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ON VE-DEGREE AND EV-DEGREE BASED TOPOLOGICAL INDICES OF CERTAIN OTIS BISWAPPED NETWORK

Nida Zahra[1] and Muhammad Ibrahim

ABSTRACT. The optical transpose interconnection system (OTIS) arrange has numerous application in designed for equal just as in conveyed arrange. Distinctive interconnection networks has contemplated identified with topological descriptors in [[1, 2]]. The present article is a contribution to Ve-degree and Ev-degree base topological indices of biswapped network with premise diagram as path and complete graph. In addition, some delicated recipes are too gotten for various kinds of topological records for the OTIS biswapped network by taking the path and complete graph on n vertices as premise of diagram.

1. INTRODUCTION

The job of graph theory is quickly expanding step by step particularly in science. Cheminformatic is another developing science identified with chemistry, arithmetic and software engineering and their segments incorporates quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) and the part can add to the examination on physicochemical properties of synthetic compound. Doling out numbers to a sub-atomic

[1]*corresponding author*

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diagram has decent properties in science. A drawing, a grouping of numbers, a polynomial, a numeric number, or a network are on the whole approaches to perceived a graph. A topological record is a numeric amount related with a graph that portrays the structure of the diagram and is invariant under the graph automorphism. To investigation of quantitative structure-property relationship (QSPR), the interconnection structure network can be represented mathematically by a graph and quantitative structure-activity relationship (QSAR), numerous topological lists are broadly utilized and are vital in present day science and organic chemistry. To acquire a hugeness connection, it is fundamental that suitable descriptors are utilized, regardless of whether they are hypothetical, exact or gotten from promptly accessible exploratory qualities of structure. Various use of graph theory can be found in auxiliary science. Its first what's more, extremely well known application in chemistry was the breaking point of the paraffin by Wiener [3]. The Wiener index as a initial topological descriptor was instigate by him. As yet in mathematical and chemical studies, the Wiener, Zagreb and Randic descriptors are mostly used [4,5]. There are numerous topological indices are presented following this investigation that clarified physico-chemical property [6,7]. Although, all above efforts has been ended under traditional degree ideas. Not long ago, in graph theory, the two advance degree notions is commence by Chellali et al. [8] namely, ve-degree and ev-degree. The traditional degree based notions have been interpreted to ev-degree and ve-degree Randic descriptors and ev and ve-degree Zagreb descriptors in [9]. it is investigated that the ve-degree Zagreb descriptor has intense divination power than the traditional Zagreb descriptor.

The primary goal of the optical transpose interconnection system (OTIS) organize is to develop an effective network for new optoelectronic PC design. The property of this organize gives advantage for both optical and electronic innovations [10]. In OTIS systems, the processors are sorted out by groups. Electronic interconnects are used between processors inside a similar group, while optical connections are used for inter cluster correspondence. There are numerous calculations for directing, determination/arranging [11–18], for numerical calculation [19], Fourier transformation [20], matrix augmentation [21], and picture processing [22]. The interconnection structure network can be represented mathematically by a graph. As diagram has vertices which can be spoken to by processor nodes and the edges speak to the joins between these nodes. The topology of a diagram decides the manner by which vertices are associated by edges. We can acquired certain properties of a system by utilizing the structure of the diagram. The most extreme separation between any two vertices in the system is the diameter of the system. The quantities of connections associated with a vertex decide the degree of that vertex. For regular systems, the degree of the considerable number of vertices must be same.

2. PRELIMINARIES

Here are some basic concepts about graph. Let $G = (V, E)$ be a graph with node set V and link set E. A **graph** $G = (V, E)$ with two nonempty sets V and E. The elements of V are called vertices and the elements of E are called edges. The **degree** of a vertex, **open neighborhood**, and **closed neighborhood** of the vertex v, we refer [23, 24]. The cardinality of the nodes with the union of the closed neighborhoods of the nodes of a and b is referred to the ev**-degree** of a link $e = ab \in E$ which is represented by $deg_{ev}(e)$. The cardinality of the different links which are incident to any node with the union of the closed neighborhoods of the any node a from the close neighborhood of a is referred to the ve**-degree** of $a \in V$ which is denoted by $deg_{ve}(a)$. The definitions of different indices relevant to ve-degree and ev-degree are given the followings:

2.1. ev-degree Zagreb index.

$$M^{ev}(G) = \sum_{e \in E} deg_{ev}(e)^2.$$

2.2. The first ve-degree Zagreb α index.

$$M_1^{\alpha ve}(G) = \sum_{v \in V} deg_{ve}(v)^2.$$

2.3. The first ve-degree Zagreb β index.

$$M_1^{\beta ve}(G) = \sum_{uv \in E} (deg_{ve}(u) + deg_{ve}(v)).$$

2.4. The second ve-degree Zagreb index.

$$M_2^{ve}(G) = \sum_{uv \in E} (deg_{ve}(u) \times deg_{ve}(v)).$$

2.5. ve-degree Randic index.

$$R^{ve}(G) = \sum_{uv \in E} (deg_{ve}(u) \times deg_{ve}(v))^{-\frac{1}{2}}.$$

2.6. ev-degree Randic index.

$$R^{ev}(G) = \sum_{e \in E} deg_{ve}(e)^{-\frac{1}{2}}.$$

2.7. ve-degree atom bond connectivity index.

