

MINIMAL AND MAXIMAL OPEN SETS IN NANO TOPOLOGICAL SPACE

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ABSTRACT. This paper deals with nano minimal open sets and nano maximal open sets. Thereafter nano maximal, nano minimal open sets, nano minimal continuous and nano maximal continuous are studied. Also we study the interrelationship among these concepts. Finally we obtain a result which is not possible in classical topology.

1. INTRODUCTION

Pawlak introduced "rough set theory" [7], a mathematical tool for dealing with vagueness or uncertainty. Since 1982, the theory and applications of rough sets have impressively developed. Nakaoko and Oda [4] introduced and studied the concept of minimal open sets and maximal open sets which are the subclasses of open sets. The complements of minimal open sets and maximal open sets are minimal and maximal closed sets. Using minimal open set in topological space we can obtain nano topology. Lellis Thivagar et al [2] interjected a new space called nano topological space whose elements are called nano open sets. It is termed as nano topology since it has atmost five open sets whatever may be the size of universe. This work is extended to some real life applications of nano

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topology in terms of basis. In this paper we introduced and study the concepts of nano minimal and nano maximal sets in nano topology.

2. PRELIMINARIES

This section is devoted to preliminaries.

Definition 2.1. [2] Let \mathcal{U} be a nonempty finite set of objects called the universe and R be an equivalence relation on \mathcal{U} named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (\mathcal{U}, R) is said to be the approximation space. Let $X \subseteq \mathcal{U}$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.

That is, $L_R(X) = \bigcup_{x \in \mathcal{U}} \{x : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x .

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \bigcup_{x \in \mathcal{U}} \{x : R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [2] Let \mathcal{U} be an universe, R be an equivalence relation on \mathcal{U} and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:

(i) \mathcal{U} and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on \mathcal{U} called the nano topology on \mathcal{U} with respect to X . We call $(\mathcal{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

Proposition 2.1. [1] Let \mathcal{U} be a nonempty finite universe and $X \subset \mathcal{U}$, \mathcal{U}/R be an indiscernibility relation on \mathcal{U} then

- (i) **Nano Type-1**(\mathcal{NT}_1): If $L_R(X) = U_R(X) = X$, then the nano topology, $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X)\}$.
- (ii) **Nano Type-2**(\mathcal{NT}_2): If $L_R(X) = \emptyset$ and $U_R(X) \neq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset, U_R(X)\}$.
- (iii) **Nano Type-3**(\mathcal{NT}_3): If $L_R(X) \neq \emptyset$ and $U_R(X) = \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), B_R(X)\}$.
- (iv) **Nano Type-4**(\mathcal{NT}_4): If $L_R(X) = \emptyset$ and $U_R(X) = \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset\}$, is the indiscrete nano topology on \mathcal{U} .
- (v) **Nano Type-5**(\mathcal{NT}_5): If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \emptyset$ and $U_R(X) \neq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$.

Theorem 2.1. [1] Let \mathcal{U} be a non empty finite universe and $X \subseteq \mathcal{U}$. Let $\tau_R(X)$ be the nano topology on \mathcal{U} with respect to X . Then $[\tau_R(X)]^c$ whose elements are A^c for $A \in \tau_R(X)$, is a nano topology on \mathcal{U} .

Remark 2.1. [1] $[\tau_R(X)]^c$ is called the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called nano closed sets. Thus, from the above theorem, we note that a subset A of \mathcal{U} is nano closed in $\tau_R(X)$ if and only if $\mathcal{U}-A$ is nano open in $\tau_R(X)$.

Definition 2.3. [1] If $(\mathcal{U}, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq \mathcal{U}$ and if $A \subseteq \mathcal{U}$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $\mathcal{Nint}(A)$. That is, $\mathcal{Nint}(A)$ is the largest nano open subset of A . The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $\mathcal{Ncl}(A)$. That is, $\mathcal{Ncl}(A)$ is the smallest nano closed set containing A .

Definition 2.4. [4] A proper nonempty open subset G of a topological space X is said to be a maximal open set if any open set which contains G is X or G .

Definition 2.5. [4] A proper nonempty open subset G of X is said to be minimal open set if any open set which is contained in G is \emptyset or G .

Definition 2.6. [2] Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and $A \subseteq \mathcal{U}$. Then A is said to be

- (i) nano semiopen if $A \subseteq \mathcal{Ncl}(\mathcal{Nint}(A))$;
- (ii) nano pre-open if $A \subseteq \mathcal{Nint}(\mathcal{Ncl}(A))$.

