

NEW IMPROVED METHOD FOR SOLVING THE FUZZY LINEAR PROGRAMMING PROBLEMS WITH VARIABLES GIVEN AS FUZZY NUMBERS

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ABSTRACT. The present paper aims to propose an alternative solution approach in obtaining the fuzzy optimal solution to a fuzzy linear programming problem with variables given as fuzzy numbers with minimum uncertainty. In this paper, the fuzzy linear programming problems with variables given as fuzzy numbers is transformed into equivalent interval linear programming problems with variables given as interval numbers. The solutions to these interval linear programming problems with variables given as interval numbers are then obtained with the help of linear programming technique. A set of six random numerical examples has been solved using the proposed approach.

1. INTRODUCTION

Linear programming is a most widely and successfully used decision tool in the quantitative analysis of practical problems where rational decisions have to be made. In order to solve a Linear Programming Problem, the decision parameters of the model must be fixed at crisp values. But to model real-life problems and perform computations we must deal with uncertainty and inexactness. These

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uncertainty and inexactness are due to measurement inaccuracy, simplification of physical models, variations of the parameters of the system, computational errors etc. Interval and fuzzy analysis are an efficient and reliable tool that allows us to handle such problems effectively.

Several researchers have carried out investigations on the fuzzy linear programming problems with variables given as interval numbers, triangular fuzzy numbers, trapezoidal fuzzy numbers, pentagonal fuzzy numbers, hexagonal fuzzy numbers, heptagonal fuzzy numbers, octagonal fuzzy numbers, nonagonal fuzzy numbers, decagonal fuzzy numbers, hendecagonal fuzzy numbers and dodecagonal fuzzy numbers.

G. Ramesh and K. Ganesan [4] proposed the solution concepts of primal and dual linear programming problems involving interval numbers without converting them to classical linear programming problems. Seyed Hadi Nasseri, Ali Ebrahimnejad and Bing-Yuan Cao [7] proposed a method to find the Fuzzy Optimal Solution of Fully Fuzzy Linear Programming Problems with Equality Constraints Having LR Flat Fuzzy Numbers. Someshwar Siddi and Raghunatha Reddy [32] proposed a method for solving Fuzzy Linear Programming Problem with pentagonal fuzzy number by using a ranking function and compared the solutions with fully Fuzzy Linear Programming Problem. Sanjivani M. Ingle and Kirtiwant P. Ghadle [33] presented fully fuzzy linear programming problem with hexagonal fuzzy number is solved by new ranking function. They converted the fully fuzzy linear programming problem to a crisp valued problem then can be solved using Simplex / Big-M method. K. Slevakumari and R. Tamilarasi [34] presented a paper aims at solving linear programming problems in which the parameters are octagonal fuzzy numbers with the help of robust ranking method. Shu-ping Wan et al. [21–31] solved fuzzy linear programming problems, Interval-valued fuzzy linear programming problems, Intuitionistic fuzzy linear programming problems and Interval-valued intuitionistic fuzzy linear programming problems.

In this paper, a new improved method for solving the fuzzy linear programming problems with variables given as fuzzy numbers is proposed. This new method finds the fuzzy optimal solution of fuzzy linear programming problems with variables given as fuzzy numbers. Moreover, the new method improves the existing methods for solving the interval Transportation Problems and Fully Fuzzy Transportation Problems with minimum uncertainty [18, 38, 39].

In general, most of the existing techniques provide only crisp solutions for the fuzzy linear programming problems with variables given as fuzzy numbers.

In contrast to most existing approaches [6, 7, 24, 26, 27, 29, 32–34], our method of transforming a fuzzy number into interval numbers is the first. So, our method proposed is the first. Also, the fuzzy optimal solution, obtained by using the new method mentioned, will always exactly satisfy the centers of all the constraints and some constraints with minimum uncertainty.

The contributions of the present study are summarized as follows: (a) We introduce new technique for improve the methods for solving the interval linear programming problems with variables given as interval numbers (2.6). (b) We introduce a formulation of fuzzy linear programming problems (2.12) with variables given as fuzzy numbers. (c) According to the proposed approach, the (2.12) is converted into classical linear programming problems and/or Interval linear programming problems. The integration of the interval optimal solutions of the sub-problems provides the fuzzy optimal solution of the problem (2.12). (d) An algorithm for the new proposed method and is developed to find the fuzzy optimal solution of the problem (2.12). (e) The complexity of computation is greatly reduced compared with commonly used existing methods in the literature.

The rest of this paper is organized as follows. In Section 2, some basic definition, arithmetic operations and interval linear programming problems are reviewed. Furthermore, we attempt to introduce a formulation of fuzzy linear problem with Fuzzy or Interval numbers. In Section 3, we propose a simple method for solving Fuzzy Linear Programming problems and a new fuzzy arithmetic on fuzzy or interval numbers. In Section 4, six numerical examples are presented to illustrate the proposed method. Advantages of the proposed method over the existing methods are discussed in Section 5. Finally, concluding remarks and future research directions are presented in Section 6.

2. MATERIALS AND METHODS

2.1. A new interval arithmetic.

In this section, some arithmetic operations for two intervals are presented [4].

Let $\mathcal{R} = \{\bar{a} = [a^1, a^2] : a^1 \leq a^2, a^1, a^2 \in \mathbb{R}\}$ be the set of all proper intervals. We shall use the terms “interval” and “interval number” interchangeably. The midpoint and width (or half-width) of an interval number $\bar{a} = [a^1, a^2]$ are defined as $m(\bar{a}) = \frac{a^2+a^1}{2}$ and $w(\bar{a}) = \frac{a^2-a^1}{2}$.

The interval number $\bar{a}$ can also be expressed in terms of its midpoint and width as

$$(2.1) \quad \bar{a} = [a^1, a^2] = \langle m(\bar{a}), w(\bar{a}) \rangle = \left\langle \frac{a^2 + a^1}{2}, \frac{a^2 - a^1}{2} \right\rangle.$$

For any two intervals $\bar{a} = [a^1, a^2] = \langle m(\bar{a}), w(\bar{a}) \rangle$ and $\bar{b} = [b^1, b^2] = \langle m(\bar{b}), w(\bar{b}) \rangle$, the arithmetic operations on $\bar{a}$ and $\bar{b}$ are defined as:

$$(2.2) \quad \text{Addition: } \bar{a} + \bar{b} = [a^1, a^2] + [b^1, b^2] = \langle m(\bar{b}) + m(\bar{a}), w(\bar{a}) + w(\bar{b}) \rangle,$$

$$(2.3) \quad \text{Subtraction: } \bar{a} - \bar{b} = [a^1, a^2] - [b^1, b^2] = \langle m(\bar{a}) - m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle,$$

$$(2.4) \quad \text{Multiplication: } \alpha \bar{a} = \alpha [a^1, a^2] = \alpha \langle m(\bar{a}), w(\bar{a}) \rangle \\ = \begin{cases} \langle \alpha m(\bar{a}), \alpha w(\bar{a}) \rangle & \text{if } \alpha \geq 0 \\ \langle \alpha m(\bar{a}), -\alpha w(\bar{a}) \rangle & \text{if } \alpha < 0 \end{cases}$$

$$\bar{a} \times \bar{b}$$

$$(2.5) \quad = \begin{cases} \langle m(\bar{a})m(\bar{b}) + w(\bar{a})w(\bar{b}), m(\bar{a})w(\bar{b}) + m(\bar{b})w(\bar{a}) \rangle & \text{if } a^1 \geq 0 \text{ and } b^1 \geq 0 \\ \langle m(\bar{a})m(\bar{b}) + m(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) + w(\bar{b})w(\bar{a}) \rangle & \text{if } a^1 < 0 \text{ and } b^1 \geq 0 \\ \langle m(\bar{a})m(\bar{b}) + m(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) - w(\bar{a})w(\bar{b}) \rangle & \text{if } a^2 < 0 \text{ and } b^1 \geq 0. \end{cases}$$