$$ABC^{ve}(G) = \sum_{uv \in E} \sqrt{\frac{deg_{ve}(u) + deg_{ve}(v) - 2}{deg_{ve}(u) \times deg_{ve}(v)}}.$$

2.8. ve-degree geometric-arithmetic index.

$$GA^{ve}(G) = \sum_{uv \in E} \frac{2\sqrt{deg_{ve}(u) \times deg_{ve}(v)}}{deg_{ve}(u) + deg_{ve}(v)}.$$

2.9. ve-degree harmonic index.

$$\mathcal{H}^{ve}(G) = \sum_{uv \in E} \frac{2}{deg_{ve}(u) + deg_{ve}(v)}.$$

2.10. ve-degree sum-connectivity index.

$$\chi^{ve}(G) = \sum_{uv \in E} (deg_{ve}(u) + deg_{ve}(v))^{-\frac{1}{2}}.$$

3. Topological indices of Biswapped networks

The biswapped interconnection network denoted by $Bsw(G)$ is a graph obtained from a base graph G whose construction for the vertex and edge set is defined as follows:

$$V(Bsw(G)) = \{\langle 0, a, b\rangle, \langle 1, a, b\rangle | a, b \in V(G)\}$$

$$E(Bsw(G)) = \{(\langle 0, a, b_1\rangle, \langle 0, a, b_2\rangle), (\langle 1, a, b_1\rangle, \langle 1, a, b_2\rangle) | (b_1, b_2) \in E(G), \\ a \in V(G)\} \cup \{(\langle 0, a, b\rangle, \langle 1, b, a\rangle) | a, b \in V(G)\}.$$

The ve-degree and ve-degree base results related to path and complete graph as basis graph for the biswapped network are discussed in the followings:

4. RESULTS FOR BISWAPPED NETWORKS $Bsw(P_n)$

We will take path graph as base graph which is denoted by P_n of n vertices and $Bsw(P_n)$ be the biswapped network. Then $|V(Bsw(P_n))| = 2n^2$ and $|E(Bsw(P_n))| = 3n^2 - 2n$ respectively. Figure 1 shows a biswapped network with the basis graph P_6.

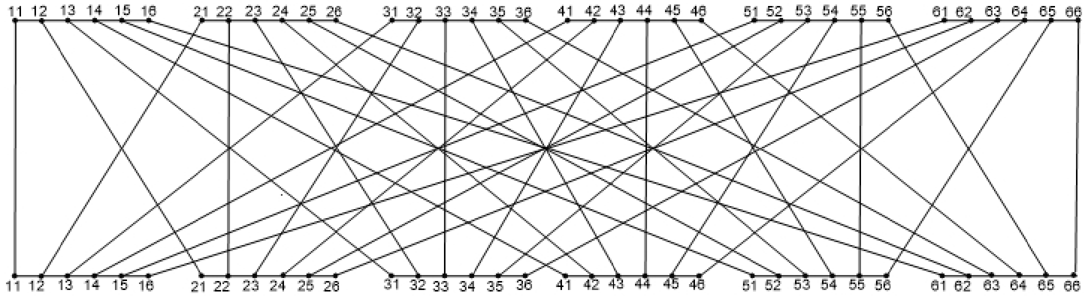

FIGURE 1. OTIS Biswapped network $Bsw(P_6)$.

In Table 1, We partition the edges, based on ev-degree of the $Bsw(P_n)$ for $n \geq 5$. In Table 2 and 3, we partition the vertices and edges based on ve-degree of $Bsw(P_n)$ for $n \geq 5$.

TABLE 1. Edge partition of $Bsw(P_n)$

Number of edges	**Degree of its end vertices**	ev**-degrees**
4	$(2,2)$	4
$\lfloor \frac{(n+3)^2}{2} \rfloor$	$(2,3)$	5
$\lceil \frac{5n^2-10n-17}{2} \rceil$	$(3,3)$	6

4.1. **Computing indices for $Bsw(P_n)$ formulae.**

We will calculate the ev-degree and ve-degree base indices in this section as defined above and we get their respective formulas.

- ev-degree Zagreb index.

TABLE 2. Vertex partition of $Bsw(P_n)$

Number of vertices	**Degrees**	ve-**degrees**
8	2	5
$4(n-2)$	2	6
8	3	7
$8(n-3)$	3	8
$2(n-2)(n-4)$	3	9

TABLE 3. The ve-degree of the end vertices of edges of $Bsw(P_n)$.

Number of edges	**Degree of its end vertices**	ve-**degrees of its end vertices**
4	$(2,2)$	$(5,5)$
8	$(2,3)$	$(5,7)$
8	$(2,3)$	$(6,7)$
$8(n-3)$	$(2,3)$	$(6,8)$
8	$(3,3)$	$(7,8)$
$4(n-4)$	$(3,3)$	$(8,8)$
$8(n-3)$	$(3,3)$	$(8,9)$
$3n^2-22n+36$	$(3,3)$	$(9,9)$

By using the values calculated in Table 1, the said indices are calculated as:

$$\mathcal{M}^{ev}(Bsw(P_n)) = \sum_{e\in E(Bsw(P_n))} deg_{ev}(e)^2,$$

$$\begin{aligned}
&\mathcal{M}^{ev}(Bsw(P_n))\\
&= 4\times 4^2 + \left\lfloor \frac{(n+3)^2}{2} \right\rfloor \times 5^2 + \left\lceil \frac{5n^2-10n-17}{2} \right\rceil \times 6^2\\
&= 64 + 25\left\lfloor \frac{(n+3)^2}{2} \right\rfloor + 36\left\lceil \frac{5n^2-10n-17}{2} \right\rceil .
\end{aligned}$$

- The first ve-degree Zagreb α index.

