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A STOCHASTIC MODEL FOR CHRONIC KIDNEY DISEASE

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ABSTRACT. Mathematical and Statistical model help researchers to examine various laws and dupes in aiding decision making associated with claims to our community. It possesses the potential to expect and evaluate likely results and represent various scenarios before they have to be examined in sensibility. This study considers a stochastic model to figure out a kidney affected human system. Using a shock model concept to describe, we presented a complex subject with five factors; F1-Blood glucose random, F2-Blood urea, F3-Serum creatinine, F4-Hemoglobin and F5-Age group based on certain medical data to examine the impact of factors illustrating the Survival rate of expectation. Our model studies in the background with these five factors influencing a patient's human system.Based on shock model approach with a survival function, Laplace transformation and inter-arrival time, the expected survival estimate is observed and draws relevant inferences on these factors to the community. The results show that in initial stage (stage 1) when the disease is found, the easier to ratify. Whereas in stage 2 and stage 3, the risk of survival is double the chance as measured in stage 1. In conclusion, as age increases, the risk is been developing.

1. INTRODUCTION

It should be noted that in the general population, the previous goals are different because they are based on determining comorbidities rather than age, race, or

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gender.Kidney disease does not take place overnight, develops in different stages. People having syndromes may not detect thing amiss in the early stage. If detected and treated kidney problem can be slowed or avoided. Chronic Kidney Disease (CKD) is well recognized as a major public health issue with enormous financial implications for human health systems [2], [14]. CKD is associated with a decreased quality of life, as well as an increased risk of death and cardiovascular failure [6]. A rapid decline in kidney function is linked to an increased risk of death and cardiovascular problems[[18], [22]]. CKD is comorbid with much prevalent illnesses comprising hypertension, diabetes, anemia and mineral/bone complications [20]. Diabetes and hypertension are the starting elements of CKD[9]. According to the National Kidney Foundation's guidelines, CKD can be divided into five stages based on estimated Glomerular Filtration Rate (eGFR) grade intervals. Globally, the prevalence of CKD ranges from 10.5% to 13.1% [21]. To measure how effectively the kidneys are operating, the status of creatinine in the blood is tested. Measured rate later applied to determine the predicted glomerular filtration rate (eGFR). Natural kidney function in normal adults falls off by age; example, adults of 20-30 years have an eGFR of 115 mL/min/1.73m2 whereas it considers declined to 85 mL/min/1.73m2, in 60-69-year age group Epidemic models can be applied as analytical devices investigate to transmit infectious diseases. An epidemic model provides an accessible outline to contagious illness data, but an important purpose is to make an idea of the biological and sociology processes of infection transmission [15]. Current epidemic models represent many hypotheses on disease transmission process. These hypotheses are to the extent approximations for the original process.

2. BACKGROUND

Initial recognition and focused intervention of CKD drew substantial concern from analysts and investigators since both have the possible to bring the number of cases continuing to End Stage Renal Disease and less the fatality rate associated to CKD and related healthcare costs[16].A comprehensive research assessment of statistical approaches adopted for examining risk aspects of CKD evolution is showed by [3]. One of their findings, which was reported in longitudinal examples when the entire renal route was operated on over time, was that linear

mixed designs can be used to identify both risk factors and their linked confidence ranges.Stochastic models help in understanding the mechanism of diseases to explain relationships between developing and progressing in disease stages and other relevant covariates. Applications of stochastic processes in medicine and their use in controlling disease-related morbidity and mortality have been attempted by some authors [4]. The progression of CKD can be visualized using a Markov process that only allows forward transitions from one state to another over time [13]. The progression of disease is continuous, and the timing of transitions is a random variable. The appropriate model to describe the course of CKD progression is a homogeneous continuous time multistage model based on Markov processes [1]. With the hidden Markov model (HMM), the true stages of disease are assumed to be hidden (unobservable or latent). A disease marker can be used to determine the actual stage of the disease. Given the genuine stages, it is expected that the obtained perceptions are independent. Speech and signal processing have both benefited from HMM [7].HMM was used by Satten and Longini [19] to describe HIV infection at different stages based on CD4 cell numbers. The HMM model was used by Jackson et al. to depict the loss in lung function post lung transplantation [11].

