ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **10** (2021), no.6, 2909–2917
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.6.16

IMPLICATION - BASED FUZZY TERNARY SEMIGROUPS FOR ISOLATED SYSTOLIC HYPERTENSION USING PENTAGONAL FUZZY NUMBER MATRIX

G. Sheeja[1] and E. Rupavathi

ABSTRACT. In this paper, medical diagnosis concerning isolated systolic hypertension and a study on implication operators is introduced, but fuzzy pentagonal number matrix concepts developed to give a numerical example and graphical representation are also established.

1. INTRODUCTION

Investigation of fuzzy sets and fuzzy set operators is an essential part of the study in both mathematical research fields and in the knowledge based medical field. Also an investigation on fuzzy set theory and ternary semigroups continued by several authors [2, 3, 7]. The concept of fuzzy theorem was introduced by Zadeh [10] in 1965. Furthermore several authors continued their innovative research in medical field together with mathematical studies in [8]. In particular the survey on pentagonal fuzzy related matrices and implication theory with respect to medical knowledge paper studied by several authors [1, 4–6, 9]. In this paper the description under medical diagnosis and the pentagonal fuzzy

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 26A33.

Key words and phrases. Fuzzy ternary semi group, Fuzzy ternary subsemigroup, pentagonal fuzzy number matrix, implication based fuzzy ternary subsemigroup.

Submitted: 03.06.2021; *Accepted:* 21.06.2021; *Published:* 30.06.2021.

number matrix are the consequences of the consideration for the Luckasiewicz operator based on implication.

2. PRELIMINARIES

Definition 2.1. *A set B is said to be a fuzzy if for a non-zero set $S(s)$ defined to be $B = \{S, \gamma_S(S), s \in S\}$. Here $\gamma_S : S \to [0, 1]$, a membership functions of S and $\gamma_S(S)$ degree of the membership respectively. Also a fuzzy set paired B, by choosing the particular element in the universe S and its degree of membership that is each s is assigned a value in the range (0, 1) indicating the extent to which s as the attribute S which is represented as $S = \{S_1, \gamma_S(S_1), \cdots, (S_k, \gamma_S(S_k))\}$.*

Definition 2.2. *Given a fuzzy set $B = \mathcal{R} \to [0, 1]$ defined on $\mathcal{R}$, real number is said to be a fuzzy number if its satisfies certain conditions: B is of normal height, convex fuzzy set and piecewise continuous.*

Definition 2.3. *Suppose that $B = (b_1, b_2, b_3, b_4, b_5)$ this representation is interpreted as membership function $\mu_{\overline{B}}(x_i)$, a pentagonal fuzzy number where b_1, b_2, b_3, b_4, b_5 are real numbers so its membership function is*
Consider the three pentagonal fuzzy number matrices of same order as $(r_{ij})_{m\times n}$

(1) $(p_1^{'}, p_2^{'}, p_3^{'}, p_4^{'}, p_5^{'}) + (q_1^{'}, q_2^{'}, q_3^{'}, q_4^{'}, q_5^{'}) + (r_1^{'}, r_2^{'}, r_3^{'}, r_4^{'}, r_5^{'}) = (p_1^{'} + q_1^{'} + r_1^{'}, p_2^{'} + q_2^{'} + r_2^{'}, p_3^{'} + q_3^{'} + r_3^{'}, p_4^{'} + q_4^{'} + r_4^{'}, p_5^{'} + q_5^{'} + r_5^{'})$

(2) $(p_1^{'}, p_2^{'}, p_3^{'}, p_4^{'}, p_5^{'}) - (q_1^{'}, q_2^{'}, q_3^{'}, q_4^{'}, q_5^{'}) - (r_1^{'}, r_2^{'}, r_3^{'}, r_4^{'}, r_5^{'}) = (p_1^{'} - q_1^{'} - r_1^{'}, p_2^{'} - q_2^{'} - r_2^{'}, p_3^{'} - q_3^{'} - r_3^{'}, p_4^{'} - q_4^{'} - r_4^{'}, p_5^{'} - q_5^{'} - r_5^{'})$

Definition 2.4. *The generalization of implication with fuzzy is the very classical fuzzy logic like t- norm and t-conorm respectively. Also operators with implication like gaines rescher, godel contrapositive and luckasiewicz operators are the part of our study which is introduced in [7]. The four operators with the membership function are*