By using the values calculated in Table 2, the said indices are calculated as:

$$\mathcal{M}_1^{\alpha ve}(Bsw(P_n)) = \sum_{v\in V(Bsw(P_n))} deg_{ve}(v)^2,$$

$$\begin{aligned}
&\mathcal{M}_1^{\alpha ve}(Bsw(P_n)) \\
&= 8\times 5^2 + 4(n-2)\times 6^2 + 8\times 7^2 + 8(n-3)\times 8^2 + 2(n-2)(n-4)\times 9^2 \\
&= 200 + 144n - 288 + 392 + 512n - 1536 + 162n^2 - 648n - 324n + 1296 \\
&= 162n^2 - 316n + 64.
\end{aligned}$$

- The First ve-degree Zagreb β index.

By using the values calculated in Table 3, the said indices are calculated as:

$$\mathcal{M}_1^{\beta ve}(Bsw(P_n)) = \sum_{uv\in E(Bsw(P_n))} (deg_{ve}(u) + deg_{ve}(v)),$$

$$\begin{aligned}
&\mathcal{M}_1^{\beta ve}(Bsw(P_n)) \\
&= 4\times 10 + 8\times 12 + 8\times 13 + 8(n-3)\times 14 + 8\times 15 + 4(n-4)\times 16 \\
&\quad + 8(n-3)\times 17 + (3n^2 - 22n + 36)\times 18 \\
&= 40 + 96 + 104 + 112n - 336 + 120 + 64n - 256 + 136n - 408 \\
&\quad + 54n^2 - 396n + 648 \\
&= 54n^2 - 84n + 8.
\end{aligned}$$

- The second ve-degree Zagreb index

By using the values calculated in Table 3, the said indices are calculated as:

$$\mathcal{M}_2^{ve}(Bsw(P_n)) = \sum_{uv\in E(Bsw(P_n))} (deg_{ve}(u) \times deg_{ve}(v)),$$

$$\begin{aligned}
&\mathcal{M}_2^{ve}(Bsw(P_n)) \\
&= 4 \times 25 + 8 \times 35 + 8 \times 42 + 8(n-3) \times 48 + 8 \times 56 + 4(n-4) \times 64 \\
&\quad + 8(n-3) \times 72 + (3n^2 - 22n + 36) \times 81 \\
&= 100 + 280 + 336 + 384n - 1152 + 448 + 256n - 1024 + 576n - 1728 \\
&\quad + 243n^2 - 1782n + 2916 \\
&= 243n^2 - 566n + 176.
\end{aligned}$$

- The ve-degree Randic index

By using the values calculated in Table 3, the said indices are calculated as:

$$\mathcal{R}^{ve}(Bsw(P_n)) = \sum_{uv\in E(Bsw(P_n))} (deg_{ve}(u) \times deg_{ve}(v))^{-\frac{1}{2}},$$

$$\begin{aligned}
&\mathcal{R}^{ve}(Bsw(P_n)) \\
&= 4 \times 25^{-\frac{1}{2}} + 8 \times 35^{-\frac{1}{2}} + 8 \times 42^{-\frac{1}{2}} + 8(n-3) \times 48^{-\frac{1}{2}} + 8 \times 56^{-\frac{1}{2}} \\
&\quad + 4(n-4) \times 64^{-\frac{1}{2}} + 8(n-3) \times 72^{-\frac{1}{2}} + (3n^2 - 22n + 36) \times 81^{-\frac{1}{2}} \\
&= \frac{4}{5} + \frac{8}{\sqrt{35}} + \frac{8}{\sqrt{42}} + \frac{2}{\sqrt{3}}n - \frac{6}{\sqrt{3}} + \frac{4}{\sqrt{14}} + \frac{1}{2}n - 2 + \frac{4}{3\sqrt{2}}n - \frac{4}{\sqrt{2}} \\
&\quad + \frac{1}{3}n^2 - \frac{22}{9}n + 4
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{3}n^2 + \left(\frac{2}{\sqrt{3}} + \frac{1}{2} + \frac{4}{3\sqrt{2}} - \frac{22}{9}\right)n \\
&\quad + \left(\frac{4}{5} + \frac{8}{\sqrt{35}} + \frac{8}{\sqrt{42}} - \frac{6}{\sqrt{3}} + \frac{4}{\sqrt{14}} - 2 - \frac{4}{\sqrt{2}} + 4\right) \\
&= \frac{1}{3}n^2 + \left(\frac{2}{\sqrt{3}} + \frac{4}{3\sqrt{2}} - \frac{35}{18}\right)n \\
&\quad + \left(\frac{14}{5} + \frac{8}{\sqrt{35}} + \frac{8}{\sqrt{42}} - \frac{6}{\sqrt{3}} + \frac{4}{\sqrt{14}} - \frac{4}{\sqrt{2}}\right)
\end{aligned}
$$

- The ev-degree Randic index.

