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AN OPTIMUM PRODUCTION INVENTORY MODEL FOR DETERIORATING PRODUCT WITH SHORTAGES IS PARTIALLY BACKLOGGED

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ABSTRACT. The author analyses a manufacture inventory prototypical for fading products with persistent demand and partially backlogged shortages. And also the model gives the length of the cycle time for the next order level. The impartial of method is to get the optimizing the shortage time, total repeated time and the average total inventory cost. The proposed strategy is projected with the help of arithmetical examples.

1. INTRODUCTION

By and large, weakening is described as the waste, harm, outdated nature, dissipation, pilferage and loss of utilization of put away items and it brings about diminishing handiness of the first one. Mishra, U et al. [1] fostered a reasonable creation inventory model for a controllable fossil fuel byproducts rate under deficiencies. Sana, S. S [2] concentrated on a creation inventory model in a defective creation process. Wang, and Lai, [3] concentrated on an ideal creation inventory model for breaking down things with various sooq interest. Hammami, R et al. [4] created fossil fuel byproducts in a more of multifid echelon creation inventory model with prime-time imperatives. Muniyappen et al. [7] fostered an

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(EOQ) framework for falling apart things with expansion and time worth of cash prompting time-subordinate decaying rate and defer installments. Srivathsan, S., and Kamath, M. [6] concentrated on Performance displaying of a two-echelon production network under various degrees of upstream inventory data sharing. D Khamrod [15] concentrated on inventory models with rain checks and damaged things inferred algebraically and AGM. Muniyappen [8] investigated a creation for seller purchaser synchronization with amount markdown, putting in a rain check and adjust for secure lifespan items. Muniyappen [9] created creation for seller purchaser direction with amount rebate, putting in a rain check and revamp for static time items. Kirubhashankar et al. [11] concentrated on a creation inventory model for weakening item with value breaks. Ismail. An et al. [10] concentrated Optimum purchaser seller inventory model with coordination and value breaks. The time changing interest considered in a large portion of the papers referenced above expects request rate to be either expanding or diminishing all through with time, while practically speaking, it settles at the developed phase of the item life cycle once the item has been acknowledged on the lookout. This sort of adjustment has been named as "incline type" and has been considered in the writing since Ritchie [13]. Yao, M et al. [2] concentrated on survey of multi-provider inventory models in store network the board. Birim, S [5] introduced assessing seller oversaw inventory frameworks: how impetuses can help store network accomplices. Esmaeili et al. [16] read up an inventory model for only merchant vender production network under various situations and exchange credit. Muniyappen, P et al. [12] concentrated on an Ideal EOQ framework for purchaser merchant with value disruptions and static croft charge. Sana [14] considered a creation inventory model of flawed value items in a layer of three production network. Mishra, U. [17] fostered an enhancing a paces of-creation inventory model on market place cost and promoting charge with decaying things. Shah, N. H., and Vaghela [18] concentrated on an Imperfect creation inventory model for time and exertion subordinate interest under expansion and most extreme unwavering quality. Beraand Jana [19] analyzed the Multi-thing blemished creation fluffy conditions. Rastogi [20] examined a creation for disintegrating items with selling value subordinate utilization proportion and deficiencies under climate. Barron, Y [21] concentrated on the exhibition investigation of a reflected liquid creation/inventory model. Adventure, R. S et al [22] examined an Investigating fossil fuel byproduct in a creation inventory model under flawed creation, assessment blunders and administration level limitation. Singh, D [23] examined the creation of weakening things with stock, selling cost, and holding cost with excess.

2. INVENTORY MODEL FORMULATION

Notation.

(1) D describes the demand rate that is constant
(2) K>D defines the Production rate
(3) t_s denotes shortage time
(4) θ is the deterioration rate
(5) T is the total time
(6) k denotes charge of production per unit time
(7) h denotes holding cost per unit time
(8) s denotes cost shortage per unit time
(9) ϕ denotes the opportunity costper unit time
(10) $I1(t)$ is the inventory stage at time $0 \leq t \leq t_s$
(11) $I2(t)$is the inventory level at time $t_s \leq t \leq T$
(12) Backlogging parameter is$\frac{1}{(1+\delta(T-t))}$

Mathematical model as follows.

