2)^T \in X : \|u\| < B\}$. Then it is an open bounded subset of X satisfying the conditions (i) of Theorem 2.2. Let $u \in \partial\Omega \cap$ Ker $L = \partial\Omega \cap \mathbb{R}^2$. Then u is a constant in $\mathbb{R}^2$ with $|u_1| + |u_2| = B$.

From the definition of B, we note that $QN \neq 0$.

Finally, we prove that conditions (ii) and (iii) of Theorem 2.2is also satisfied. To this end, we define F :Dom $L \times [0, 1] \to X$ by

$$
\begin{aligned}
F(u_i, u_2, \mu) = &\begin{pmatrix} \bar{r}_1 - \bar{a}_1 \exp(u_1) \\ \frac{-\bar{r}_2 - \bar{a}_2 \exp(u_2) + \bar{b}_2 \exp(u_1)}{m(t)+m_1(t)\exp(u_1)+m_2(t)\exp(u_2)} \end{pmatrix} \\
&+ \mu \begin{pmatrix} \frac{\bar{b}_1 \exp(u_1)}{m(t)+m_1(t)\exp(u_1)+m_2(t)\exp(u_2)} \\ 0 \end{pmatrix},
\end{aligned} \tag{2.23}
$$

where $\mu \in [0, 1]$. By comparing F with (2.19), we have that $F(u_1, u_2, \mu) \neq 0$ on $\partial\Omega \cap$ ker L.

By using the invariance property and a homotopy of topological degree and some direct computation, we get

$$
\deg(JQN(u_1, u_2)^T, \Omega \cap KerL, (0,0)^T) = \deg(F, \Omega \cap \text{ ker } L, (0, 0^T)) = 1,
$$

where $\deg(\cdot,\cdot,\cdot)$ is the brouwer degree and the isomorphism of J of Im Q onto Ker L can be chosen to be the identity mapping, since IM Q = Ker L. Hence the conditions (ii) and (iii) of Theorem 2.2 hold. Thus (1.2) has atleast one ω-periodic solutions. □

Next we consider the Predator-Prey systems with distributed deviating arguments

$$
\begin{aligned}
x'(t) &= x(t)\left[r_1(t) - a_1(t)\int_{-\tau}^{0} x(t+\theta)d\mu(\theta)\right] \\
&\quad - \frac{b_1(t)x(t)y(t)}{m(t)+m_1(t)x(t)+m_2(t)y(t)}, \\
y'(t) &= y(t)\left[-r_2(t) - a_2(t)\int_{-\tau}^{0} x(t+\theta)d\mu(\theta)\right. \\
&\quad \left. + \frac{b_2(t)x_1(t)x_2(t)}{m(t)+m_1(t)x(t)+m_2(t)y(t)}\right]
\end{aligned}
\tag{2.24}
$$

Theorem 2.3. *If the conditions of Theorem 2.1 are satisfied, then* (2.24) *has at least one positive* ω*-periodic solution.*

Proof. The proof is similar to the proof of Theorem2.2 and hence is omitted here. □

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[1] DEPARTMENT OF MATHEMATICS
ANNA UNIVERSITY BIT-CAMPUS
TIRUCHIRAPPALLI,TAMIL NADU
INDIA.
Email address: mahasenthil2@gmail.com

[2] DEPARTMENT OF MATHEMATICS
BHARATHIDASAN UNIVERSITY
TIRUCHIRAPPALLI,TAMIL NADU
INDIA.
Email address: vpm@bdu.ac.in