By using the values calculated in Table 1, the said indices are calculated as:

$$
\mathcal{R}^{ev}(Bsw(P_n)) = \sum_{e \in E(Bsw(P_n))} deg_{ev}(e)^{-\frac{1}{2}},
$$

$$
\begin{aligned}
&\mathcal{R}^{ev}(Bsw(P_n)) \\
&= 4 \times 4^{-\frac{1}{2}} + \left\lfloor \frac{(n+3)^2}{2} \right\rfloor \times 5^{-\frac{1}{2}} + \left\lceil \frac{5n^2 - 10n - 17}{2} \right\rceil \times 6^{-\frac{1}{2}} \\
&= \frac{4}{\sqrt{4}} + \left\lfloor \frac{(n+3)^2}{2} \right\rfloor \frac{1}{\sqrt{5}} + \left\lceil \frac{5n^2 - 10n - 17}{2} \right\rceil \frac{1}{\sqrt{6}} \\
&= 2 + \left\lfloor \frac{(n+3)^2}{2} \right\rfloor \frac{1}{\sqrt{5}} + \left\lceil \frac{5n^2 - 10n - 17}{2} \right\rceil \frac{1}{\sqrt{6}}
\end{aligned}
$$

- The atom bond connectivity index

By using the values calculated in Table 3, the said indices are calculated as:

$$\mathcal{ABC}^{ve}(Bsw(P_n)) = \sum_{uv\in E(Bsw(P_n))} \sqrt{\frac{deg_{ve}(u)+deg_{ve}(v)-2}{deg_{ve}(u)\times deg_{ve}(v)}},$$

$$\begin{aligned}
&\mathcal{ABC}^{ve}(Bsw(P_n))\\
&= 4\times\sqrt{\frac{10-2}{25}} + 8\times\sqrt{\frac{12-2}{35}} + 8\times\sqrt{\frac{13-2}{42}} + 8(n-3)\times\sqrt{\frac{14-2}{48}}\\
&\quad + 8\times\sqrt{\frac{15-2}{56}} + 4(n-4)\times\sqrt{\frac{16-2}{64}} + 8(n-3)\times\sqrt{\frac{17-2}{72}}\\
&\quad + (3n^2-22n+36)\times\sqrt{\frac{18-2}{81}}\\
&= \frac{8\sqrt{2}}{5} + \frac{8\sqrt{2}}{\sqrt{7}} + \frac{8\sqrt{11}}{\sqrt{42}} + 4n - 12 + \frac{4\sqrt{13}}{\sqrt{14}} + \frac{\sqrt{14}}{2}n - 2\sqrt{14} + \frac{4\sqrt{15}}{3\sqrt{2}}n\\
&\quad - \frac{4\sqrt{15}}{\sqrt{2}} + \frac{4}{3}n^2 - \frac{88}{9}n + 16\\
&= \frac{4}{3}n^2 + \left(4 + \frac{\sqrt{14}}{2} + \frac{4\sqrt{15}}{3\sqrt{2}} - \frac{88}{9}\right)n\\
&\quad + \left(\frac{8\sqrt{2}}{5} + \frac{8\sqrt{2}}{\sqrt{7}} + \frac{8\sqrt{11}}{\sqrt{42}} - 12 + \frac{4\sqrt{13}}{\sqrt{14}} - 2\sqrt{14} - \frac{4\sqrt{15}}{\sqrt{2}} + 16\right)\\
&= \frac{4}{3}n^2 + \left(\frac{\sqrt{14}}{2} + \frac{4\sqrt{15}}{3\sqrt{2}} - \frac{52}{9}\right)n\\
&\quad + \left(\frac{8\sqrt{2}}{5} + \frac{8\sqrt{2}}{\sqrt{7}} + \frac{8\sqrt{11}}{\sqrt{42}} + \frac{4\sqrt{13}}{\sqrt{14}} - 2\sqrt{14} - \frac{4\sqrt{15}}{\sqrt{2}} + 4\right)
\end{aligned}$$

- The geometric arithmetic index

By using the values calculated in Table 3, the said indices are calculated as:

$$\mathcal{GA}^{ve}(Bsw(P_n)) = \sum_{uv\in E(Bsw(P_n))} \frac{2\sqrt{deg_{ve}(u)\times deg_{ve}(v)}}{deg_{ve}(u)+deg_{ve}(v)},$$

$$\begin{aligned}
&\mathcal{GA}^{ve}(Bsw(P_n))\\
&= 4\times\frac{2\sqrt{25}}{10} + 8\times\frac{2\sqrt{35}}{12} + 8\times\frac{2\sqrt{42}}{13} + 8(n-3)\times\frac{2\sqrt{48}}{14} + 8\times\frac{2\sqrt{56}}{15}\\
&\quad + 4(n-4)\times\frac{2\sqrt{64}}{16} + 8(n-3)\times\frac{2\sqrt{72}}{17} + (3n^2-22n+36)\times\frac{2\sqrt{81}}{18}\\
&= 4 + \frac{4\sqrt{35}}{3} + \frac{16\sqrt{42}}{13} + \frac{32\sqrt{3}}{7}n - \frac{96\sqrt{3}}{7} + \frac{32\sqrt{14}}{15} + 4n - 16 + \frac{96\sqrt{2}}{17}n\\
&\quad - \frac{288\sqrt{2}}{17} + 3n^2 - 22n + 36\\
&= 3n^2 + \left(\frac{32\sqrt{3}}{7} + 4 + \frac{96\sqrt{2}}{17} - 22\right)n\\
&\quad + \left(4 + \frac{4\sqrt{35}}{3} + \frac{16\sqrt{42}}{13} - \frac{96\sqrt{3}}{7} + \frac{32\sqrt{14}}{15} - 16 - \frac{288\sqrt{2}}{17} + 36\right)\\
&= 3n^2 + \left(\frac{32\sqrt{3}}{7} + \frac{96\sqrt{2}}{17} - 18\right)n\\
&\quad + \left(\frac{4\sqrt{35}}{3} + \frac{16\sqrt{42}}{13} - \frac{96\sqrt{3}}{7} + \frac{32\sqrt{14}}{15} - \frac{288\sqrt{2}}{17} + 24\right)
\end{aligned}$$