$$\begin{aligned}\frac{dI_1(t)}{dt} + \theta I_1(t) &= K - D; \theta \leq t \leq t_s \\ \frac{dI_2(t)}{dt} &= \frac{-D}{1+\delta(T-t)}; t_s \leq t \leq T.\end{aligned}$$

With limit Circumstances are $I_1(t_s) = 0$ and $I_2(T) = 0$.
The inventory levels in the different stages are

$$\begin{aligned}I_1(t) &= \frac{(K-D)}{\theta(1-e^{\theta}(t_s-t))}; 0 \leq t \leq t_s \\ I_2(t) &= \frac{D}{\delta}\{\log[1+\delta(T-t)] - \log[1+\delta(T-t_s)]\}; t_s \leq t \leq T.\end{aligned}$$

Production quantity is $Q_0 = \frac{(K-D)}{\theta}(1-e^{\theta t_2})$, $I(0) = I_1(0) + I_2(0)$.

Average total inventory cost, which is the sum of holding cost, format cost, production cost, deficiency cost and opportunity cost

$$\begin{aligned} & TC(t_s, T) \\ = & \frac{A}{T} + \frac{h}{T}\{\frac{(K-D)}{\theta^2}\left[1 - e^{\theta t_s}\right] + \frac{K-D}{\theta} t_s + \frac{D}{\delta^2}\delta(T - t_s) - \log[1 + \theta(T - t_s)]\} \\ + & \frac{k}{T}\{\frac{K-D}{\theta}(1 - e^{\theta t_s})\} + \frac{Ds}{T\delta^2}\{\theta(T - t_s) - \log[1 + \delta(T - t_s)]\} \\ + & \frac{D\phi}{T\delta}\{\delta(T - t_s) - \log[1 + \delta(T - t_s)]\} \end{aligned}$$

Average setup cost is $\frac{A}{T}$.

Average holding cost is

$$\frac{h}{T}\{\int_0^{t_s} I_1(t)dt + \int_{t_s}^{T} I_2(t)dt\}$$
$$= \frac{h}{T}\left\{\frac{K-D}{\theta^2}\left[1 - e^{\theta t_s}\right] + \frac{K-D}{\theta t_s} + \frac{D}{\delta^2}\delta(T - t_s) - \log[1 + \delta(T - t_s)]\right\}.$$

Average shortage cost is $\frac{s}{T}\int_{t_s}^{T} I_2(t)dt = \frac{Ds}{T\delta^2}\delta(T - t_s) - \log[1 + \delta(T - t_s)]$.

Opportunity cost is $\frac{\phi}{T}\int_{t_s}^{T} D\left(1 - \frac{1}{1+\delta(T-1)}\right) dt = \frac{D\phi}{T\delta}\delta(T - t_s) - \log[1 + \delta(T - t_s)]$.

Average production cost is $\frac{k}{T}Q_0 = \frac{k}{T}\{\frac{K-D}{\theta}(1 - e^{\theta t_s})\}$.

For optimality,

$$\frac{\partial TC(t_s, T)}{\partial t_s} = 0, \quad \frac{\partial TC(t_s, T)}{\partial T} = 0$$

and

$$\frac{\partial^2 TC(t_s, T)}{\partial t_s^2} > 0, \quad \frac{\partial^2 TC(t_s, T)}{\partial T^2} > 0,$$

$$\frac{\partial TC(t_s, T)}{\partial t_s} = \frac{1}{T}\left\{\frac{K-D}{\theta}\left[h\left(1 - \frac{e^{\theta t_s}}{\theta^2}\right) - \frac{k}{\theta}e^{\theta t_s}\right] - \frac{D(T - t_s)(h + s + \delta\pi)}{1 + \delta(T - t_s)}\right\}$$

$$\frac{\partial^2 TC(t_s, T)}{\partial t_s^2} = \frac{1}{T}\left\{-\frac{K-D}{\theta^3}(h + k\theta)e^{\theta t_s} + \frac{D(h + s + \delta\pi)}{(1 + \delta(T - t_s))^2}\right\}$$

$$\frac{\partial TC(t_s, T)}{\partial t_s} = 0.$$

This gives

$$t_s = \frac{\frac{(K-D)}{\theta}\left[\frac{k}{\theta} - h\left(1 - \frac{1}{\theta^2}\right)\right] + DT(h + s + \delta\pi)}{\frac{(K-D)}{\theta}\left[\frac{(h-k\theta)}{\theta} + D(h + s + \delta\pi)\right]}$$

and

$$\frac{\partial TC(t_s,T)}{\partial T} = \frac{-1}{T^2}\left\{X + Y\left[\delta(T-t_s) - \log(1+\delta(T-t_s))\right]\right\} + \frac{1}{T}\left\{\frac{Y\delta^2(T-t_s)}{1+\delta(T-t_s)}\right\}.$$