$NSO(\mathcal{U}, \tau_R(X))$, $NPO(\mathcal{U}, \tau_R(X))$ respectively denote the families of all nano semiopen, nano preopen subsets of \mathcal{U} .

Definition 2.7. [2] Let $(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R_1}(Y))$ be two nano topological space then a mapping $f: \mathcal{U} \longrightarrow \mathcal{V}$ is said to be nano continuous on \mathcal{U} if the inverse image of every nano open set in \mathcal{V} is nano open in \mathcal{U} .

3. MINIMAL OPEN AND MAXIMAL OPEN SETS IN NANO TOPOLOGY

In this section we introduce nano minimal open sets and nano maximal open sets.

Definition 3.1. A nonempty nano open set A of \mathcal{U} is said to be a nano minimal open set if and only if any nano open set which is contained in A is \emptyset or A . A proper nonempty nano closed subset F of \mathcal{U} is said to be nano minimal closed which is contained in F is \emptyset or F . The family of all nano minimal open sets in a nano topological space \mathcal{U} is denoted by $NMIO(\mathcal{U}, X)$.

Definition 3.2. A proper nonempty nano open subset A of a nano topological space \mathcal{U} is said to be nano maximal open set if any nano open set which contains A is \mathcal{U} or A . A proper nonempty nano closed subset F of \mathcal{U} is said to be NMAC if any nano closed set which contains F is F or \mathcal{U} . The family of all nano maximal open sets in a nano topological space \mathcal{U} is denoted by $NMAO(\mathcal{U}, X)$.

Example 1. Let $\mathcal{U} = \{a, b, c, d\}$ and $X = \{a, b\}$, $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d\}\}$, $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}, \{a, b, c\}, \{b, c\}\}$. Here $NMAO(\mathcal{U}, X) = \{a, b, c\}$.

Theorem 3.1.

- (i) Let G be a NMIO and W be an nano open set. Then $G \cap W = \emptyset$ or $G \subseteq W$.
- (ii) Let G and H be two NMIOs. Then $G \cap H = \emptyset$ or $G = H$.

Proof.

- (i) Let W be an nano open set such that $G \cap W \neq \emptyset$. Since G is NMIO and $G \cap W \subset G$, we have $G \cap W = G$. Then $G \subset W$.
- (ii) If $G \cap H \neq \emptyset$, then we see that $G \subset H$ and $H \subset G$ by (i). Therefore $G = H$.

□

Theorem 3.2. Every NMIO is nano open.

Proof. Since by definition of NMIOS it is obvious that every NMIOS is nano open set. \square

Remark 3.1. Converse of the above Theorem 3.2 is not true by the following example.

Example 2. Let $\mathcal{U} = \{a, b, c, d\}$, $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a\}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}\}$. Here $NMIO(\mathcal{U}, X) = \{a\}$ and \mathcal{U} is not NMIO.

Theorem 3.3. Let F be a NMIO. Then $Nint(F) = F$ or $Nint(F) = \emptyset$.

Proof. If F is NMIOS then the subset of F is \emptyset and itself. Since if F is nano open then $Nint(F) = F$ and by the definition of NMIOS. Then the statement is obvious. \square

Theorem 3.4. Let G be a NMAO and N be a subset of \mathcal{U} with $G \subset N$. Then N is a NPO.

Proof. If $N = G$, then N is a nano open set. Therefore N is a NPO set. Otherwise, $G \subset N$, then $Nint(Ncl(N)) = Nint(\mathcal{U}) = \mathcal{U} \supset N$. Therefore N is a NPO. \square

Theorem 3.5. Let G be a NMAO and x be an element of $\mathcal{U} - G$. Then, $\mathcal{U} - G \subset H$ for any nano open neighbourhood H of x .

Proof. Since $x \in \mathcal{U} - G$, we have $H \not\subseteq G$, for any nano open neighbourhood H of x . Then $H \cup G = \mathcal{U}$. Therefore, $\mathcal{U} - G \subset H$. \square

Theorem 3.6. Let G be NMAO and S a nonempty nano subset of $\mathcal{U} - G$. Then, $Ncl(G) = \mathcal{U} - G$.