- The harmonic index

By using the values calculated in Table 3, the said indices are calculated as:

$$\mathcal{H}^{ve}(Bsw(P_n)) = \sum_{uv\in E(Bsw(P_n))} \frac{2}{deg_{ve}(u)+deg_{ve}(v)},$$

$$\begin{aligned}
&\mathcal{H}^{ve}(Bsw(P_n))\\
&= 4\times\frac{2}{10}+8\times\frac{2}{12}+8\times\frac{2}{13}+8(n-3)\times\frac{2}{14}+8\times\frac{2}{15}\\
&\quad +4(n-4)\times\frac{2}{16}+8(n-3)\times\frac{2}{17}+(3n^2-22n+36)\times\frac{2}{18}\\
&= \frac{4}{5}+\frac{4}{3}+\frac{16}{13}+\frac{8}{7}n-\frac{24}{7}+\frac{16}{15}+\frac{1}{2}n-2+\frac{16}{17}n-\frac{48}{17}+\frac{1}{3}n^2-\frac{22}{9}n+4\\
&= \frac{1}{3}n^2+\frac{299}{2142}n+\frac{1382}{7735}.
\end{aligned}$$

- The sum-connectivity index

By using the values calculated in Table 3, the said indices are calculated as:

$$\chi^{ve}(Bsw(P_n)) = \sum_{uv\in E(Bsw(P_n))} (deg_{ve}(u)+deg_{ve}(v))^{-\frac{1}{2}},$$

$$\begin{aligned}
&\chi^{ve}(Bsw(P_n))\\
&= 4\times 10^{-\frac{1}{2}}+8\times 12^{-\frac{1}{2}}+8\times 13^{-\frac{1}{2}}+8(n-3)\times 14^{-\frac{1}{2}}+8\times 15^{-\frac{1}{2}}\\
&\quad +4(n-4)\times 16^{-\frac{1}{2}}+8(n-3)\times 17^{-\frac{1}{2}}+(3n^2-22n+36)\times 18^{-\frac{1}{2}}\\
&= \frac{4}{\sqrt{10}}+\frac{4}{\sqrt{3}}+\frac{8}{\sqrt{13}}+\frac{8}{\sqrt{14}}n-\frac{24}{\sqrt{14}}+\frac{8}{\sqrt{15}}+n-4+\frac{8}{\sqrt{17}}n-\frac{24}{\sqrt{17}}\\
&\quad +\frac{1}{\sqrt{2}}n^2-\frac{22}{3\sqrt{2}}n+\frac{12}{\sqrt{2}}\\
&= \frac{1}{\sqrt{2}}n^2+\left(\frac{8}{\sqrt{14}}+1+\frac{8}{\sqrt{17}}-\frac{22}{3\sqrt{2}}\right)n\\
&\quad +\left(\frac{4}{\sqrt{10}}+\frac{4}{\sqrt{3}}+\frac{8}{\sqrt{13}}-\frac{24}{\sqrt{14}}+\frac{8}{\sqrt{15}}-4-\frac{24}{\sqrt{17}}+\frac{12}{\sqrt{2}}\right).
\end{aligned}$$

RESULTS FOR BISWAPPED NETWORKS $Bsw(K_n)$

We will take complete graph as base graph which is denoted by K_n of n vertices and $Bsw(K_n)$ be the biswapped network. Then $|V(Bsw(K_n))| = 2n^2$ and $|E(Bsw(P_n))| = n^3$ respectively. Figure 2 shows a biswapped network with the basis graph K_4.

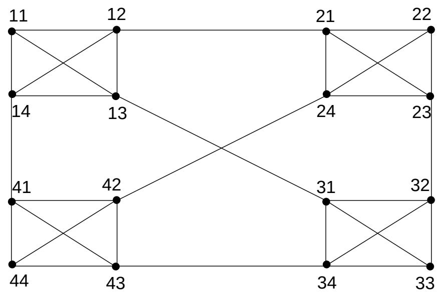

FIGURE 2. OTIS biswapped network $Bsw(K_4)$.

In Table 4, We partition the edges, based on ev-degree of the $Bsw(Kn)$.

TABLE 4. Edge partition of $Bsw(Kn)$

Number of edges	**Degree of its end vertices**	ev**-degrees**
n^2	(n, n)	$2n$
$n^2(n-1)$	(n, n)	$n+2$

In Table 5 and 6, we partition the vertices and edges based on ve-degree of $Bsw(Kn)$.

TABLE 5. Vertex partition of $Bsw(Kn)$

Number of vertices	**Degrees**	ve**-degrees**
$2n^2$	n	$\frac{n^2+3n-2}{2}$

TABLE 6. The ve-degree of the end vertices of edges of $Bsw(Kn)$.