2.2. Formulation of linear programming problem with variables given as Interval numbers.

We consider the Linear Programming Problem involving Interval numbers as follows [4]:

$$(2.6) \quad \begin{cases} \text{Max / Min } \bar{Z}^{pq}(\bar{x}^{pq}) \approx \sum_{j=1}^n c_j \bar{x}_j^{pq} \\ \text{Subject to the constraints} \\ \sum_{j=1}^n a_{ij} \bar{x}_j^{pq} \begin{pmatrix} \approx \\ \succeq \\ \preceq \end{pmatrix} \bar{b}_i^{pq}, \text{ for } i = 1, 2, \dots, m \end{cases}$$

where :

- p and q are integers ($\mathbb{N}$) with $q \geq p$,
- $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$ are unrestricted interval numbers and

- c_j and a_{ij} are real numbers ($\mathbb{R}$).

Objective function transformation:

$$\bar{Z}^{pq}(\bar{x}^{pq}) = \langle m(Z^{pq}(\bar{x}^{pq})), w(\bar{Z}^{pq}(\bar{x}^{pq})) \rangle.$$

$$\bar{Z}^{pq}(\bar{x}^{pq}) \approx \sum_{j=1}^n c_j [x_j^p, x_j^q] = \sum_{j=1}^n \langle m(c_j \bar{x}_j^{pq}), w(c_j \bar{x}_j^{pq}) \rangle$$

where

$$(2.7) \quad m(c_j \bar{x}_j^{pq}) = c_j m(\bar{x}_j^{pq}) \text{ and } w(c_j \bar{x}_j^{pq}) = \begin{cases} c_j w(\bar{x}_j^{pq}) & \text{if } c_j \geq 0 \\ -c_j w(\bar{x}_j^{pq}) & \text{if } c_j < 0. \end{cases}$$

Transformation of constraints: $\sum_{j=1}^n a_{ij} \bar{x}_j^{pq} \begin{pmatrix} \approx \\ \succeq \\ \preceq \end{pmatrix} \bar{b}_j^{pq} \text{ for } i = 1, 2, \dots, m.$

We have $\bar{b}_i^{pq} = \langle m(\bar{b}_i^{pq}), w(\bar{b}_i^{pq}) \rangle$ and

$$\sum_{j=1}^n a_{ij} \bar{x}_j^{pq} = \sum_{j=1}^n \langle m(a_{ij} \bar{x}_j^{pq}), w(a_{ij} \bar{x}_j^{pq}) \rangle = \langle \sum_{j=1}^n a_{ij} m(\bar{x}_j^{pq}), \sum_{j=1}^n w(a_{ij} \bar{x}_j^{pq}) \rangle.$$

Then

$$\langle \sum_{j=1}^n a_{ij} m(\bar{x}_j^{pq}), \sum_{j=1}^n w(a_{ij} \bar{x}_j^{pq}) \rangle \begin{pmatrix} \approx \\ \succeq \\ \preceq \end{pmatrix} \langle m(\bar{b}_i^{pq}), w(\bar{b}_i^{pq}) \rangle.$$

We can write the following remark (2.7).

Remark 2.1.

- (i) $\sum_{j=1}^n a_{kj} \bar{x}_j^{pq} = \bar{b}_k^{pq}$ if and only if $\sum_{j=1}^n a_{kj} m(\bar{x}_j^{pq}) = m(\bar{b}_k^{pq})$ and $\sum_{j=1}^n w(a_{kj} \bar{x}_j^{pq}) = w(\bar{b}_k^{pq})$ for $k \in [1, m]$.
- (ii) $\sum_{j=1}^n a_{kj} \bar{x}_j^{pq} \neq \bar{b}_k^{pq}$ if and only if $\sum_{j=1}^n a_{kj} m(\bar{x}_j^{pq}) = m(\bar{b}_k^{pq})$ and $\sum_{j=1}^n w(a_{kj} \bar{x}_j^{pq}) \neq w(\bar{b}_k^{pq})$ for $k \in [1, m]$.

Remark 2.2.

- (i) $\sum_{j=1}^n a_{kj} \bar{x}_j^{pq} = \bar{b}_k^{pq}$ if and only if the slack variable $x_{n+k}^{pq} = 0$ for $k \in [1, m]$.
- (ii) $\sum_{j=1}^n a_{kj} \bar{x}_j^{pq} \neq \bar{b}_k^{pq}$ if and only if the slack variable $x_{n+k}^{pq} \neq 0$ for $k \in [1, m]$.

From Remark 2.1 and 2.2, we can say that

$$(2.8) \quad \left\{ \begin{array}{l} \text{Max / Min } \bar{Z}^{pq}(\bar{x}^{pq}) \approx \langle \sum_{j=1}^n m(c_j \bar{x}_j^{pq}), \sum_{j=1}^n w(c_j \bar{x}_j^{pq}) \rangle \\ \text{Subject to the constraints} \\ \langle \sum_{j=1}^n a_{ij} m(\bar{x}_j^{pq}), \sum_{j=1}^n w(a_{ij} \bar{x}_j^{pq}) \rangle \begin{pmatrix} \approx \\ \succeq \\ \preceq \end{pmatrix} \langle m(\bar{b}_i^{pq}), w(\bar{b}_i^{pq}) \rangle. \end{array} \right.$$

From (2.7) and (2.8), we can get:

$$(2.9) \quad \left\{ \begin{array}{l} \text{Max / Min } m(\bar{Z}^{pq}(\bar{x}^{pq})) \approx \sum_{j=1}^n c_j m(\bar{x}_j^{pq}) \\ \text{Subject to the constraints} \\ \sum_{j=1}^n a_{ij} m(\bar{x}_j^{pq}) \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} m(\bar{b}_i^{pq}), \quad \text{for } i \in 1, 2, \dots, m \end{array} \right.$$

where $\sum_{j=1}^n w(a_{kj} \bar{x}_j^{pq}) = w(\bar{x}_k^{pq})$ or $\sum_{j=1}^n w(a_{kj} \bar{x}_j^{pq}) \neq w(\bar{b}_k^{pq})$ for $k \in [1, m]$. So, (2.9) is equivalent to

$$(2.10) \quad \left\{ \begin{array}{l} \text{Max / Min } m(\bar{Z}^{pq}(\bar{x}^{pq})) \approx \sum_{j=1}^n c_j m(\bar{x}_j^{pq}) \\ \text{Subject to the constraints} \\ \sum_{j=1}^n a_{ij} \bar{x}_j^{pq} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^q + b_i^p}{2}, \quad \text{for } i \in 1, 2, \dots, m \end{array} \right.$$

where $\bar{x}_j^{pq} = m(\bar{x}_j^{pq}) = \frac{x_j^q + x_j^p}{2}$, $w(\bar{w}_j^{pq}) = \frac{x_j^q - x_j^p}{2}$, $m(\bar{b}_i^{pq}) = \frac{b_i^q + b_i^p}{2}$ and $w(\bar{b}_i^{pq}) = \frac{b_i^q - b_i^p}{2}$.