Proof. Since $\emptyset \neq S \not\subseteq \mathcal{U} - G$, we have $W \cap S \neq \emptyset$ for any element x of $\mathcal{U} - G$ and any nano open neighbourhood W of x by previous theorem. Then, $\mathcal{U} - G \subset Ncl(S)$. Since $\mathcal{U} - G$ is nano closed set and $S \subset \mathcal{U} - G$, we see that $Ncl(S) \subset Ncl(\mathcal{U} - G) = \mathcal{U} - G$. Therefore $\mathcal{U} - G = Ncl(S)$. \square

4. CHARACTERIZATIONS

In this section characterization of NMIOS and NMAOS are discussed.

Theorem 4.1. Let \mathcal{U} be a nano topological space and $F \subset \mathcal{U}$. F is NMIC if and only if $\mathcal{U} - F$ is NMAO.

Proof. If F is NMIC then nonempty proper closed set which is contained in F is \emptyset and itself. So the other nonempty proper closed sets are supersets of F . $\mathcal{U} - F$ is nano open and it is a super set of all other nonempty proper nano open sets. Hence $\mathcal{U} - F$ is NMAOS. Conversely let $\mathcal{U} - F$ is NMAOS that is F is nano closed and F is NMIC. \square

Theorem 4.2. *Let \mathcal{U} be a NTS and $F \subset \mathcal{U}$. G is NMIOS if and only if $\mathcal{U} - G$ is NMAC.*

Proof. Proof is obvious. \square

Theorem 4.3.

- (i) *Let G be a NMAO and W is a nano open set. Then $G \cup W = \mathcal{U}$ or $W \subset G$.*
- (ii) *Let G and H be NMAO. Let $G \cup H = \mathcal{U}$ or $G = H$.*

Proof.

- (i) Let W be a nano open set such that $G \cup W \neq \mathcal{U}$. Since G is a NMAOS and $G \subset G \cup W$, we have $G \cup W = G$. Therefore, $W \subset G$.
- (ii) If $G \cup H \neq \mathcal{U}$, then $G \subset H$ and $H \subset G$ by (i). Therefore $G = H$.

\square

Theorem 4.4. *Let G be a nonempty nano open set. Then the following three conditions are equivalent.*

- (i) *G is a NMIOS.*
- (ii) *$G \subseteq \text{Ncl}(S)$ for any nonempty nano open subset S of G .*
- (iii) *$\text{Ncl}(G) = \text{Ncl}(S)$ for any nonempty nano open subset S of G .*

Proof.

(i) \implies (ii): Let S be a nonempty nano open subset S of G . For any element x of G and any nano open neighbourhood H of x , we have $S = G \cap S \subset H \cap S$. Then we have $H \cap S \neq \emptyset$ and hence x is an element of $\text{Ncl}(S)$. It follows that $G \subset \text{Ncl}(S)$.

(ii) \implies (iii): For any nonempty subset S of G , we have $\text{Ncl}(S) \subset \text{Ncl}(G)$. By (ii) we see $\text{Ncl}(G) \subset \text{Ncl}(\text{Ncl}(S)) = \text{Ncl}(S)$. Therefore we have $\text{Ncl}(S) = \text{Ncl}(G)$ for any nonempty subset S of G .

(iii) \implies (i): Suppose that G is not NMIOS. Then there exists a nonempty nano open set V such that $V \not\subseteq G$ and hence there exists an element $a \in G$ such that $a \notin V$. Then we have $\text{Ncl}(\{a\}) \neq \text{Ncl}(G)$. \square

Theorem 4.5. *Let G be a NMIOS and M be a nonempty subset of \mathcal{U} . If there exists an nano open neighbourhood W of M such that $W \subset \text{Ncl}(M \cup G)$, then $M \cup S$ is a NPO for any nonempty subset S of \mathcal{U} .*

Proof. By Theorem 4.4(iii), we have $\text{Ncl}(M \cup S) = \text{Ncl}(M) \cup \text{Ncl}(S) = \text{Ncl}(M) \cup \text{Ncl}(G) = \text{Ncl}(M \cup G)$. Since $W \subset \text{Ncl}(M \cup G) = \text{Ncl}(M \cup S)$ by assumption, we have $\text{Nint}(W) \subset \text{Nint}(\text{Ncl}(M \cup S))$. Since W is an nano open neighbourhood of M , namely W is an nano open set such that $M \subset W$, we have $M \subset W = \text{Nint}(W) \subset \text{Nint}(\text{Ncl}(M \cup S))$. Moreover, we have $\text{Nint}(G) \subset \text{Nint}(\text{Ncl}(M \cup G))$, for $\text{Nint}(G) = G \subset \text{Ncl}(G) \subset \text{Ncl}(M) \cup \text{Ncl}(G) = \text{Ncl}(M \cup G)$. Since G is an nano open set, we have $S \subset G = \text{Nint}(G) \subset \text{Nint}(\text{Ncl}(M \cup G)) = \text{Nint}(\text{Ncl}(M \cup S))$. Therefore $M \cup S \subset \text{Nint}(\text{Ncl}(M \cup S))$. \square