(1) *Gaines-Rescher implication operator (i_{gr}):* $i_{gr}(a^{'}, b^{'}) = \begin{cases} 1 & \text{if } a^{'} \leq b^{'} \\ 0 & \text{otherwise} \end{cases}$;

(2) *Godel limplication operator (i_g):* $i_g(a^{'}, b^{'}) = \begin{cases} 1 & \text{if } a^{'} \leq b^{'} \\ b^{'} & \text{otherwise} \end{cases}$ *z;*

(3) *Contrapositive of godel implication operator (i_{cg}):*

$$i_{cg}(a^{'}, b^{'}) = \begin{cases} 1 & \textit{if } a^{'} \leq b^{'} \\ 1 - a^{'} & \textit{otherwise} \end{cases};$$

(4) *Luckasiewics implication operator (i_{li}):*

$$i_{li}(a^{'}, b^{'}) = \begin{cases} 1 & \textit{if } a^{'} \leq b^{'} \\ 1 - a^{'} + b^{'} & \textit{otherwise} \end{cases}.$$

3. Pentagonal numbered matrix related to fuzzy algorithm concerning isolated systolic hypertension model:

Here the relation between the patients (R) and diseases (S) affected can be identified under medical diagnosis with the knowledge of fuzzy environment and G denote the treatment stages that corresponds four implication operator such as gaines rescher, godel contrapositive and luckasiewicz operator respectively. For the pentagonal fuzzy number matrix $A = (p_{ij})_{m\times n}$ where $p_{ij} = (p_{ijl}, p_{ijm}, p_{ijn}, p_{ijr}, p_{ijs})$ with $0 \leq p_{ijl} \leq p_{ijm} \leq p_{ijn} \leq p_{ijr} \leq p_{ijs} \leq 1$ where $0 \leq \frac{p_{ijl}}{10} \leq \frac{p_{ijm}}{10} \leq \frac{p_{ijn}}{10} \leq \frac{p_{ijr}}{10} \leq \frac{p_{ijs}}{10} \leq 1$.

Step i: Firstly consider pentagonal fuzzy number matrix (R, S) over R where the function $f_1 : S \to f(R)$, this pentagonal matrix is denoted by F_0 describing the pentagonal fuzzy event matrix or anxious patient - medicine pentagonal fuzzy number matrix.

Step ii: Following the above (i) , let us take another pentagonal numbered matrix related to fuzzy (f_2, R) over G and the function $f_2 : R \to f^{'}(G)$ a new pentagonal numbered matrix related to fuzzy is obtained and is denoted by F_G which related anxious patient- treatment stages pentagonal fuzzy number matrix.

Step iii: The membership function for pentagonal numbered matrix related to fuzzy for the above step i and step ii is $(F_0)_{mem}$ and $(F_G)_{mem}$.

Step iv: Using max-min composition the relation matrices $F^{'}, F^{''}$ are calculated using the formulas $F^{'} = (F_G)_{mem} * (F_0)_{mem}, F^{''} = (F_G)_{mem} * (j - (F_0)_{mem})$, j a pentagonal numbered matrix related to fuzzy membership function in which

all the entries are (1,1,1,1,1) which is $((F_0)_{mem})^c$ called non anxious-patient-medicine pentagonal numbered matrix related to fuzzy. Further $F''' = (J - (F_G)_{mem}) * (F_0)_{mem}$, $(J-(F_G)_{mem})$ is $((F_G)_{mem})^c$ called the non treatment stages-medicine pentagonal fuzzy number matrix. Also F', F'', F''', F^{iv} calculated using the definition of subtraction of pentagonal numbered matrix related to fuzzy and the elements is of the form $q_{ij} = (q_{ijl}, q_{ijm}, q_{ijn}, q_{ijr}, q_{ijs})$. But the elements of $F^v = F' - F'''$ is of the form $r_{ij} = (r_{ijl}, r_{ijm}, r_{ijn}, r_{ijr}, r_{ijs})$ where $0 \leq q_{ijl} \leq q_{ijm} \leq q_{ijn} \leq q_{ijr} \leq q_{ijs} \leq 1, 0 \leq r_{ijl} \leq r_{ijm} \leq r_{ijn} \leq r_{ijr} \leq r_{ijs} \leq 1$.

Step v: Calculate F^{vi} the conclusion is over and the algorithm will be processed only if the numerical example given below with the knowledge of medical model which is taken for our study.