Optimal solution according to the choice of the decision maker with minimum uncertainty:

$$(2.11) \quad \text{Max / Min } m(\bar{Z}^{pq}(\bar{x}^{pq})) \approx \sum_{j=1}^n c_j \bar{x}_j^{pq}$$

with $\bar{x}_j^{pq} = [x_j^p, x_j^q] = [x_j^{pq} - w(\bar{x}_j^{pq}), x_j^{pq} + w(\bar{x}_j^{pq})]$.

For $x_{n+k}^{pq} = 0$, we have $\sum_{j=1}^n a_{kj} \bar{x}_j^{pq} = \bar{b}_k^{pq}$ and $\sum_{j=1}^n a_{kj} w(\bar{x}_j^{pq}) = w(\bar{b}_k^{pq})$, $k \in [1, m]$.

2.3. Formulation of linear programming problem with variables given as fuzzy numbers.

The fuzzy linear programming formulation of a Linear Programming Problem with variables given as fuzzy numbers can be written as follows as follows [6, 7]:

$$(2.12) \quad \left\{ \begin{array}{l} \text{Max / Min } \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j \\ \text{Subject to the constraints} \\ \sum_{j=1}^n a_{ij} \tilde{x}_j^{pq} \left(\begin{array}{l} \approx \\ \succeq \\ \preceq \end{array} \right) \tilde{b}_i, \text{ for } i = 1, 2, \dots, m \end{array} \right.$$

where $\tilde{x}_j = (x_j^1 \leq x_j^2 \leq \dots \leq x_j^t)$ and $\tilde{b}_i = (b_i^1 \leq b_i^2 \leq \dots \leq b_i^t)$ are unrestricted fuzzy numbers and c_j and a_{ij} are real numbers.

We use $t \in \mathbb{N}_{\geq 1}$ to extend the algorithm to all types of numbers (real numbers, interval numbers and fuzzy numbers).

3. RESULTS

In this section, a solution procedure for solving the problem (2.6) via (2.10) is developed in the following steps:

- Step 1.** Construct the fuzzy linear programming problem (2.12), and then convert it into an interval linear programming problem (2.6) based on the new arithmetic of fuzzy or interval numbers.
- Step 2.** Convert the problem (2.6) into the corresponding classical linear programming problems (2.10) based on the new arithmetic of fuzzy or interval numbers, and then solving (2.10):

$$\left\{ \begin{array}{l} \text{Max / Min } Z^{pq}(x^{pq}) \approx \sum_{j=1}^n c_j x_j^{pq} \\ \text{Subject to the constraints} \\ \sum_{j=1}^n a_{ij} x_j^{pq} \left(\begin{array}{l} = \\ \leq \\ \geq \end{array} \right) \frac{b_i^q + b_i^p}{2} \text{ and } x_j^{pq} \geq 0. \end{array} \right.$$

- Step 3.** Determine $w(\bar{x}_j^{pq})$ with $\bar{x}_j^{pq} = [x_j^{pq} - w(\bar{x}_j^{pq}), x_j^{pq} + w(\bar{x}_j^{pq})] = [x_j^p, x_j^q]$ for $j = 1, \dots, n$, by applying the following conditions:

$$\sum_{j=1}^n a_{kj} w(\bar{x}_j^{pq}) = w(\bar{b}_k^{pq}).$$

if, and only if, the slack variable

$$x_{n+k}^{pq} = 0 \text{ for } k \in [1, m].$$

Considering the following cases:

Case 1. t is odd or even:

(i) If t is odd, then $p = q = (t + 1)/2$ and $w(\bar{x}_j^{pq}) = 0$, and go to

Case 2.

(ii) If t is even, then $p = t/2$ and $q = (t + 2)/2$ do:

(a) If $x_j^{pq} = 0$, then $w(\bar{x}_j^{pq}) = 0$. Else, choose between (b) or (c) or (d):

(b) **Very important decision:** if $\sum_{j=1}^n a_{kj}w(\bar{x}_j^{pq}) = w(\bar{b}_k^{pq})$ for all $k \in [1, m]$, then the current solution is optimal and go to **Case 2.**

(c) **Very important decision:** if $\sum_{j=1}^n a_{kj}w(\bar{x}_j^{pq}) = w(\bar{b}_k^{pq})$ for some $k \in [1, m]$, then the current solution is optimal and go to **Case 2.**

(d) **Important decision:** choose an index k such that

$$\sum_{j=1}^n a_{kj}w(\bar{x}_j^{pq}) = w(\bar{b}_k^{pq}), \text{ then go to Case 2.}$$

Case 2. For $p = q \neq (t + 1)/2$, $p \neq t/2$ and $q \neq (t + 2)/2$, then choose between

(a) or (b) or (c):

(a) **Very important decision:** if $\sum_{j=1}^n a_{kj}w(\bar{x}_j^{pq}) = w(\bar{b}_k^{pq})$ for all $k \in [1, m]$ with $|x_j^{pq} - x_j^{(p+1)(q-1)}| + w(\bar{x}_j^{(p+1)(q-1)}) \leq w(\bar{x}_j^{pq})$, then the current solution is optimal.

(b) **Very important decision:** if $\sum_{j=1}^n a_{kj}w(\bar{x}_j^{pq}) = w(\bar{b}_k^{pq})$ for some $k \in [1, m]$ with $|x_j^{pq} - x_j^{(p+1)(q-1)}| + w(\bar{x}_j^{(p+1)(q-1)}) \leq w(\bar{x}_j^{pq})$, then the current solution is optimal.

(c) **Important decision:** choose an index k such that $\sum_{j=1}^n a_{kj}w(\bar{x}_j^{pq}) = w(\bar{b}_k^{pq})$ with $|x_j^{pq} - x_j^{(p+1)(q-1)}| + w(\bar{x}_j^{(p+1)(q-1)}) \leq w(\bar{x}_j^{pq})$ otherwise $w(\bar{x}_j^{pq}) = |x_j^{pq} - x_j^{(p+1)(q-1)}| + w(\bar{x}_j^{(p+1)(q-1)})$.

3.1. Solution procedure for Linear Programming Problem with variables given as Interval numbers ($t = 2$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Interval numbers as follows (2.6), (2.12) and ($t = 2$) [4] where $\bar{x}_j^{12} = [x_j^1, x_j^2]$ and $\bar{b}_i^{12} = [b_i^1, b_i^2]$ with $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The interval optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \bar{Z}^{12}(\bar{x}^{12}) \approx \sum_{j=1}^n c_j \bar{x}_j^{12}$.