Theorem 4.6. *Let G be a NMIO. Then any nonempty subset S of G is a NPO.*

Proof. By Theorem 4.4(ii), we have $\text{Nint}(G) \subset \text{Nint}(\text{Ncl}(S))$. Since G is nano open, we have $S \subset G = \text{Nint}(G) \subset \text{Nint}(\text{Ncl}(S))$. \square

Theorem 4.7. *Let G be a NMAO and H is proper subset of \mathcal{U} with $G \subset H$. Then $\text{Nint}(H) = G$.*

Proof. If $H = G$, then $\text{Nint}(H) = \text{Nint}(G) = G$. Otherwise, $H \neq G$, and hence $G \not\subset H$. It follows that $G \subset \text{Nint}(H)$. Since G is nano maximal open set, we have also $\text{Nint}(H) \subset G$. Therefore, $\text{Nint}(H) = G$. \square

Theorem 4.8. *If G is both NMAO and NMAC and H is nano clopen, then either $G \cup H = \mathcal{U}$ or $H \subset G$.*

Proof. If G is both NMAO and NMAC implies $G \subset \mathcal{U}$ or G also G is nano clopen set since every NMAO is nano open $G \cup H = \mathcal{U}$ or $H \subset G$ hence the theorem is proved. \square

Theorem 4.9. *If G is both NMAO and NMIC, H is nano open and E is nano closed, then either the following is true.*

- (i) $H \subset G \subset E$.
- (ii) $H \subset G$ and $G \cap E = \emptyset$.
- (iii) $G \cup H = \mathcal{U}$ and $G \subset E$.
- (iv) $G \cup H = \mathcal{U}$ and $G \cap E = \emptyset$.

Proof. Consider G is NMAO, since if G is NMAO and H is a nano open set then $G \cup H = \mathcal{U}$ or $H \subset G$, we get $H \subset G$ or $G \cup H = \mathcal{U}$. Consider G is NMIC and since if G is NMIC and E is an nano open set, then $G \cap E = \emptyset$ and $G \subset E$. $H \subset G$ and $G \subset E \implies H \subset G \subset E$. The remaining combinations are $H \subset G$, $G \cap E = \emptyset$, $G \cup H = \mathcal{U}$, $G \subset E$ and $G \cup H = \mathcal{U}$, $G \cap E = \emptyset$. \square

Remark 4.1. In classical topology it is not necessary that every nonempty proper open set is minimal open or maximal open. Whereas in Nano topology every nonempty nano open sets are either NMIO or NMAO sets.

Theorem 4.10. Every nonempty proper nano open subsets of $(\mathcal{U}, \tau_R(X))$ is either NMIO or NMAO.

Proof.

- (i) If \mathcal{U} is an \mathcal{NT}_5 space then $L_R(X) \cap B_R(X) = \emptyset$ so $L_R(X)$ and $B_R(X)$ is NMIO since $L_R(X) \subset U_R(X)$ and $B_R(X) \subset U_R(X)$ so $U_R(X)$ is NMAO.
- (ii) If \mathcal{U} is \mathcal{NT}_1 space then $L_R(X) = U_R(X) = X$. Here $\emptyset \subset L_R(X)$ and $L_R(X) \subset L_R(X)$. Also $L_R(X) \subset \mathcal{U}$ and hence by definition $L_R(X)$ is NMIO and NMAO.
- (iii) If \mathcal{U} is \mathcal{NT}_2 space then the nonempty nano open set exist here is $U_R(X)$. By (ii) $U_R(X)$ is both NMIO and NMAO.
- (iv) If \mathcal{U} is \mathcal{NT}_3 space then nonempty nano open sets are $L_R(X)$ and $B_R(X)$. As we know that $L_R(X) \cap B_R(X) = \emptyset$. So by definition $\emptyset \subset L_R(X)$ and $L_R(X) \subset L_R(X)$, $\emptyset \subset B_R(X)$ and $B_R(X) \subset B_R(X)$. Also $\mathcal{U} \supset L_R(X)$ and $\mathcal{U} \supset B_R(X)$. Hence by definition $L_R(X)$ and $B_R(X)$ is NMAO and NMIO.