Let $X = A + \frac{K-D}{\theta}\left[h\left(\frac{1-e^{\theta t_s}}{\theta} + t_s\right) + k(1-e^{\theta t_s})\right]$ and $Y = \frac{D}{\delta^2}(h+s+\delta\pi)$. Then,

$$\begin{aligned}\frac{\partial^2 TC(t_s,T)}{\partial T^2} &= \frac{2}{T^3}X + Y[\delta(T-t_s) - \log(1+\delta(T-t_s))] - \frac{1}{T^2}\left\{\frac{Y\delta^2(T-t_s)}{1+\delta(T-t_s)}\right\}\\ &\quad + \frac{TY\delta^2(1+\delta(T-t_s)) - Y\delta^2(T-t_s)(1+\delta(2T-t_s))}{T62(1+\delta(T-t_s))^2}\\ \frac{\partial TC(t_s,T)}{\partial T} &= 0.\end{aligned}$$

This gives.

$$T = \frac{2Y\delta t_s - \delta t_s - X - 2Y\delta^2 t_s^2}{t_s + 2Y\delta + \delta - 4Y\delta^2 t_s}.$$

Algorithm.

1. Fixing a trial value of T.
2. From the Equation (7), findt_s.
3. From the Equation (9), find T.
4. Repeat the steps 2–3 until $TC_n \leq TC_{n-1}$.Set $t_s^* = t_s$and $T^* = T$
5. Compute the corresponding $TC(t_s^*, T^*)$

3. Mathematical Example

Example 1. *Let* $A = 100$*,* $P = 2000$*,* $D = 1000$*,* $h = 0.6$*,*$\theta = 0.05$*,* $k = 3$*,* $s = 0.5$*,* $\delta = 2$*,*$\pi = 0.2$*,* $T = 0.05$. *Hence optimum decision variables are*

$$t_s^* = 0.2994, \quad T^* = 1.9380 \quad \textit{and} \quad TC(t_s^*, T^*) = 169.5797.$$

Example 2. *Let* $A = 200$*,* $P = 200$*,* $D = 100$*,* $h = 1.2$*,*$\theta = 0.08$*,* $k = 20$*,* $s = 30$*,* $\delta = 2$*,* $\pi = 15$*,* $T = 0.05$. *Hence optimum decision variables are*

$$t_s^* = 0.0834, \quad T^* = 0.0963 \quad \textit{and} \quad TC(t_s^*, T^*) = 34z0.0704.$$

4. CONCLUSION

The author developed production inventory model for weakening goods with constant claim and partially backlogged deficiencies. This method considered as shortages are partially backlogged. Additionally, mathematical examples are given to reveal the proposed model and a process is proposed to find optimum decision parameters. The model can further extended in different situation such as stock dependent demand, credit periods, price break, quantity discount etc.

REFERENCES

[1] U. MISHRA, J.Z. WU, B. SARKAR: *A sustainable production-inventory model for a controllable carbon emissions rate under shortages*, Journal of Cleaner Production, **256** (2020), art. id. 120268.

[2] S.S. SANA: *A production–inventory model in an imperfect production process*, European Journal of Operational Research, **200**(2) (2010), 451-464.

[3] Y. HE, S.Y. WANG, K.K. LAI: *An optimal production-inventory model for deteriorating items with multiple-market demand*, European Journal of Operational Research, **203**(3) (2010), 593-600.

[4] R. HAMMAMI, I. NOUIRA, Y. FREIN: *Carbon emissions in a multi-echelon production-inventory model with lead time constraints*, International Journal of Production Economics, **164** (2015), 292-307.