Number of edges	**Degree of its end vertices**	ve**-degrees of its end vertices**
n^3	(n, n)	$(\frac{n^2+3n-2}{2}, \frac{n^2+3n-2}{2})$

4.2. **Computing indices for $Bsw(Kn)$ formulas.** In this section we will calculate ev-degree and ve-degree based indices such as the ev-degree Zagreb index, first ve-degree Zagreb α index, first ve-degree Zagreb β index, the second ve-degree Zagreb index, ve-degree Randic index, ev-degree Randic index, ve-degree atom-bond connectivity (ve-$\mathcal{ABC}$) index, ve-degree geometric-arithmetic (ve-$\mathcal{GA}$) index, ve-degree harmonic (ve-$\mathcal{H}$) index and ve-degree sum-connectivity (ve-χ) for $Bsw(Kn)$ formulas.

- ev-degree Zagreb index

By using the values calculated in Table 4, we compute the ev-degree based Zagreb index:

$$\mathcal{M}^{ev}(Bsw(Kn)) = \sum_{e \in E(Bsw(Kn))} deg_{ev}(e)^2,$$

$$\begin{aligned}\mathcal{M}^{ev}(Bsw(Kn)) &= n^2 \times (2n)^2 + n^2(n-1) \times (n+2)^2 \\ &= 4n^4 + (n^3 - n^2)(n^2 + 4 + 4n) \\ &= n^5 + 7n^4 - 4n^2.\end{aligned}$$

- The first ve-degree Zagreb α index

By using the values calculated in Table 5, the said indices are calculated as:

$$\mathcal{M}_1^{\alpha ve}(Bsw(Kn)) = \sum_{v \in V(Bsw(Kn))} deg_{ve}(v)^2,$$

$$\begin{aligned}\mathcal{M}_1^{\alpha ve}(Bsw(Kn)) &= 2n^2 \times (\frac{n^2+3n-2}{2})^2 \\ &= \frac{1}{2}n^6 + 3n^5 + \frac{5}{2}n^4 - 6n^3 + 2n^2.\end{aligned}$$

- The first ve-degree Zagreb β index

By using the values calculated in Table 6, the said indices are calculated as:

$$\mathcal{M}_1^{\beta ve}(Bsw(Kn)) = \sum_{uv \in E(Bsw(Kn))} (deg_{ve}(u) + deg_{ve}(v)),$$

$$\begin{aligned}\mathcal{M}_1^{\beta ve}(Bsw(Kn)) &= n^3 \times n^2 + 3n - 2 \\ &= n^5 + 3n^4 - 2n^3.\end{aligned}$$

- The second ve-degree Zagreb index

By using the values calculated in Table 6, the said indices are calculated as:

$$\mathcal{M}_2^{ve}(Bsw(Kn)) = \sum_{uv\in E(Bsw(Kn))} (deg_{ve}(u) \times deg_{ve}(v)),$$

$$\mathcal{M}_2^{ve}(Bsw(Kn)) = n^3 \times \frac{n^2+3n-2}{2}^2$$
$$= \frac{1}{4}n^7 + \frac{3}{2}n^6 + \frac{5}{4}n^5 - 3n^4 + n^3.$$

- The ve-degree Randic index

By using the values calculated in Table 6, the said indices are calculated as:

$$\mathcal{R}^{ve}(Bsw(Kn)) = \sum_{uv\in E(Bsw(Kn))} (deg_{ve}(u) \times deg_{ve}(v))^{-\frac{1}{2}},$$

$$\mathcal{R}^{ve}(Bsw(Kn)) = n^3 \times ((\frac{n^2+3n-2}{2})^2)^{-\frac{1}{2}}$$
$$= \frac{2n^3}{n^2+3n-2}.$$

- The ev-Ddegree Randic index

By using the values calculated in Table 4, the said indices are calculated as:

$$\mathcal{R}^{ev}(Bsw(Kn)) = \sum_{e\in E(Bsw(Kn))} deg_{ev}(e)^{-\frac{1}{2}},$$

$$\mathcal{R}^{ev}(Bsw(Kn)) = n^2 \times 2n^{-\frac{1}{2}} + n^2(n-1) \times (n+2)^{-\frac{1}{2}}.$$

- The atom bond connectivity index

By using the values calculated in Table 6, the said indices are calculated as:

$$\mathcal{ABC}^{ve}(Bsw(Kn)) = \sum_{uv\in E(Bsw(Kn))} \sqrt{\frac{deg_{ve}(u)+deg_{ve}(v)-2}{deg_{ve}(u)\times deg_{ve}(v)}},$$

$$\mathcal{ABC}^{ve}(Bsw(Kn)) = n^3 \times \sqrt{\frac{n^2+3n-2-2}{(\frac{n^2+3n-2}{2})^2}}$$
$$= \frac{2n^3\sqrt{n^2+3n-4}}{n^2+3n-2}.$$

- The geometric arithmetic index

By using the values calculated in Table 6, the said indices are calculated as:

$$\mathcal{GA}^{ve}(Bsw(Kn)) = \sum_{uv\in E(Bsw(Kn))} \frac{2\sqrt{deg_{ve}(u)\times deg_{ve}(v)}}{deg_{ve}(u)+deg_{ve}(v)},$$

$$\mathcal{GA}^{ve}(Bsw(Kn)) = n^3 \times \frac{2\sqrt{(\frac{n^2+3n-2}{2})^2}}{n^2+3n-2}$$
$$= n^3.$$

- The harmonic index

By using the values calculated in Table 6, the said indices are calculated as:

$$\mathcal{H}^{ve}(Bsw(Kn)) = \sum_{uv\in E(Bsw(Kn))} \frac{2}{deg_{ve}(u)+deg_{ve}(v)},$$

$$\mathcal{H}^{ve}(Bsw(Kn)) = n^3 \times \frac{2}{n^2+3n-2}$$
$$= \frac{2n^3}{n^2+3n-2}.$$

- The Sum-Connectivity Index.