3. DATA PREPARATION AND CLEANING

In this article the progression of CKD using data with age group observed between 30 to 90 [10]. Patients with all kinds of primary renal disease are included in the study.A population of 400 was observed with almost twenty variables; fourteen nominal variables (Specific Gravity, Albumin, Sugar, Red blood cells, Pus cell, pus cell clumps, Bacteria, Hypertension, Diabetes Mellitus, Coronary Artery Disease, Appetite, Pedal Edema, Class) and eleven numerical variables (Age, Blood Pressure, Blood glucose random, Blood urea, Serum creatinine, Sodium, Potassium, Hemoglobin, Packed cell volume, White blood cell counts, Red blood cell counts) were present.The goals are to look at the factors that influence renal disease outcomes and progression in CKD patients, with a particular focus on age-related risk factors. The model development in our view is performed with five factors affecting CKD, F1-Blood glucose random, F2-Blood urea, F3-Serum creatinine, F4-Hemoglobin and F5-Age group. Theoretical modeling of epidemics is

necessary when the number of virulent entities is small or when the anxiety in transmission, recovery, births, deaths, or the environment changes the epidemic outcome. The uncertainty identified with individual changes such as automatic transmission, healing, births or deaths is pointed out as demographic variability. The volatility related to the environment, such as conditions referred to terrestrial or marine sites, is pointed out as environmental uncertainty. Environmental variability is effective in designing zoonotic infectious diseases, vector-borne epidemics, and waterborne diseases [**?**]. Practice of statistical models and structure understanding approaches has been developing in evaluating health and disease issues. Length of the infections period not seen being defined is most usually considered an exponential family. Distribution of exponential family is effective because of its consistent hazard rate and its lack of memory property.

4. MODEL DEVELOPMENT

Mathematical models of outbreaks of infectious illnesses may be organized into two broad classes: deterministic and stochastic. The term "stochastic" relates to being or including a random variable. Stochastic processes vary in the basic hypotheses about the time and the state variables. Stochastic models will serve us determine and expect the concealed components. Lifetime data with increasing, decreasing, and upside-down bathtub shaped breakdown rates can be modelled using the Shifted Exponential Distribution (SED) [8].Because of its simplicity and mathematical feasibility, the Shifted Exponential Distribution is the most widely used distribution for lifetime data analysis. However, we rarely come across engineering systems in the real world that have a consistent hazard rate throughout their lifetime. As a result, it seems reasonable to assume hazard rate as a function of time, leading to the construction of an alternative lifetime data analysis model. In general, the threshold Y follows (SED) with parameter θ. It can be shown that,

$$P(X_i \leq Y) = \int_0^\infty g_k(x)\bar{H}(x)dx = \int_0^\infty g_k(x)e^{-\frac{x-\theta}{\beta}} = \int_0^\infty g_k^*(x)e^{-\frac{(x-\theta)}{\beta}}.$$

On simplifications we get,

$$= [g^*\frac{1-\theta}{\beta}]^k \tag{4.1}$$

When a patient is dealing with risks from any of the five criteria, age is the possibility of surviving in various stages. A system of different stages gives included material as to turn up the predicted survival of the patients. Patients showed whether they have ever been proved with CKD.The possibility that the accumulated threshold will break only after time t is given by the survival function.

$S(t) = P(T \geq t) =$ The chance that cumulative damage last longer than $t = \sum_{k=0}^{\infty} P$ { exactly k decisions in $(0,t] * P$(cumulative threshold $(0,t]$}.