3.2. Solution procedure for Linear Programming Problem with variables given as Triangular fuzzy numbers ($t = 3$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Triangular fuzzy numbers as follows (2.12) and ($t = 3$) [6, 7] where $\tilde{x}_j = (x_j^1, x_j^2, x_j^3) = (x_j^2; \bar{x}_j^{13})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3) = (b_i^2; \bar{b}_i^{13})$ with $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.3. Solution procedure for Linear Programming Problem with variables given as Trapezoidal fuzzy numbers ($t = 4$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Trapezoidal fuzzy numbers as follows (2.12) and ($t = 4$) [6, 7] where $\tilde{x}_j = (\bar{x}_j^1, x_j^2, x_j^3, x_j^4) = (\bar{x}_j^{23}; \bar{x}_j^{14})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4) = (\bar{b}_i^{23}; \bar{b}_i^{14})$ with $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.4. Solution procedure for Linear Programming Problem with variables given as Pentagonal fuzzy numbers ($t = 5$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Pentagonal fuzzy numbers as follows (2.12) and ($t = 5$) [17] where $\tilde{x}_j = (\bar{x}_j^1, x_j^2, x_j^3, x_j^4, x_j^5) = (x_j^3; \bar{x}_j^{24}; \bar{x}_j^{15})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5) = (b_i^3; \bar{b}_i^{24}; \bar{b}_i^{15})$ with $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.5. Solution procedure for Linear Programming Problem with variables given as Hexagonal fuzzy numbers ($t = 6$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Hexagonal fuzzy numbers as follows (2.12) and ($t = 6$) [18] where $\tilde{x}_j = (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6) = (\bar{x}_j^{34}; \bar{x}_j^{24}; \bar{x}_j^{15})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6) = (\bar{b}_i^{34}; \bar{b}_i^{25}; \bar{b}_i^{16})$ with $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.6. Solution procedure for Linear Programming Problem with variables given as Heptagonal fuzzy numbers ($t = 7$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Heptagonal fuzzy numbers as follows (2.12) and ($t = 7$) [20] where $\tilde{x}_j = (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7) = (\bar{x}_j^4; \bar{x}_j^{35}; \bar{x}_j^{26}; \bar{x}_j^{17})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7) = (\bar{b}_i^4; \bar{b}_i^{35}; \bar{b}_i^{26}; \bar{b}_i^{17})$ with $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.7. Solution procedure for Linear Programming Problem with variables given as Octagonal fuzzy numbers ($t = 8$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Octagonal fuzzy numbers as follows (2.12) and ($t = 8$) [19] where $\tilde{x}_j = (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8) = (\bar{x}_j^{45}; \bar{x}_j^{36}; \bar{x}_j^{27}; \bar{x}_j^{18})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8) = (\bar{b}_i^{45}; \bar{b}_i^{36}; \bar{b}_i^{27}; \bar{b}_i^{18})$ with $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.8. Solution procedure for Linear Programming Problem with variables given as Nonagonal fuzzy numbers ($t = 9$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Nonagonal fuzzy numbers as follows (2.12) and ($t = 9$) [21] where $\tilde{x}_j = (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9) = (\bar{x}_j^{56}; \bar{x}_j^{46}; \bar{x}_j^{37}; \bar{x}_j^{28}; \bar{x}_j^{19})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8, b_i^9) = (\bar{b}_i^{56}; \bar{b}_i^{46}; \bar{b}_i^{37}; \bar{b}_i^{28}; \bar{b}_i^{19})$ with $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.9. Solution procedure for Linear Programming Problem with variables given as Decagonal fuzzy numbers ($t = 10$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Decagonal fuzzy numbers as follows (2.12) and ($t = 10$) [22] where $\tilde{x}_j = (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9, x_j^{10}) = (\bar{x}_j^{56}; \bar{x}_j^{47}; \bar{x}_j^{38}; \bar{x}_j^{29}; \bar{x}_j^{110})$

and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8, b_i^9, b_i^{10}) = (\bar{b}_i^{56}, \bar{b}_i^{47}, \bar{b}_i^{38}, \bar{b}_i^{29}, \bar{b}_i^{110})$ with $\tilde{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.10. Solution procedure for Linear Programming Problem with variables given as Hendecagonal fuzzy numbers ($t = 11$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Hendecagonal fuzzy numbers as follows (2.12) and ($t = 11$) [23] where $\tilde{x}_j = (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9, x_j^{10}, x_j^{11}) = (x_j^6, \bar{x}_j^{57}, \bar{x}_j^{48}, \bar{x}_j^{39}, \bar{x}_j^{210}, \bar{x}_j^{111})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8, b_i^9, b_i^{10}, b_i^{11}) = (b_i^6, \bar{b}_i^{57}, \bar{b}_i^{48}, \bar{b}_i^{39}, \bar{b}_i^{210}, \bar{b}_i^{111})$ with $\tilde{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

3.11. Solution procedure for Linear Programming Problem with variables given as Dedocagonal fuzzy numbers ($t = 12$).

For all the rest of this paper, we will consider the following linear programming problem with variables given as Dedocagonal fuzzy numbers as follows (2.12) and ($t = 12$) [23] where $\tilde{x}_j = (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9, x_j^{10}, x_j^{11}, x_j^{12}) = (\bar{x}_j^{67}, \bar{x}_j^{58}, \bar{x}_j^{49}, \bar{x}_j^{310}, \bar{x}_j^{211}, \bar{x}_j^{112})$ and $\tilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8, b_i^9, b_i^{10}, b_i^{11}, b_i^{12}) = (\bar{b}_i^{67}, \bar{b}_i^{58}, \bar{b}_i^{49}, \bar{b}_i^{310}, \bar{b}_i^{211}, \bar{b}_i^{112})$ with $\tilde{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$.

The fuzzy optimal solution according to the choice of the decision maker with minimum uncertainty is $Max / Min \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$.

4. NUMERICAL EXAMPLES

Example 1. Consider the following interval number linear programming problem [4]:

$$\left\{ \begin{array}{l} Min \bar{Z}^{12}(\bar{x}^{12}) \approx 26\bar{x}_1^{12} + 7\bar{x}_2^{12} \\ \text{Subject to the constraints} \\ 6\bar{x}_1^{12} + 4\bar{x}_2^{12} \geq [29, 31] \\ 5\bar{x}_1^{12} + 2\bar{x}_2^{12} \geq [22, 24] \\ 3\bar{x}_1^{12} + 5\bar{x}_2^{12} \geq [28, 30]. \end{array} \right.$$

Step 1. Solving (2.6) via (2.10). We have $p = 1$, $q = 2$. We get

$$\left\{ \begin{array}{l} \text{Min } Z^{12}(x^{12}) = 26x_1^{12} + 7x_2^{12} \\ \text{Subject to the constraints} \\ 6x_1^{12} + 4x_2^{12} \geq 30 \\ 5x_1^{12} + 2x_2^{12} \geq 23 \\ 3x_1^{12} + 5x_2^{12} \geq 29. \end{array} \right.$$

Optimal solution: $x_1^{12} = 0$ and $x_2^{12} = \frac{23}{2}$. Slack variables values: $x_3^{12} = 16$, $x_4^{12} = 0$ and $x_5^{12} = \frac{57}{2}$.

Very important decision: For $x_4^{12} = 0$, we have $5w(\bar{x}_1^{12}) = +2w(\bar{x}_2^{12}) = 1$. We get $w(\bar{x}_1^{12}) = 0$ and $w(\bar{x}_2^{12}) = \frac{1}{2}$. Therefore, we get $\bar{x}_1^{12} = [0, 0]$ and $\bar{x}_2^{12} = [11, 12]$.