Example 1. *We consider the set $T = \{t_1, t_2, t_3, t_4\}$, R denote the relation matrix*

$$(r_0)_{mem} = \begin{pmatrix} (2.4, 5.6, 8.2, 9.1, 9.3) & (1.6, 4.85, 7.55, 9, 9.2) \\ (1.90, 5.25, 7.95, 8.7, 8.9) & (1.7, 4.65, 8.15, 8.4, 8.6) \\ (2.10, 4.95, 7.65, 8.13, 8.7) & (2.0, 5.0, 8.0, 8.1, 9.11) \\ (2.20, 5.45, 7.50, 8.2, 8.65) & (2.30, 4.75, 7.85, 8.22, 9.22) \end{pmatrix}$$

Step 2: *The four implication operator are*

Case i: Gaines Rescher implication operator:
$F(t_1) = \{(p_1, (2.1, 5.5, 8, 8.5, 9.1)); (p_2, (2, 5.0, 8.09, 8.7, 9.34));$
$(p_3, (2.4, 5.49, 8.19, 8.6, 9))\}$

Case ii: G.I. P implication operator
$F(t_2) = \{(p_1, (2.01, 5.6, 8.1, 8.9, 9.01)); (p_2, (2.4, 5.58, 8.3, 8.95, 9.33));$
$(p_3, (2, 5.55, 8.22, 8.93, 9.13))\}$

Case iii: Contraposition of G I P:
$F(t_3) = \{(p_1, (1.95, 5.25, 8, 8.8, 9.09)); (p_2, (1.85, 5.35, 8.23, 8.7, 9.15));$
$(p_3, (2.15, 5.45, 8.05, 9.2, 9.35))\}$

Case iv: The Luckasiewics implication operator:
$F(t_4) = \{(p_1, (2.25, 5.25, 7.93, 9.1, 9.3)); (p_2, (2.35, 5.46, 8.2, 9, 9.21));$
$(p_3, (2.18, 5.56, 7.85, 8.99, 9.23))\}$

It is a ternary semi group. Suppose consider (F, T) another pentagonal Fuzzy

number matrix then the relation matrix r_T and its transpose

$$(r_T)_{mem} = \begin{pmatrix} t_1 & t_2 \\ (0.21, 0.55, 0.8, 0.85, 0.91) & (0.201, 0.56, 0.81, 0.89, 0.901) \\ (0.2, 0.5, 0.809, 0.87, 0.934) & (0.24, 0.558, 0.83, 0.895, 0.933) \\ (0.24, 0.549, 0.819, 0.86, 0.9) & (0.2, 0.555, 0.822, 0.893, 0.913) \end{pmatrix}$$

$$\begin{pmatrix} t_3 & t_4 \\ (0.195, 0.525, 0.8, 0.88, 0.909) & (0.225, 0.525, 0.793, 0.91, 0.93) \\ (0.185, 0.535, 0.823, 0.87, 0.915) & (0.235, 0.546, 0.82, 0.9, 0.921) \\ (0.215, 0.545, 0.803, 0.92, 0.935) & (0.218, 0.556, 0.785, 0.899, 0.923) \end{pmatrix}$$

$$(r_0)_{mem} = \begin{pmatrix} (0.24, 0.56, 0.82, 0.91, 0.93) & (0.16, 0.485, 0.755, 0.9, 0.922) \\ (0.19, 0.525, 0.795, 0.87, 0.89) & (0.17, 0.465, 0.815, 0.84, 0.86) \\ (0.21, 0.495, 0.765, 0.813, 0.87) & (0.2, 0.5, 0.8, 0.81, 0.911) \\ (0.22, 0.545, 0.75, 0.82, 0.865) & (0.23, 0.475, 0.785, 0.822, 0.922) \end{pmatrix}$$

$$(r_2) = \begin{pmatrix} (0.652, 1.01, 0.696, 0.289, 0.406) & (0.726, 1.12, 0.67, 0.55, 0.35) \\ (0.676, 1.00, 0.714, 0.44, 0.409) & (0.696, 1.11, 0.69, 0.55, 0.35) \\ (0.684, 1.03, 0.701, 0.522, 0.495) & (0.77, 1.444, 0.68, 0.56, 0.355) \end{pmatrix}$$

$$(r_3) = \begin{pmatrix} (0.681, 0.977, 0.623, 0.404, 0.31) & (0.61, 0.88, 0.628, 0.39, 0.315) \\ (0.676, 0.989, 0.562, 0.397, 0.263) & (0.596, 0.89, 0.56, 0.39, 0.269) \\ (0.671, 0.953, 0.600, 0.368, 0.294) & (0.569, 1.864, 0.67, 0.364, 0.29) \end{pmatrix}$$