The survival function S(t) which is the probability that an individual survives for a time t. Renewal process observes that

$$\begin{aligned} P(T \geq t) &= \sum_{0}^{\infty} F_k(t) P(X_i \geq y) \\ &= \sum_{0}^{\infty} F_k(t) - F_{(k+1)}(t) \left[g^* \frac{1-\theta}{\beta} \right]^k \\ &= \left[1 - g^* \frac{1-\theta}{\beta} \right] \sum_{k=1}^{\infty} [F_k(t)] \left[g^* \frac{1-\theta}{\beta} \right]^{[k-1]} \end{aligned} \tag{4.2}$$

$P(T \geq t) = L(t) =$the distribution function of life time (t). Using convolution theorem for Laplace transforms, $F_0(t) = 1$and on simplification, it can be shown that,

$$L(t) = 1 - \left[1 - g^* \left(\frac{1-\theta}{\beta} \right) \right] \sum_{k=1}^{\infty} F_k(t) \left[g^* \left(\frac{1-\theta^{(k-1)}}{\beta} \right) \right]. \tag{4.3}$$

By taking Laplace-Stieltjes transform from equation (4.3). Let the random variable with inter-arrival time follows exponential with parameter,$f^*(s) = \frac{c}{c+s}$, substituting we get,

$$\begin{aligned} l^*(s) &= \frac{\left[1 - g^* \left(\frac{1-\theta}{\beta} \right) \right] f^*(S)}{\left[1 - g^* \left(\frac{1-\theta}{\beta} \right) \right] f^*(S)} \\ &= \frac{\left[1 - g^* \left(\frac{1-\theta}{\beta} \right) \right] c}{\left[c + s - g^* \left(\frac{1-\theta}{\beta} \right) c \right]}. \end{aligned} \tag{4.4}$$

After first and second derivatives on simplification we get the expected time of SED with parameterθ. It can be shown that,

$$E(T) = \frac{1+\mu\beta-\theta}{c[1-\theta]}. \tag{4.5}$$

5. RESULTS

The three parameters which were selected for our model estimation are Blood glucose random, Serum creatinine and Hemoglobin. A selectively twelve observation was estimated in equation (6) to see the goodness of fit with age group between 48 to 76.The parameters assigned for the model fit are; Blood glucose random - (μ), Serum creatinine - (β), Hemoglobin - (θ). The inter-arrival (c) is been the Age group which access the survival rate at the three stages of the CKD patients. From Table 2 and Fig. 2, we can predict that as the age group increases; in the first stage when the age of the patient was 48 the survival rate of the patients was found 81.53, as the age increased the survival rate also increased and at age 76 the survival rate showed a high risk of 2056.83 (i.e., expected chance of risk is high as age increased). In stage 2, the patient with age 48 found the survival rate at a high risk of 288.12 and also in the third stage the risk increased to 830.00 for the patient with age 48. This case is observed for all different age of patients. Jing Zhao et al. [12] established and justified a prediction model of estimated glomerular filtration rate (EGFR) by data got from a local health organization. Age, gender, body mass index, obesity, hypertension, and diabetes, which brought about a mean coefficient of conviction of 0.95. The scientists discovered that a model based on real-world electronic medical record parameters can predict future kidney functions and help with clinical decision-making.Noura Anwar and Mahmoud Riad [17] designed a theoretical model that illustrates the evolution process of CKD, measures the meantime invested in each stage of illness that precedes improving end-stage renal failure and to measure the life expectancy of a CKD patient. They have suggested a positive construct of the transition probability matrix of CKD process with five states, the initial four of them illustrate the 2nd, 3rd, 4th, and end-stage renal disease of CKD conforming to the Kidney Disease Outcomes Quality Initiative analysis, and the last state is death.