Step 2. The interval optimal solution according to the choice of the decision maker with minimum uncertainty is $\text{Min } \bar{Z}^{12}(\bar{x}^{12}) \approx [77, 84]$ where $\bar{x}_1^{12} = [0, 0]$ and $\bar{x}_2^{12} = [11, 12]$. Then the corresponding dual problem is given by: $\text{Max } \bar{W}^{12}(\bar{x}^{12}) \approx [77, 84]$ where $y_1 = 0$, $y_2 = \frac{7}{2}$ and $y_3 = 0$.

We see that both primal and dual problems have interval optimal solutions and the two interval optimal values are equal.

In contrast to [4], the centers of all constraints are saturated $\sum_{j=1}^n a_{ij}x_j^{12} = b_i^{12}$ and the second constraint is saturated: $\sum_{j=1}^n a_{2j}x_j^{12} = b_2^{12}$.

Example 2. Consider the following linear programming problem with variables given as Triangular fuzzy numbers [5]

$$\left\{ \begin{array}{l} \text{Max } \tilde{Z}(\tilde{x}) \approx 4\tilde{x}_1 + 3\tilde{x}_2 \\ \text{Subject to the constraints} \\ \tilde{x}_1 + 2\tilde{x}_2 \preceq (4, 8, 12), \\ 2\tilde{x}_1 + \tilde{x}_2 \preceq (6, 9, 12). \end{array} \right.$$

Step 1. Solving (2.6) via (2.10). We have $p = 2$, $q = 2$. We get

$$\left\{ \begin{array}{l} \text{Max } Z^2(x^2) = 4x_1^2 + 3x_2^2 \\ \text{Subject to the constraints} \\ x_1^2 + 2x_2^2 \leq 8, \\ 2x_1 + x_2 \leq 9. \end{array} \right.$$

Optimal solution: $x_1^2 = \frac{10}{3}$ and $x_2^2 = \frac{7}{3}$. Slack variables values: $x_3^2 = 0$ and $x_4^2 = 0$.

Step 2. Solving (2.6) via (2.10). We have $p = 1$, $q = 3$. We get

$$\left\{ \begin{array}{l} \text{Max } Z^{13}(x^{13}) = 4x_1^{13} + 3x_2^{13} \\ \text{Subject to the constraints} \\ x_1^{13} + 2x_2^{13} \leq 8 \\ 2x_1^{13} + x_2^{13} \leq 9. \end{array} \right.$$

Optimal solution: $x_1^{13} = \frac{10}{3}$ and $x_2^{13} = \frac{7}{3}$. Slack variables values: $x_3^{13} = 0$ and $x_4^{13} = 0$.

Very important decision: For $x_3^{13} = 0$ and $x_4^{13} = 0$, we have $w(\bar{x}_1^{13}) + 2w(\bar{x}_2^{13}) = w(\bar{b}_1^{13}) = 4$ and $2w(\bar{x}_1^{13}) + w(\bar{x}_2^{13}) = w(\bar{b}_2^{13}) = 3$. We get $w(\bar{x}_1^{13}) = \frac{2}{3}$ and $w(\bar{x}_2^{13}) = \frac{5}{3}$ with $|x_1^{13} - x_1^2| \leq \frac{2}{3}$ and $|x_2^{13} - x_2^2| \leq \frac{5}{3}$. Therefore, we get $\bar{x}_1^{13} = [\frac{8}{3}, 4]$ and $\bar{x}_2^{13} = [\frac{2}{3}, 4]$.

Step 3. The optimal solution according to the choice of the decision maker with minimum uncertainty is $\text{Max } \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$ with $\tilde{x}_j = (x_j^2; \bar{x}_j^{13}) = (x_j^1, x_j^2, x_j^3) : \text{Max } \tilde{Z}(\tilde{x}) \approx (\frac{38}{3}, \frac{61}{3}, 28)$ where $\bar{x}_1 = (\frac{8}{3}, \frac{10}{3}, 4)$ and $\bar{x}_2 = (\frac{2}{3}, \frac{7}{3}, 4)$. Then the corresponding dual problem is given by: $\text{Min } \tilde{W}(y) \approx (\frac{38}{3}, \frac{61}{3}, 28)$ where $y_1 = \frac{2}{3}$ and $y_2 = \frac{5}{3}$.

We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to [5] the centers of all constraints are saturated and all constraints are saturated.

Example 3. Consider the following linear programming problem with variables given as Trapezoidal fuzzy numbers [6, 7]:

$$\left\{ \begin{array}{l} \text{Max } \tilde{Z}(\tilde{x}) \approx 50\tilde{x}_1 + 20\tilde{x}_2 + 25\tilde{x}_3 \\ \text{Subject to the constraints} \\ 9\tilde{x}_1 + 3\tilde{x}_2 + 5\tilde{x}_3 \preceq (335, 340, 370, 395), \\ 2\tilde{x}_1 + 4\tilde{x}_2 + 3\tilde{x}_3 \preceq (220, 245, 265, 270), \\ 5\tilde{x}_1 + 2\tilde{x}_3 \preceq (135, 140, 160, 165), \\ \tilde{0} \preceq \tilde{x}_1 \preceq (22, 24, 26, 28), \\ \tilde{0} \preceq \tilde{x}_2 \preceq (24, 26, 32, 42), \\ \tilde{0} \preceq \tilde{x}_3 \preceq (29, 34, 38, 39). \end{array} \right.$$

Step 1. Solving (2.6) via (2.10). We have $p = 2$, $q = 3$. We get

$$\left\{ \begin{array}{l} \text{Max } Z^{23}(x^{23}) = 50x_1^{23} + 20x_2^{23} + 25x_3^{23} \\ \text{Subject to the constraints} \\ 9x_1^{23} + 3x_2^{23} + 5x_3^{23} \leq 355, \\ 2x_1^{23} + 4x_2^{23} + 3x_3^{23} \leq 255, \\ 5x_1^{23} + 2x_3^{23} \leq 150, \\ 0 \leq x_1^{23} \leq 25, \\ 0 \leq x_2^{23} \leq 29, \\ 0 \leq x_3^{23} \leq 36. \end{array} \right.$$

Optimal solution: $x_1^{23} = 25$, $x_2^{23} = 29$ and $x_3^{23} = \frac{43}{5}$. *Slack variables values:* $x_4^{23} = 0$, $x_5^{23} = \frac{273}{5}$, $x_6^{23} = \frac{39}{5}$, $x_7^{23} = 0$, $x_8^{23} = 0$ and $x_9^{23} = \frac{137}{5}$.

Very important decision: For $x_7^{23} = 0$ and $x_8^{23} = 0$, we have $w(\bar{x}_1^{23}) = 1$ and $w(\bar{x}_2^{23}) = 3$. We have $x_9^{23} \neq 0$ then $w(\bar{x}_3^{23}) < 2$. We take $w(\bar{x}_3^{23}) = \frac{3}{5}$. Therefore, we get $\bar{x}_1^{23} = [24, 26]$, $\bar{x}_2^{23} = [26, 32]$ and $x_3^{23} = [8, \frac{46}{5}]$.