$$(r_4) = \max(r_2, r_3)$$

$$= \begin{pmatrix} (0.652, 1.01, 0.69, 0.289, 0.46) & (0.601, 0.886, 0.62, 0.39, 0.31) \\ (0.676, 1.00, 0.714, 0.442, 0.49) & (0.696, 1.11, 0.69, 0.55, 0.359) \\ (0.684, 1.03, 0.701, 0.522, 0.495)) & (0.707, 1.144, 0.68, 0.56, 0.355) \end{pmatrix}$$

$$(r_5) = \begin{pmatrix} (-0.48, 0.14, 1.804, 2.711, 2.794) & (-0.4513, 0.114, 1.892, 2.974) \\ (-0.494, 0.14, 1.856, 2.558, 2.891) & (-0.536, -0.09, 1.88, 2.424, 2.981) \\ (-0.494, 1.36, 1.859, 2.51, 2.89) & (-0.537, -0.084, 0.868, 2.43, 2.95)) \end{pmatrix}$$

$$(r_6) = \begin{pmatrix} 2.794 & 2.974 \\ 2.891 & 2.981 \\ 2.890 & 2.950 \end{pmatrix}; \ (r_7) = \begin{pmatrix} -0.480 & -0.451 \\ -0.496 & -0.536 \\ -0.494 & -0.530 \end{pmatrix}$$

4. THE MEMBERSHIP FUNCTION FOR FOUR IMPLICATION OPERATORS:

(1) GAINESS RESCHER IMPLICATION OPERATOR:

$$\mu_{LOW(g,r)} = \begin{cases} 1 & x \leq 2.4 \\ \dfrac{2.5 - x}{0.1} & 2.4 \leq 2.5 \\ 0 & x \geq 5.6 \end{cases}$$

(2) GODEL IMPLICATION OPERATOR:

$$\mu_{LOW-MIDDLE(g,i)} = \begin{cases} 0 & x \leq 5.6, x \geq 7.95 \\ \dfrac{x - 5.6}{0.1} & 5.6 \leq x \leq 5.7 \\ \dfrac{7.7 - x}{0.1} & 6.7 \leq x \leq 7.7 \\ 1 & 5.7 \leq x \leq 6.7 \end{cases}$$

(3) CONTRAPOSITIVE OF GODEL IMPLICATION OPERATOR:

$$\mu_{MIDDLE-HIGH(c,g)} = \begin{cases} 0 & x \leq 7.95 \\ \dfrac{x - 7.95}{0.1} & 7.95 \leq x \leq 8.05 \\ 1 & x \geq 8.7 \end{cases}$$

(4) LUCKASIEWICZ IMPLICATION OPERATOR:

$$\mu_{HIGH(l,i)} = \begin{cases} 0 & x \leq 8.7 \\ \dfrac{x - 8.8}{0.1} & 8.8 \leq x \leq 8.9 \\ 1 & x \geq 1 \end{cases}$$

5. NUMERICAL GRAPHICAL SOLUTION

(1) GAINESS RESCHER IMPLICATION:

- In this fig, for the pentagonal fuzzy number matrix the Gainess Rescher Implication Operators shows that the patient affected with disease named anemia that has represented in the form of scatted dots.
- Where the dosage level is very low.

(2) GODEL IMPLICATION OPERATOR:

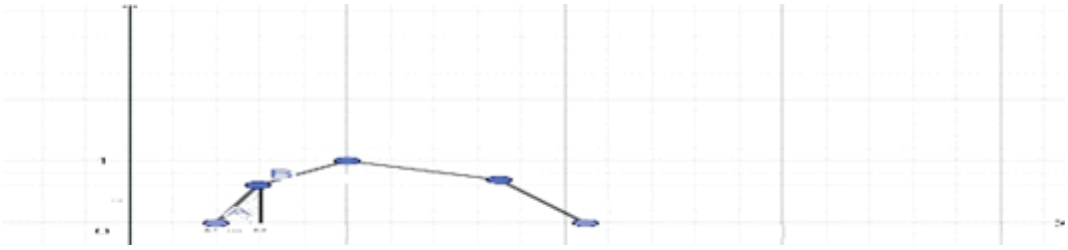

FIGURE 1.

