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2D PROBLEM FOR A SPHERE IN THE FRACTIONAL ORDER THEORY THERMOELASTICITY TO AXISYMMETRIC TEMPERATURE DISTRIBUTION

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ABSTRACT. In the present article, we implement the fractional thermoelasticity theory to a 2D issue for a sphere whose surface is free from traction, subject to a provided axisymmetric temperature distribution of heat. The medium is supposed to be quiescent initially. A direct method is used to get a solution and the Laplace transform technique is used. Mathematical models for copper material are designed as a particular instance. Numerical results are computed with help of Mathcad software and graphically represented and the fractional-order parameter effect has been explained.

Nomenclature:

t	Time
T	Absolute temperature
ρ	Density
λ, μ	Lamé's constants
e	Cubical dilation = div **u**
γ	$= (3\lambda + 2\mu)\alpha_t$
σ_{ij}	Stress tensor components

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$\mathbf{u} = (u, \vartheta, 0)$	Displacement vector
k	Thermal conductivity
k^*	Material constant
c	Speed of propagation of isothermal elastic waves
T_0	Reference temperature $\mid (T - T_0)/T_0 \mid \ll 1$
α	Constant such that$0 \leq \alpha \leq 1$
α_t	Coefficient of linear thermal expansion
c_E	Specific heat per unit mass in the absence of deformation

1. INTRODUCTION

The generalization of the classical coupled theory of thermoelasticity was presented by Biot [1]. Shulman and Lord [2] presented for the isotropic body the generalized dynamic thermoelasticity theory with one period of relaxation. Naghdi and Green [3] introduced the thermoelastic material behavior without energy dissipation with nonlinear and linear theories. Zong [4] resolved quasi-static thermoelastic issues with time-dependent boundary conditions for multi-layered spheres. Povstenko [5, 9, 10] solved some thermoelastic problems based on the equation of heat conduction in 1D as well as 2D with a time-fractional derivative and associated thermal stresses. In four distinct thermoelasticity theories, Roushan Kumar and Mukhopadhyay [6] explored general thermoelastic interactions in unbounded elastic media and spherical cavities. Avijit and Kanoria [8] presented the wide spread thermoelasticity theories of a hollow-sphere with a thermal shock problem. In the fractional calculus technique, Sherief et. al. [11] introduced the novel coupling thermoelasticity and widespread thermoelasticity with one relaxation cycle. Magdy [12] developed the novel Magneto thermoelasticity model for a different consideration of the fractional derivative heat conduction. Sherief et. al. [13] introduced a 1D thermal shock issue with a theory of fractional-order for a half-space using the Laplace methods and prediction theory compared with coupled as well as generalized thermoelasticity theories. In terms of fractional order thermoelasticity, Eman [14] solved the thermoelastic issue directly of an infinitely long circular cylinder. Raslan et al. [15] addressed the 1D issue utilizing the Laplace transform technique of the thermoelasticity fractional order of an infinitely long cylindrical cavity. Bayatet. al. [16] analyzed the unsteady state thermomechanical problem of the FGM thick sphere. Raslan [17] resolved a 2D problem of axisymmetric temperature distribution fractional thermoelasticity order theory of a thick plate. Tripathi [18] showed the impact of axisymmetric supply of heat on the diffusion phenomena of an infinite and finite thick thermoelastic platform, and the theory of widespread thermoelastic diffusion with a one-time interval of relaxation. Magdy [19] developed a 3D thermoelasticity model with time-dependent thermal shock issue, utilizing a fractional thermoelasticity order theory, for a half-space. Many thermoelastic issues have recently been addressed [20–40]

This study aims for estimating the temperature distribution, displacement, as well as stress for a sphere, where the surface is free of traction and exposed to a specified axisymmetric heat temperature distribution. The medium is supposed to be quiescent initially. A direct method is used to get a solution and the Laplace transform technique is used. Mathematical models for copper material are designed as a particular instance. Numerical results are computed with help of Mathcad software and represented graphically as well as the fractional order parameter effect has been explained.

2. Formulation of the problem

We take anisotropic, homogeneous, thermoelastic solid sphere of radius a and are supposed to be quiescent initially. The spherical polar coordinate (r, ϑ, ϕ) is introduced with the cavity center as the origin. The sphere surface is free from traction while subject to axisymmetric heat distribution. Due to the ϑ axis rotational symmetry, all independent quantities of the coordinate ϕ. Therefore, the vector displacement has form $\mathbf{u} = (u, \vartheta, 0)$.