Step 2. Solving (2.6) via (2.10). We have $p = 1$, $q = 4$. We get

$$\left\{ \begin{array}{l} \text{Max } Z^{14}(x^{14}) = 50x_1^{14} + 20x_2^{14} + 25x_3^{14} \\ \text{Subject to the constraints} \\ 9x_1^{14} + 3x_2^{14} + 5x_3^{14} \leq 365, \\ 2x_1^{14} + 4x_2^{14} + 3x_3^{14} \leq 245, \\ 5x_1^{14} + 2x_3^{14} \leq 150, \\ 0 \leq x_1^{14} \leq 25, \\ 0 \leq x_2^{14} \leq 33, \\ 0 \leq x_3^{14} \leq 34. \end{array} \right.$$

Optimal solution: $x_1^{14} = 25$, $x_2^{14} = 33$ and $x_3^{14} = \frac{41}{5}$. *Slack variables values:* $x_4^{14} = 0$, $x_5^{14} = \frac{151}{5}$, $x_6^{14} = \frac{43}{5}$, $x_7^{14} = 0$, $x_8^{14} = 0$ and $x_9^{14} = \frac{129}{5}$.

Very important decision: For $x_7^{14} = 0$ and $x_8^{14} = 0$, we have $w(\bar{x}_1^{14}) = 3$ and $w(\bar{x}_2^{14}) = 9$. We have $x_9^{14} \neq 0$, then $w(\bar{x}_3^{14}) < 5$. We take $w(\bar{x}_3^{14}) = 2$. Therefore, we get $\bar{x}_1^{14} = [22, 28]$, $\bar{x}_2^{14} = [24, 42]$ and $x_3^{14} = [\frac{31}{5}, \frac{51}{5}]$.

Step 3. The optimal solution according to the choice of the decision maker with minimum uncertainty is $\text{Max } \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^n c_j \tilde{x}_j$ with $\tilde{x}_j = (\bar{x}_j^{23}; \bar{x}_j^{14}) = (x_j^1, x_j^2, x_j^3, x_j^4)$:

$$\text{Max } \tilde{Z}(\tilde{x}) \approx (1735, 1920, 2170, 2495)$$

where

$$\tilde{x}_1 = (22, 24, 26, 28), \quad \tilde{x}_2 = (24, 26, 32, 42), \quad \text{and} \quad \tilde{x}_3 = \left(\frac{31}{5}, 8, \frac{46}{5}, \frac{51}{5}\right).$$

Then the corresponding dual problem is given by: $\text{Min } \tilde{W}(y) \approx (1905, 1950, 2140, 2325)$ where $y_1 = 5, y_2 = 0, y_3 = 0, y_4 = 5, y_5 = 5$ and $y_6 = 0$.

We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to [6, 7], the centers of all constraints are saturated and the fourth and fifth are saturated.

Example 4. Consider the following linear programming problem with variables given as Pentagonal fuzzy numbers [35]

$$\text{Max } \tilde{Z}(\tilde{x}) \approx 5\tilde{x}_1 + 15\tilde{x}_2$$

subject to the constraints :

$$\begin{cases} 4\tilde{x}_1 + 8\tilde{x}_2 \preceq (28, 44, 65, 68, 73) \\ \tilde{x}_1 + 7\tilde{x}_2 \preceq (18, 22, 33, 38, 46) \end{cases}$$

Step 1. Solving (2.6) via (2.10). We have $p = 3, q = 3$. We get

$$\text{Max } Z^{33}(x^{33}) = 5x_1^{33} + 15x_2^{33}$$

subject to the constraints :

$$\begin{cases} 4x_1^{33} + 8x_2^{33} \leq 65 \\ x_1^{33} + 7x_2^{33} \leq 33 \end{cases}$$

Optimal solution : $x_1^{33} = \frac{191}{20}$ and $x_2^{33} = \frac{67}{20}$. Slack variables values: $x_3^{33} = 0$ and $x_4^{33} = 0$.

Step 2. : Solving (2.6) via (2.10). We have $p = 2, q = 4$. We get

$$\text{Max } Z^{24}(x^{24}) = 5x_1^{24} + 15x_2^{24}$$

subject to the constraints :

$$\begin{cases} 4x_1^{24} + 8x_2^{24} \leq 56 \\ x_1^{24} + 7x_2^{24} \leq 30 \end{cases}$$

Optimal solution : $x_1^{24} = \frac{38}{5}$ and $x_2^{24} = \frac{16}{5}$. *Slack variables values:* $x_3^{24} = 0$ and $x_4^{24} = 0$.

Important decision: For $x_4^{24} = 0$, we have $w(\bar{x}_1^{24}) + 7w(\bar{x}_2^{24}) = w(\bar{b}_2^{24}) = 8$. We get $w(\bar{x}_1^{24}) = \frac{26}{5}$, $w(\bar{x}_2^{24}) = \frac{2}{7}$ with $|x_1^{24} - x_1^3| = \frac{39}{20} \leq \frac{26}{5}$ and $|x_2^{24} - x_2^3| = \frac{3}{20} \leq \frac{2}{5}$. Therefore, we get $\bar{x}_1^{24} = [\frac{12}{5}, \frac{64}{5}]$ and $\bar{x}_2^{24} = [\frac{14}{5}, \frac{18}{5}]$.

Step 3. Solving (2.6) via (2.10). we have $p = 1$, $q = 5$. We get

$$\text{Max } Z^{15}(x^{15}) = 5x_1^{15} + 15x_2^{15}$$

subject to the constraints :

$$\begin{cases} 4x_1^{15} + 8x_2^{15} \leq \frac{101}{2} \\ x_1^{15} + 7x_2^{15} \leq 32 \end{cases}$$

Optimal solution : $x_1^{15} = \frac{39}{8}$ and $x_2^{15} = \frac{31}{8}$. *Slack variables values:* $x_3^{15} = 0$ and $x_4^{15} = 0$. *Important decision:* For $x_4^{15} = 0$, we get $w(\bar{x}_1^{15}) = |x_1^{15} - x_1^{24}| + w(\bar{x}_1^{24}) = \frac{317}{40}$ and $w(\bar{x}_2^{15}) = |x_2^{15} - x_2^{24}| + w(\bar{x}_2^{24}) = \frac{43}{40}$. Therefore, we get $\bar{x}_1^{15} = [\frac{-61}{20}, \frac{256}{20}]$ and $\bar{x}_2^{15} = [\frac{56}{20}, \frac{99}{20}]$.

Step 4. The optimal solution according to the choice of the decision maker with minimum uncertainty is $\tilde{Z}(\tilde{x}) \approx (\frac{107}{4}, 54, 98, 118, \frac{553}{4})$ with $\tilde{x}_1 = (\frac{-61}{20}, \frac{12}{5}, \frac{191}{20}, \frac{64}{5}, \frac{256}{20})$ and $\tilde{x}_2 = (\frac{56}{20}, \frac{14}{5}, \frac{67}{20}, \frac{18}{5}, \frac{99}{20})$. Then the corresponding dual problem is given by: $\bar{w}(y) \approx (46, 66, 98, 106, 119)$ with $y_1 = 1$ and $y_2 = 1$.

We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to [35], the centers of all constraints are saturated and the second constraint is saturated.