- In fig 2, there is slight change in the dosage level where the patients with diseases kidney related problems are affected more in the human body and that is compared with the ISH in which represented in the graph.
- Also it is clearly shown that when compared to Gaines Rescher Operator there is some improvement in Godel that the patients are cured.

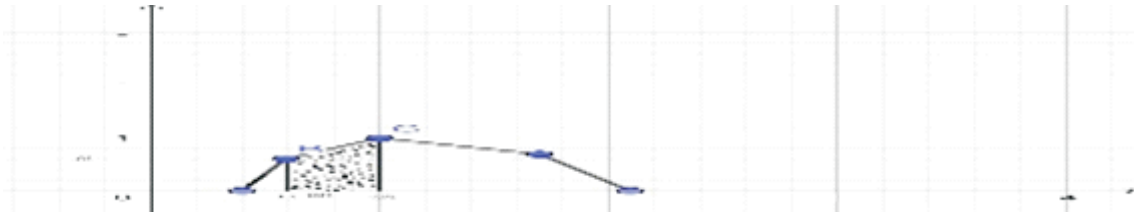

FIGURE 2.

(3) CONTRAPOSITIVE OF GODEL IMPLICATION OPERATOR:

- In fig 3, the word Contrapositive itself describe how the patients can be easily cured when the dosage is increased more and that like patients.
- We have taken is cholesterol, here compared to Godel Operator the Contrapositive is much better and those patients can be cured who are all affected with ISH.

(4) LUCKASIEWICZ IMPLICATION OPERATOR:

- In fig 4, the graph shows how the patients can be completely cured when the dosage is given at correct level.
- So that every human body can withstand their conditions according to the patients affected with ISH and in the form of mathematical

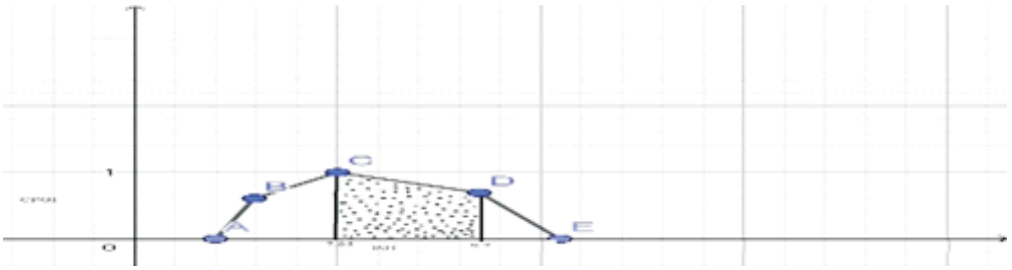

FIGURE 3.

Luckasiewicz operator also fuzzy membership function for pentagonal fuzzy number matrix.

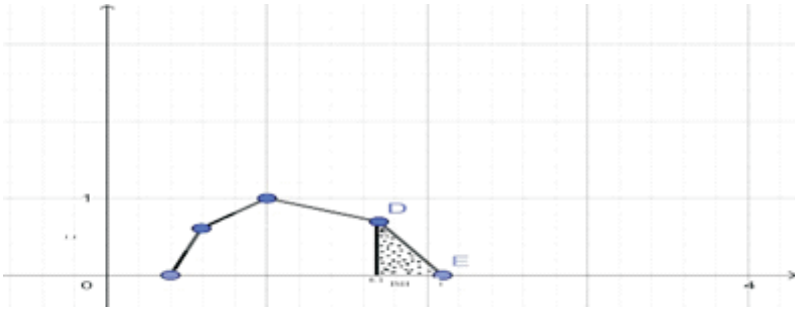

FIGURE 4.

6. CONCLUSION

The paper emerges a new study of fuzzy ternary semigroup and many results concerning pentagonal fuzzy number matrix according to their medical diagnostic model is also established. But, ISH model help the study to find how the patients can be cured in the way of mathematical learning. The survey of ISH model for the range of the data are taken and we concluded and generalized in the form of pentagonal fuzzy number matrix. In particular, the four implication operator we considered for our study clearly shows that the Luckasiewicz implication operator is in relation with the ISH model where the dosage will be normal for the human body as compared with the other operators. Also network graph is given.

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DEPARTMENT OF MATHEMATICS
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
KATTANKULATHUR - 603 203, TAMIL NADU,
INDIA.
Email address: sheejag.dhe@gmail.com

DEPARTMENT OF MATHEMATICS
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
KATTANKULATHUR - 603 203, TAMIL NADU,
INDIA.
Email address: rupaanjalie@gmail.com