Table -1: Survival rate of the patient with age group

Age	Stage-1	Stage-2	Stage-3
76	2056.83	4647.48	8297.05
75	1179.03	4167.11	2155.39
71	1245.74	2511.65	3938.39
68	1074.47	2441.71	4383.78
67	723.07	1653.22	2985.84
65	632.35	1436.73	2578.94
62	532.34	1218.77	2204.06
60	489.25	1133.32	2072.59
59	486.14	1357.15	3060.89
54	288.73	818.16	1871.80
50	134.61	235.20	298.32
48	81.5	288.12	830.00

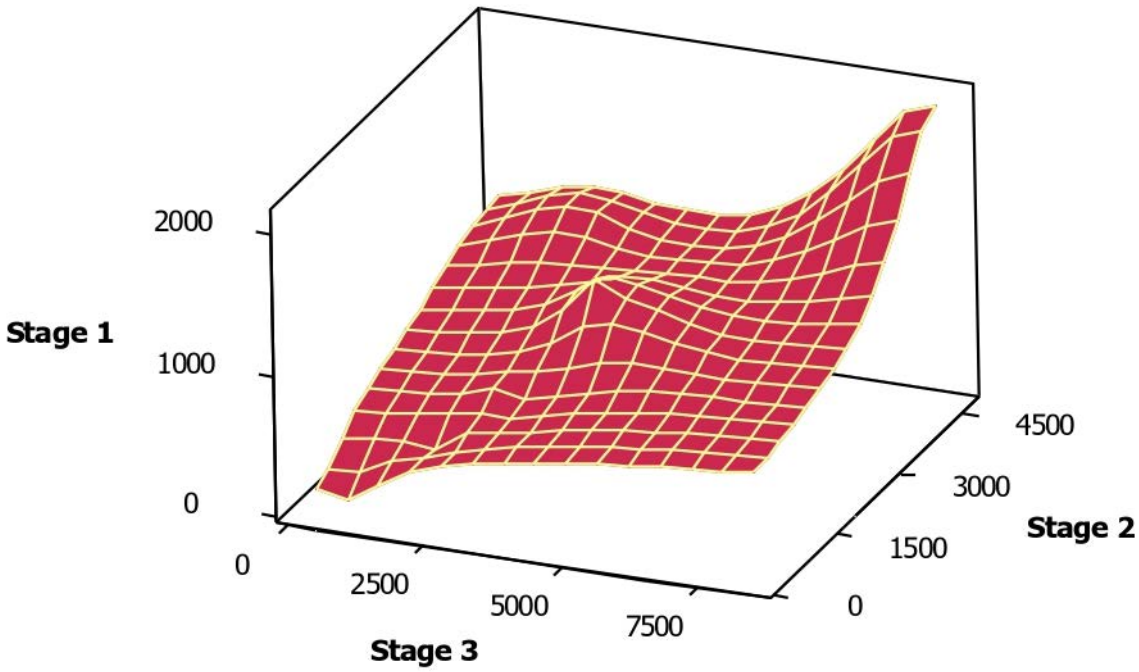

FIGURE 1. Survival of CKD patient at three different stages

6. CONCLUSION

This model proves that when the CKD patient has been affected with different factors of CKD then immediate attention should be given to the immune system

so that we can rectify the disease at the early stage (ie., at stage 1). As observed in Table 1 and Fig 1, if unnoticed and no treatment made at the initial stage survival rate of the patients increases drastically in stage 2, as well as in stage 3 also. Misclassification of levels can occur as a result of a failure to determine the subject's history. The model aids us in determining the critical relationship of advanced CKD. The study concludes that misclassification of stages in the CKD process can occur due to behavior or a lack of prognostic concerns. The presence of variables such as hypertension and diabetes may improve CKD stages, as their absence adds to stage misunderstanding. The study reveals that the likelihood of stage misclassification is larger in the early stages of sickness than the later stages. When stage misclassification is investigated, the mean sojourn duration increases. In subsequent researches, stochastic models were to apply to subject-individual indicators of CKD progression. Our model has an excellent match for all illness groups, and our parameter predictions have been proven to be accurate. Both the research and the CKD's judgment support these claims in part.Various samples can be handled for the model selection method to defend against over-fitting to our data. The stochastic model where the expected time period applied for measuring parameters.

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