\square

Remark 4.2. If a nano topological space is \mathcal{NT}_1 , \mathcal{NT}_2 and \mathcal{NT}_3 then every nonempty proper nano open subset of \mathcal{U} is both NMIO and NMAO.

5. NANO MINIMAL AND NANO MAXIMAL CONTINUOUS

In this section NMIO and NMAO continuous maps are discussed.

Definition 5.1. Let \mathcal{U} and \mathcal{V} be the nano topological spaces. A map $f : \mathcal{U} \longrightarrow \mathcal{V}$ is called

- (i) nano minimal continuous if $f^{-1}(M)$ is an nano open set in \mathcal{U} for every NMIOS M in \mathcal{V} .
- (ii) nano maximal continuous if $f^{-1}(M)$ is an nano open set in \mathcal{U} for every NMAOS M in \mathcal{V} .
- (iii) nano minimal maximal continuous if $f^{-1}(M)$ is NMAOS in \mathcal{U} for every NMIOS M in \mathcal{V} .
- (iv) nano maximal minimal continuous if $f^{-1}(M)$ is NMIOS in \mathcal{U} for every NMAOS M in \mathcal{V} .

Theorem 5.1. Every nano continuous map if and only if nano minimal continuous.

Proof. Let $f : \mathcal{U} \longrightarrow \mathcal{V}$ be a nano continuous map. Let G be NMIOS in \mathcal{V} . Since every NMIO is nano open set G is nano open in \mathcal{V} . Since f is nano continuous, $f^{-1}(G)$ is an nano open set in \mathcal{U} . Hence f is nano minimal continuous. Conversely, if inverse image of NMIOS in \mathcal{V} is nano open. Since $L_R(Y) \subseteq U_R(Y)$ and $B_R(Y) \subseteq U_R(Y)$ also we know that $f^{-1}(L_R(Y))$ is nano open and $f^{-1}(B_R(Y))$ is nano open. Therefore $f^{-1}(L_R(Y)) \cup f^{-1}(B_R(Y)) = f^{-1}((L_R(Y)) \cup (B_R(Y))) = f^{-1}(U_R(Y))$ is nano open. \square

Theorem 5.2. Every nano continuous map if and only if nano maximal continuous.

Proof. Proof follows from the Theorem 5.1. \square

Remark 5.1. In classical topology minimal continuous and maximal continuous maps are independent of each other but in nano topology nano minimal continuous and nano maximal continuous maps are not independent of each other.

Theorem 5.3. Every nano minimal continuous map is nano maximal continuous but not conversely.

Proof. If $L_R(X)$ and $B_R(X)$ is and NMIO and inverse image of $L_R(X)$ and $B_R(X)$ is nano open since f is nano minimal continuous. By the property of approximation $L_R(X) \cup B_R(X) = U_R(X)$ and hence $U_R(X)$ is also nano open. Therefore f is also nano maximal continuous. \square

Remark 5.2. Converse of the Theorem 5.3 is not true by the following example.

Example 3. Let $\mathcal{U} = \{a, b, c, d\}, \mathcal{U}/R = \{\{a, b\}, \{c, d\}\}$ and $X = \{a, b\}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b\}\}$ and $\mathcal{V} = \{1, 2, 3, 4\}, \mathcal{V}/R_1 = \{\{1\}, \{2, 3\}, \{4\}\}$ and $Y = \{1, 2\}$ then $\tau_{R_1}(Y) = \{\mathcal{V}, \emptyset, \{1\}, \{1, 2, 3\}, \{2, 3\}\}$. Define a function $f: \mathcal{U} \rightarrow \mathcal{V}$ by $f(a) = 1, f(b) = 1, f(c) = 3, f(d) = 4$. Here f is nano maximal continuous but not nano minimal continuous since $f^{-1}(\{2, 3\}) = \{c\}$ is not nano open.