Example 5. Consider the following linear programming problem with variables given as Hexagonal fuzzy numbers [36]:

$$\text{Max } \tilde{Z}(\tilde{x}) \approx 16\tilde{x}_1 + 36\tilde{x}_2$$

subject to the constraints :

$$\begin{cases} 69\tilde{x}_1 + 99\tilde{x}_2 \preccurlyeq (151, 153, 155, 157, 159, 161) \\ 129\tilde{x}_1 + 159\tilde{x}_2 \preccurlyeq (271, 273, 275, 277, 279, 281) \end{cases}$$

Step 1. Solving (2.6) via (2.10). We have $p = 3$, $q = 4$. We get

$$\text{Max } Z^{34}(x^{34}) = 16x_1^{34} + 36x_2^{34}$$

subject to the constraints :

$$\begin{cases} 69x_1^{34} + 99x_2^{34} & \leq 156 \\ 129x_1^{34} + 159x_2^{34} & \leq 276 \end{cases}$$

Optimal solution: $x_1^{34} = 0$ and $x_2^{34} = \frac{52}{33}$. *Slack variables values:* $x_3^{34} = 0$ and $x_4^{34} = \frac{280}{11}$.

Very important decision: For $x_3^{34} = 0$, we have $66w(\bar{x}_1^{34}) + 99w(\bar{x}_1^{34}) = w(\bar{b}_1^{34}) = 1$. We get $w(\bar{x}_1^{34}) = 0$ ($x_1^{34} = 0$) and $w(\bar{x}_2^{34}) = \frac{1}{99}$. Therefore, we get $\bar{x}_1^{34} = [0, 0]$ and $\bar{x}_2^{34} = [\frac{155}{99}, \frac{157}{99}]$.

Step 2. Solving (2.6) via (2.10). We have $p = 2$, $q = 5$. We get

$$\text{Max } Z^{25}(x^{25}) = 16x_1^{25} + 36x_2^{25}$$

subject to the constraints :

$$\begin{cases} 69x_1^{25} + 99x_2^{25} & \leq 156 \\ 129x_1^{25} + 159x_2^{25} & \leq 276 \end{cases}$$

Optimal solution: $x_1^{25} = 0$ and $x_2^{25} = \frac{52}{33}$. *Slack variables values:* $x_3^{25} = 0$ and $x_4^{25} = \frac{280}{11}$.

Very important decision: For $x_3^{25} = 0$, we have $66w(\bar{x}_1^{25}) + 99w(\bar{x}_2^{25}) = w(\bar{b}_1^{25}) = 3$. We get $w(\bar{x}_1^{25}) = 0$ ($x_1^{25} = 0$) and $w(\bar{x}_2^{25}) = \frac{1}{33}$ with $|x_2^{25} - x_2^{32}| = 0 \leq \frac{1}{33}$. Therefore, we get $\bar{x}_1^{25} = [0, 0]$ and $\bar{x}_2^{25} = [\frac{17}{11}, \frac{53}{33}]$.

Step 3. Solving (2.6) via (2.10). We have $p = 1$, $q = 6$. We get:

$$\text{Max } Z^{16}(x^{16}) = 16x_1^{16} + 36x_2^{16}$$

subject to the constraints :

$$\begin{cases} 69x_1^{16} + 99x_2^{16} & \leq 156 \\ 129x_1^{16} + 159x_2^{16} & \leq 276 \end{cases}$$

Optimal solution: $x_1^{16} = 0$ and $x_2^{16} = \frac{52}{33}$. *Slack variables values:* $x_3^{16} = 0$ and $x_4^{16} = \frac{280}{11}$.

Very important decision : For $x_3^{16} = 0$, we have $66w(\bar{x}_1^{16}) + 99w(\bar{x}_2^{16}) = w(\bar{b}_1^{16}) = 5$. We get $w(\bar{x}_1^{16}) = 0$ ($x_1^{16} = 0$) and $w(\bar{x}_2^{16}) = \frac{5}{99}$ with $|x_2^{16} - x_2^{25}| + \frac{1}{33} = \frac{1}{33} \leq \frac{5}{99}$. Therefore, we get $\bar{x}_1^{16} = [0, 0]$ and $\bar{x}_2^{16} = [\frac{151}{99}, \frac{161}{99}]$.

Step 4. The optimal solution according to the choice of the decision maker with minimum uncertainty is $\max \tilde{Z}(\tilde{x}) \approx (\frac{604}{11}, \frac{612}{11}, \frac{620}{11}, \frac{628}{11}, \frac{636}{11}, \frac{644}{11})$ with $\tilde{x}_1 = 0$ and $\tilde{x}_2 = (\frac{151}{99}, \frac{17}{11}, \frac{155}{99}, \frac{157}{99}, \frac{53}{33}, \frac{161}{99})$. Then the corresponding dual problem is given by: $\min \bar{w}(y) \approx (\frac{604}{11}, \frac{612}{11}, \frac{620}{11}, \frac{628}{11}, \frac{636}{11}, \frac{644}{11})$ with $y_1 = \frac{4}{11}$ and $y_2 = 0$.

We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to [36], the centers of all constraints are saturated and the first constraint is saturated.

Example 6. Consider the following linear programming problem with variables given as Octagonal fuzzy numbers [37]:

$$\text{Min } \tilde{Z}(\tilde{x}) \approx 6\tilde{x}_1 + 8\tilde{x}_2$$

subject to the constraints :

$$\begin{cases} 20\tilde{x}_1 + 30\tilde{x}_2 \succeq (885, 886, 888, 890, 910, 912, 914, 915) \\ 40\tilde{x}_1 + 30\tilde{x}_2 \succeq (1190, 1191, 1193, 1195, 1205, 1207, 1209, 1210) \end{cases}$$

Step 1. Solving (2.6) via (2.10). We have $p = 4$, $q = 5$. We get

$$\min \quad Z^{45}(x^{45}) = 6x_1^{45} + 8x_2^{45}$$

subject to the constraints :

$$\begin{cases} 20x_1^{45} + 30x_2^{45} \geq 900 \\ 40x_1^{45} + 30x_2^{45} \geq 1200 \end{cases}$$

Optimal solution : $x_1^{45} = 15$ and $x_2^{45} = 20$. *Slack variables values:* $x_3^{45} = 0$ and $x_4^{45} = 0$.

Important decision : For $x_4^{45} = 0$, we have $40w(\bar{x}_1^{45}) + 30w(\bar{x}_2^{45}) = w(\bar{b}_2^{45}) = 5$. We get $w(\bar{x}_1^{45}) = \frac{1}{16}$ et $w(\bar{x}_2^{45}) = \frac{1}{12}$. Therefore, we get $\bar{x}_1^{45} = [\frac{239}{16}, \frac{241}{16}]$ and $\bar{x}_2^{45} = [\frac{239}{12}, \frac{241}{12}]$.

Step 2. Solving (2.6) via (2.10). We have $p = 3$, $q = 6$. We get

$$\text{Min } Z^{36}(x^{36}) = 6x_1^{36} + 8x_2^{36}$$

subject to the constraints :

$$\begin{cases} 20x_1^{36} + 30x_2^{36} \geq 900 \\ 40x_1^{36} + 30x_2^{36} \geq 1200 \end{cases}$$

Optimal solution: $x_1^{36} = 15$ and $x_2^{36} = 20$. *Slack variables values:* $x_3^{36} = 0$ and $x_4^{36} = 0$.

Important decision : For $x_4^{36} = 0$ we have $40w(\bar{x}_1^{36}) + 30w(\bar{x}_2^{36}) = w(\bar{b}_2^{36}) = 7$. We get $w(\bar{x}_1^{36}) = \frac{7}{80}$ and $w(\bar{x}_2^{36}) = \frac{7}{60}$ with $|x_1^{36} - x_1^{45}| + \frac{5}{80} = \frac{5}{80} \leq \frac{7}{80}$ and $|x_2^{36} - x_2^{45}| + \frac{5}{60} = \frac{5}{60} \leq \frac{7}{60}$. Therefore, we get $\bar{x}_1^{36} = [\frac{1193}{80}, \frac{1207}{80}]$ and $\bar{x}_2^{36} = [\frac{1193}{60}, \frac{1207}{60}]$.