By using the values calculated in Table 6, the said indices are calculated as:

$$\chi^{ve}(Bsw(Kn)) = \sum_{uv\in E(Bsw(Kn))} (deg_{ve}(u)+deg_{ve}(v))^{-\frac{1}{2}},$$

$$\chi^{ve}(Bsw(Kn)) = n^3 \times (n^2+3n-2)^{-\frac{1}{2}}$$

5. Numerical and Graphical Representation

We have results of ten different types of indices which are related to ve-degree and ev-degree for both numerically and graphically for the OTIS biswapped network with the basis graph as path and complete graph. We have compare their explicit formula with other indices and found their behavior about increasing or decreasing for the given ten different types of indices.

(i) The numerical and graphical representation of OTIS biswapped network of path are shown below. It can be observe (Table 7, Figure 3)that the values of all indices increase with increasing value of n except ev-degree zaghreb index. It decreases at $n = 2$ and then increase with growing values of n.

TABLE 7. Numerical Comparison of $M^{ev}, M_1^{\alpha ve}, M_1^{\beta ve}$ and M_2^{ve}

n	M^{ev}	$M_1^{\alpha ve}$	$M_1^{\beta ve}$	M_2^{ve}
1	48	-90	-22	-147
2	40	80	56	16
3	478	574	242	665
4	1096	1392	536	1800
5	1908	2534	938	3421
6	2936	4000	1448	5528
7	4158	5790	2066	8121
8	5596	7904	2792	11200
9	7228	10342	3626	14765
10	9076	13104	4568	18816

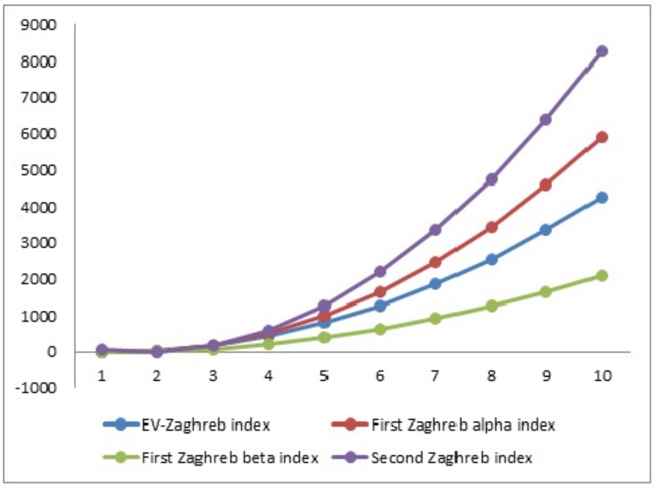

FIGURE 3. Graphical Comparison of $M^{ev}, M_1^{\alpha ve}, M_1^{\beta ve}$ and M_2^{ve}

(ii) Also, It can be seen (Table 8, Figure 4) that ve-degree Randic index and ve-degree harmonic (ve-$\mathcal{H}$) index are overlapped for all values of n.

TABLE 8. Numerical Comparison of $R^{ve}, R^{ev}, ABC^{ve}, GA^{ve}, H^{ve} and \chi^{ve}$

n	R^{ve}	R^{ev}	ABC^{ve}	GA^{ve}	H^{ve}	χ^{ve}
1	0.649	1.088	1.128	1.038	0.651	0.709
2	1.801	3.692	4.872	7.942	1.789	2.723
3	3.619	9.638	11.282	20.846	3.593	6.151
4	6.103	17.624	20.358	39.75	6.063	10.993
5	9.253	28.136	32.1	64.654	9.199	17.249
6	13.069	41.096	46.508	95.558	13.001	24.919
7	17.551	56.582	63.582	132.462	17.469	34.003
8	22.699	74.516	83.322	175.366	22.603	44.501
9	28.513	94.976	105.728	224.27	28.403	56.413
10	34.993	117.884	130.8	279.174	34.869	69.739

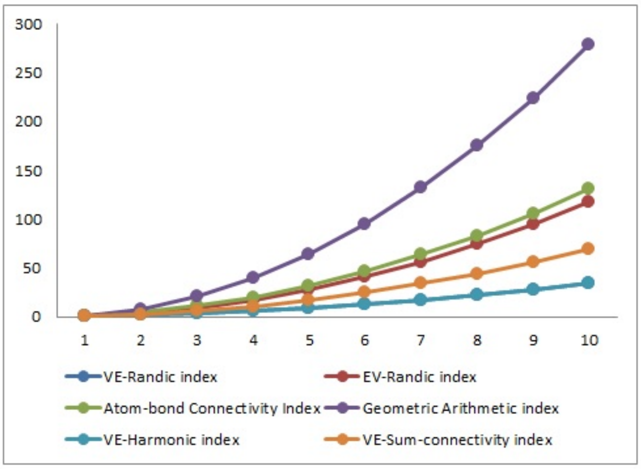

FIGURE 4. Graphical Comparison of R^{ev}, GA^{ve}, ABC^{ve} H^{ve}, R^{ve} and χ^{ve}

(iii) The graphical representation of OTIS biswapped network of complete graph are shown below. It can be perceive (Table 9, Figure 5) that the values of all indices increase with increasing value of n. But second ve-degree Zagreb index behave differently.It depicts a large increament with increasing value of n.