Example 4. Let $\mathcal{U} = \{a, b, c, d\}, X = \{a, b\}$ and $\mathcal{U}/R = \{\{a\}, \{b\}, \{c, d\}\}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b\}\}$ and $\mathcal{V} = \{1, 2, 3, 4\}, Y = \{1, 2\}$ and $\mathcal{V}/R_1 = \{\{1\}, \{2, 3\}, \{4\}\}$ then $\sigma_{R_1}(Y) = \{\mathcal{V}, \emptyset, \{1\}, \{1, 2, 3\}, \{2, 3\}\}$. Define a function $f: \mathcal{U} \rightarrow \mathcal{V}$ by $f(a) = 2, f(b) = 3, f(c) = 4, f(d) = 4$. Here f is nano maximal minimal continuous.

Theorem 5.4. Every nano minimal maximal continuous map is nano minimal continuous map but not conversely.

Proof. Let $f: \mathcal{U} \rightarrow \mathcal{V}$ be a nano minimal maximal continuous map. Let N be any nano minimal open set in \mathcal{V} . Since f is nano minimal maximal continuous, $f^{-1}(N)$ is a nano maximal open set in \mathcal{U} . Since every nano maximal open set is nano open, $f^{-1}(N)$ is nano open set in \mathcal{U} . Hence \mathcal{U} is nano minimal continuous. \square

Remark 5.3. Converse of Theorem 5.4 is not true by the following example.

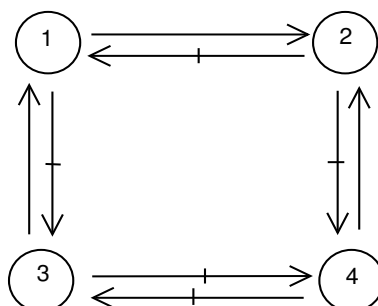
Example 5. Let $\mathcal{U} = \{a, b, c, d\}, X = \{a, b\}$ and $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d\}\}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}, \{a, b, c\}, \{b, c\}\}$, and $\mathcal{V} = \{1, 2, 3, 4\}, Y = \{1, 2\}$ and $\mathcal{V}/R_1 = \{\{1\}, \{2, 3\}, \{4\}\}$ then $\sigma_{R_1}(Y) = \{\mathcal{V}, \emptyset, \{1\}, \{1, 2, 3\}, \{2, 3\}\}$. Define a function $f: \mathcal{U} \rightarrow \mathcal{V}$ by $f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4$. Here f is nano minimal continuous but $f^{-1}(\{1\}) = \{a\}$ so $\{a\}$ is not nano maximal open.

Theorem 5.5. Every nano maximal minimal continuous map is nano maximal continuous map but not conversely.

Proof. Similar to the Theorem 5.4. \square

Example 6. In Example 5, $f^{-1}(\{1, 2, 3\}) = \{a, b, c\}$ which is nano open but not NMIO. Hence f is nano maximal continuous but not nano maximal minimal continuous.

Remark 5.4. From the above discussion and known results we have the following implications.



1.nano minimal continuous 2.nano maximal continuous 3.nano minimal maximal continuous 4.nano maximal minimal continuous.

6. CONCLUSION

In this paper we have introduced the basic concepts of NMIO and NMAO sets. The interrelations among NMIC, NMAC, NMIO and NMAO were also studied. Also nano minimal continuous and nano maximal continuous maps are discussed. Further our concepts nano minimal sets can be extended to stronger forms and also leads to some applications.

REFERENCES

- [1] L. THIVAGAR, S. DEVI: *On MultiGranular Nano Topology*, Southeast Asian Bulletin of Mathematics, **40** (2016), 875-885 .
- [2] L. THIVAGAR, C. RICHARD: *On Nano continuity in a strong form*, International Journal of Pure and Applied Mathematics, **101** (2015), 893-904.
- [3] A.S. MASHHOUR, A.E. MONSEF, S.N. EL DEEP: *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt, (1982), 47-53.
- [4] F. NAKAOKA, N. ODA: *On minimal closed sets*, Proceeding of Topological Spaces and its Applications, **5** (2003), 19-21.
- [5] N. BOURBAKI: *General Topology*, Elements of Mathematics, Springer Verlag, **101** (1989), 1-4.
- [6] V. POPA, T. NOIRI: *On the definitions of some generalized forms of continuity under minimal conditions*, Mem. Fac. Sci. Kochi Univ. Math Ser., **22** (2001), 9-19.
- [7] Z. PAWLAK: *Rough Sets*, Int. J. Comput. Inform. Sci., **11** (1982), 341-356.

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