Step 3. Solving (2.6) via (2.10). We have $p = 2$, $q = 7$. We get

$$\text{Min } Z^{27}(x^{27}) = 6x_1^{27} + 8x_2^{27}$$

subject to the constraints :

$$\begin{cases} 20x_1^{27} + 30x_2^{27} \geq 900 \\ 40x_1^{27} + 30x_2^{27} \geq 1200 \end{cases}$$

Optimal solution: $x_1^{27} = 15$ and $x_2^{27} = 20$. *Slack variables values:* $x_3^{27} = 0$ and $x_4^{27} = 0$.

Important decision : For $x_4^{27} = 0$ we have $40w(\bar{x}_1^{27}) + 30w(\bar{x}_2^{27}) = w(\bar{b}_2^{27}) = 9$. We get $w(\bar{x}_1^{27}) = \frac{9}{80}$ and $w(\bar{x}_2^{27}) = \frac{9}{60}$ with $|x_1^{27} - x_1^{36}| + \frac{7}{80} = \frac{7}{80} \leq \frac{9}{80}$ and $|x_2^{27} - x_2^{36}| + \frac{7}{60} = \frac{7}{60} \leq \frac{9}{60}$. Therefore, we get $\bar{x}_1^{27} = [\frac{1191}{80}, \frac{1209}{80}]$ and $\bar{x}_2^{27} = [\frac{1191}{60}, \frac{1209}{60}]$.

Step 4. Solving (2.6) via (2.10). We have $p = 1$, $q = 8$. We get

$$\text{Min } Z^{18}(x^{18}) = 6x_1^{18} + 8x_2^{18}$$

subject to the constraints :

$$\begin{cases} 20x_1^{18} + 30x_2^{18} \geq 900 \\ 40x_1^{18} + 30x_2^{18} \geq 1200 \end{cases}$$

Optimal solution: $x_1^{18} = 15$ and $x_2^{18} = 20$. *Slack variables values:* $x_3^{18} = 0$ and $x_4^{18} = 0$.

Important decision : For $x_4^{18} = 0$ we have $40w(\bar{x}_1^{18}) + 30w(\bar{x}_2^{18}) = w(\bar{b}_2^{18}) = 10$. We get $w(\bar{x}_1^{18}) = \frac{10}{80}$ and $w(\bar{x}_2^{18}) = \frac{10}{60}$ with $|x_1^{18} - x_1^{27}| + \frac{9}{80} = \frac{9}{80} \leq \frac{10}{80}$ and $|x_2^{18} - x_2^{27}| + \frac{9}{60} = \frac{9}{60} \leq \frac{10}{60}$. Therefore, we get $\bar{x}_1^{18} = [\frac{1190}{80}, \frac{1210}{80}]$ and $\bar{x}_2^{18} = [\frac{1190}{60}, \frac{1210}{60}]$.

Step 5. The optimal solution according to the choice of the decision maker with minimum uncertainty is $Min\tilde{Z}(\tilde{x}) \approx (\frac{5950}{24}, \frac{5955}{24}, \frac{5965}{24}, \frac{5975}{24}, \frac{6025}{24}, \frac{6035}{24}, \frac{6045}{24}, \frac{6050}{24})$ with

$$\tilde{x}_1 = (\frac{1190}{80}, \frac{1191}{80}, \frac{1193}{80}, \frac{239}{16}, \frac{241}{16}, \frac{1207}{80}, \frac{1209}{80}, \frac{1210}{80})$$

and

$$\tilde{x}_2 = (\frac{1190}{60}, \frac{1191}{60}, \frac{1193}{60}, \frac{239}{12}, \frac{241}{12}, \frac{1207}{60}, \frac{1209}{60}, \frac{1210}{60}).$$

Then the corresponding dual problem is given by:

$$Max\tilde{w}(y) \approx (\frac{7385}{30}, \frac{7393}{30}, \frac{7409}{30}, \frac{7425}{30}, \frac{7575}{30}, \frac{7591}{30}, \frac{7607}{30}, \frac{7615}{30})$$

with $y_1 = \frac{7}{30}$ and $y_2 = \frac{1}{30}$.

We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to [37], the centers of all constraints are saturated and the second constraint is saturated.

5. ADVANTAGES OF THE PROPOSED METHOD OVER THE EXISTING METHODS

To be more specific, we will concentrate on showing the advantages of the proposed method over the well-known existing methods existing methods proposed by [6, 7, 24, 26, 27, 29, 32–34].

The advantages of the new method proposed over the existing methods proposed by [6, 7, 24, 26, 27, 29, 32–34] can be summarized as follows:

- (i) The new method improves the existing methods for solving the linear programming problems with variables given as interval numbers and fuzzy numbers with minimum uncertainty.
- (ii) The new method improves the existing methods for solving the interval Transportation Problems and Fully Fuzzy Transportation Problems with minimum uncertainty [18, 38, 39].

- (iii) In contrast to most existing approaches, our method of transforming a fuzzy number into interval numbers is the first. So, our method proposed is the first.
- (iv) The proposed technique does not use the goal and parametric approaches which are difficult to apply in real life situations. These difficulties (or limitations) are overcome by the new proposed method.
- (v) To solve the (2.12) by using the existing method, there is need to use arithmetic operations of generalized fuzzy numbers. While, if the proposed technique is used for the same then there is need to use arithmetic operations of real numbers. This proves that it is much easy to apply the proposed method as compared to the existing method.
- (vi) In contrast to most existing approaches, which provide an optimal solution using ranking function, the proposed method provides a fuzzy optimal solution without using ranking function. Similarly, to the competing methods in the literature, the proposed method is applicable for all types of fuzzy numbers.
- (vii) Also, the fuzzy optimal solution, obtained by using the new method mentioned, will always exactly satisfy the centers of all the constraints and some constraints with minimum uncertainty.

6. CONCLUDING REMARKS AND FUTURE RESEARCH DIRECTIONS

6.1. Concluding remarks. The present paper aims to propose an alternative solution approach in obtaining the fuzzy optimal solution to a fuzzy linear programming problem with variables given as fuzzy numbers with minimum uncertainty. In this paper, the fuzzy linear programming problems with variables given as fuzzy numbers is transformed into equivalent interval linear programming problems with variables given as interval numbers. The solutions to these interval linear programming problems with variables given as interval numbers are then obtained with the help of linear programming technique. The comparisons numerical examples show that in all problems the proposed method provides a better solution than the existing methods [6, 7, 24, 26, 27, 29, 32–34]. So, the proposed approach can be considered as an alternative approach for solving the fuzzy linear programming problems with variables given as fuzzy numbers if decision maker is interested in finding the fuzzy optimal solution with minimum uncertainty.

6.2. Future research directions. Finally, we feel that, there are many other points of research and should be studied later on interval numbers or fuzzy numbers. Some of these points are below:

- Linear programming problem with generalized interval-valued fuzzy numbers,
- Interval-valued intuitionistic fuzzy linear programming problem,
- Fully fuzzy linear fractional programming problems with fuzzy numbers and intuitionistic fuzzy,

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