TABLE 9. Numerical Comparison of M^{ev}, $M_1^{\alpha ve}$, $M_1^{\beta ve}$ and M_2^{ve}

n	M^{ev}	$M_1^{\alpha ve}$	$M_1^{\beta ve}$	M_2^{ve}
1	4	2	2	1
2	128	128	64	128
3	774	1152	432	1728
4	2752	5408	1664	10816
5	7400	18050	4750	45125
6	16704	48672	11232	146016
7	33418	113288	23324	396508
8	61184	236672	44032	946688
9	104652	455058	77274	2047761
10	169600	819200	128000	4096000

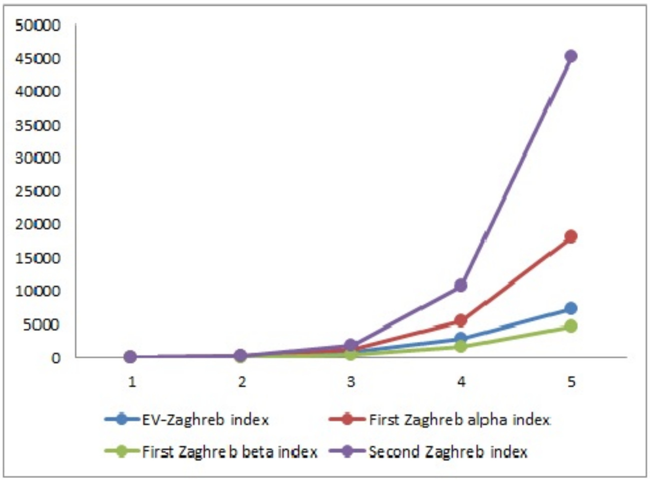

FIGURE 5. Graphical Comparison of M^{ev}, $M_1^{\alpha ve}$, $M_1^{\beta ve}$ and M_2^{ve}

(iv) In OTIS biswapped network of complete graph, ve-degree Randic index and ve-degree harmonic (ve-$\mathcal{H}$) index gives same values for all values of n (Table 10, Figure 6).

TABLE 10. Numerical Comparison of $R^{ve}, R^{ev}, ABC^{ve}, GA^{ve}, H^{ve} and \chi^{ve}$

n	R^{ve}	R^{ev}	ABC^{ve}	GA^{ve}	H^{ve}	χ^{ve}
1	1	0.707	0	1	1	0.707
2	2	4	4.899	8	2	2.828
3	3.375	11.724	12.628	27	3.375	6.75
4	4.923	25.253	24.118	64	4.923	12.551
5	6.579	45.702	39.474	125	6.579	20.278
6	8.308	74.032	58.746	216	8.308	29.954
7	10.088	111.096	81.955	343	10.088	41.595
8	11.907	157.67	109.129	512	11.907	55.21
9	13.755	214.471	140.274	729	13.755	70.807
10	15.625	282.168	175.39	1000	15.625	88.388

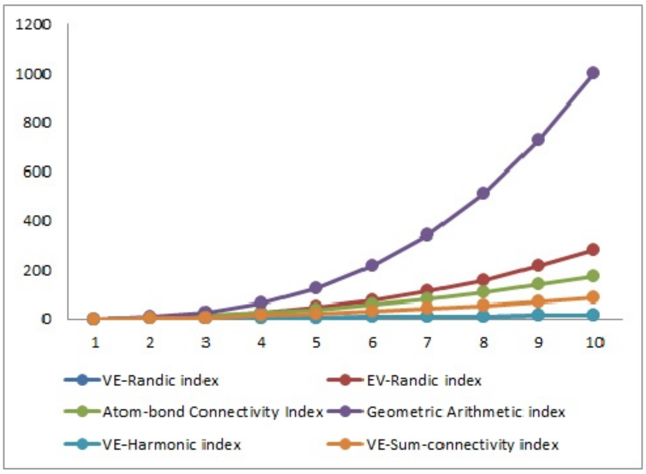

FIGURE 6. Graphical Comparison of R^{ev}, GA^{ve}, ABC^{ve} H^{ve}, R^{ve} and χ^{ve}

6. CONCLUSION

The topological descriptors play a very significant role in the study of graphs and networks which have very attractive application in many area of science especially in computer science where various graph invariants based assessments are used to deal with several challenging schemes. To predict, analyzed and approximate the biological properties of different indices related to graphs and

networks, the (QSPRs) and (QSARs) helps in finding these parameters.
The present paper is a contribution in find the ve-degree and ev-degree base topological indices for various kind of indices as found and calculated above for the OTIS biswapped network of path and complete graph as basis graph.

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CENTRE FOR ADVANCED STUDIES IN PURE AND APPLIED MATHEMATICS
BAHAUDDIN ZAKARIYA UNIVERSITY, MULTAN
MULTAN, PAKISTAN.
Email address: zahranida512@gmail.com

CENTRE FOR ADVANCED STUDIES IN PURE AND APPLIED MATHEMATICS
BAHAUDDIN ZAKARIYA UNIVERSITY, MULTAN
MULTAN, PAKISTAN.
Email address: mibtufail@gmail.com