[5] S. BIRIM, C. SOFYALIOGLU: *Evaluating vendor managed inventory systems: how incentives can benefit supply chain partners*, Journal of Business Economics and Management, **18**(1) (2017), 163-179.

[6] S. SRIVATHSAN, M. KAMATH: *Performance modeling of a two-echelon supply chain under different levels of upstream inventory information sharing*, Computers & Operations Research, **77** (2017), 210-225.

[7] P. MUNIYAPPEN, R. UTHAYAKUMAR, S. GANESH: *An EOQ model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments*, Systems Science & Control Engineering, **3**(1) (2015), 427-434.

[8] P. MUNIYAPPEN, R. UTHAYAKUMAR, S. GANESH: *A production inventory model for vendor–buyer coordination with quantity discount, backordering and rework for fixed life time products*, Journal of Industrial and Production Engineering, **33**(6) (2016), 355-362.

[9] S. HEMAMALINI, M. RAVITHAMMAL, P. &MUNIYAPPEN: *EOQ inventory model for buyer-vendor with screening, disposed cost and controllable lead time*, In AIP Conference Proceedings **2095**(1) (2019), art. id. 030010.

[10] A.M. ISMAIL, P. MUNIYAPPEN, C.K. KIRUBHASHANKAR: *Optimum buyer-vendor inventory model with coordination and price breaks*, Journal of Physics: Conference Series **1770**(1) (2021), art. id. 012104.

[11] C.K. KIRUBHASHANKAR, P. MUNIYAPPEN, A.M. ISMAIL: *A production inventory model for deteriorating product with price breaks*, Journal of Physics: Conference Series, **1770**(1) (2021), art.id 012102.

[12] P. MUNIYAPPEN, C.K. KIRUBHASHANKAR, A.M. ISMAIL: *An Optimum EOQ model for buyer-vendor with price breaks and fixed holding cost*, Journal of Physics: Conference Series, **1770**(1) (2021), art.id. 012103.

[13] K.F. YUAN, Y. GAO: *Inventory decision-making models for a closed-loop supply chain system*, International Journal of Production Research, **48**(20) (2021), 6155-6187.

[14] S.S. SANA: *A production-inventory model of imperfect quality products in a three-layer supply chain*, Decision support systems, **50**(2) (2011), 539-547.

[15] A. GHARAEI, S.H.R. PASANDIDEH, A.A. KHAMSEH: *Inventory model in a four-echelon integrated supply chain: Modeling and optimization*, Journal of Modeling in Management, **12**(4) (2017), 739-762.

[16] M. ESMAEILI, M. NASRABADI: *An inventory model for single-vendor multi-retailer supply chain under inflationary conditions and trade credit*, Journal of Industrial and Production Engineering, **38**(2) (2021), 75-88.

[17] U. MISHRA: *Optimizing a three-rates-of-production inventory model under market selling price and advertising cost with deteriorating items*, International Journal of Management Science and Engineering Management, **13**(4) (2018), 295-305.

[18] N.H. SHAH, C.R. VAGHELA: *Imperfect production inventory model for time and effort dependent demand under inflation and maximum reliability*, International Journal of Systems Science: Operations & Logistics, **5**(1) (2018), 60-68.

[19] A.K. BERA, D.K. JANA: *Multi-item imperfect production inventory model in Bi-fuzzy environments*, Opsearch, **54**(2) (2017), 260-282.

[20] M. RASTOGI, S.R. SINGH: *A production inventory model for deteriorating products with selling price dependent consumption rate and shortages under inflationary environment*, International Journal of Procurement Management, **11**(1) (2018), 36-52.

[21] Y. BARRON: *Performance analysis of a reflected fluid production/inventory model*, Mathematical Methods of Operations Research, **83**(1) (2016), 1-31.

[22] R.S. SAGA, W.A. JAUHARI, P.W. LAKSONO, A.R. DWICAHYANI: *Investigating carbon emissions in a production-inventory model under imperfect production, inspection errors and service-level constraint*, International Journal of Logistics Systems and Management, **34**(1) (2019), 29-55.

[23] D. SINGH: *Production inventory model of deteriorating items with holding cost, stock, and selling price with backlog*, International Journal of Mathematics in Operational Research, **14**(2) (2019), 290-305.

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