The motion equation [3] may be represented as

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \text{ grad div } \mathbf{u} - \gamma \text{grad} T = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}. \tag{1}$$

The form of time-fractional heat conduction is given below:

$$k^* \nabla^2 T = \frac{\partial^{1+\alpha}}{\partial t^{1+\alpha}} (\rho\, c_E T + \gamma\, T_0\, e), \tag{2}$$

where

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \vartheta} \frac{\partial}{\partial \vartheta} \left(\sin \vartheta \frac{\partial}{\partial \vartheta} \right). \tag{3}$$

We obtain upon taking the divergence of both sides of equation (1)

$$(\lambda + 2\mu)\nabla^2 e - \gamma\ \nabla^2 T = \rho \frac{\partial^2 e}{\partial t^2} \tag{4}$$

$$e = \frac{\partial u}{\partial r} + \frac{2u}{r} + \frac{1}{r \sin\vartheta} \frac{\partial(\nu\ \sin\vartheta)}{\partial\vartheta}. \tag{5}$$

The stress tensor components are given by [3]

$$\sigma_{rr} = 2\mu \frac{\partial u}{\partial r} + \lambda e - \gamma\ (T - T_0) \tag{6}$$

$$\sigma_{r\vartheta} = \mu \left(\frac{1}{r} \frac{\partial u}{\partial \vartheta} - \frac{\nu}{r} + \frac{\partial \nu}{\partial r} \right). \tag{7}$$

The following are the non-dimensional variables which are expressed as:

$$r' = c\eta r, \qquad u' = c\eta u, \qquad \nu' = c\eta\nu, \qquad t' = c^2\eta t,$$
$$\theta' = \frac{\gamma\ (T - T_0)}{\lambda + 2\mu}, \qquad \sigma'_{ij} = \frac{\sigma_{ij}}{\mu}.$$

where

$$c = \sqrt{\frac{\lambda + 2\mu}{\rho}},\ \eta = \frac{\rho c_E}{k}.$$

The stress components and governing equations adopt the form by putting non-dimensional quantities in (1)–(7)equations: (drop the primes for simplicity).

$$\nabla^2\ \mathbf{u} + (\beta^2 - 1)\ \text{grad}\ e - \beta^2 \text{grad}\ \theta = \beta^2 \frac{\partial^2\ \mathbf{u}}{\partial t^2} \tag{8}$$

$$\nabla^2 e\ -\ \nabla^2 \theta = \frac{1}{c^2}\ \frac{\partial^2 e}{\partial t^2} \tag{9}$$

$$c_T^2\ \nabla^2 \theta = \frac{\partial^{1+\alpha}}{\partial t^{1+\alpha}} (\ \theta + \varepsilon e\ ) \tag{10}$$

$$\sigma_{rr} = 2\frac{\partial u}{\partial r} + (\beta^2\ -\ 2)e\ -\ \beta^2 \theta \tag{11}$$

$$\sigma_{r\vartheta} = \frac{1}{r} \frac{\partial u}{\partial \vartheta}\ -\ \frac{\nu}{r}\ +\ \frac{\partial \nu}{\partial r} \tag{12}$$

where

$$\beta^2 = \frac{\lambda + 2\mu}{\mu},\ \varepsilon = \frac{T_0 \gamma^2}{[(\lambda + 2\mu)\ \rho c_E]},\ c_T^2 = \frac{k^*}{c^2 \rho c_E}.$$

The non-dimensional boundary conditions may be expressed as:

$$\theta(a,\vartheta,t)=f(\vartheta,t) \tag{13}$$

$$\sigma_{rr}(a,\vartheta,t)=0,\sigma_{r\vartheta}(a,\vartheta,t)=0. \tag{14}$$

It is supposed to be quiescent initially.

3. Laplace transform domain solution

Using Laplace transform characterized by following relationship which is applied,

$$\bar{f}(r,\vartheta,s)=L[f(r,\vartheta,t)]=\int_0^\infty e^{-st}f(r,\vartheta,t)dt \tag{15}$$

to equations (8–12), we obtain

$$\nabla^2\bar{u}+(\beta^2\ -\ 1)\ \text{grad}\ \bar{e}\ -\ \beta^2\ \text{grad}\ \bar{\theta}=\beta^2 s^2\bar{u} \tag{16}$$

$$(c_T^2\nabla^2\ -\ s^{\alpha+1})\bar{\theta}\ =\ \varepsilon\ s^{\alpha+1}\ \bar{e} \tag{17}$$

$$(\nabla^2\ -\ s^2)\ \bar{e}=\nabla^2\bar{\theta} \tag{18}$$

$$\bar{\sigma}_{rr}=2\frac{\partial\bar{u}}{\partial r}+(\beta^2\ -\ 2)\bar{e}\ -\ \beta^2\bar{\theta} \tag{19}$$

$$\bar{\sigma}_{r\vartheta}=\frac{1}{r}\frac{\partial\bar{u}}{\partial\vartheta}-\frac{\bar{\nu}}{r}+\frac{\partial\bar{\nu}}{\partial r}. \tag{20}$$

Eliminating $\bar{e}$ between equations (19) and (20), we get

$$\{\ c_T^2\nabla^4-(s^2c_T^2+(1+\varepsilon)s^{\alpha+1})\nabla^2+s^3s^\alpha\}\bar{\theta}=0. \tag{21}$$

After factorization of equation (21) becomes

$$(\nabla^2-k_1^2)(\nabla^2-k_2^2)\bar{\theta}=0, \tag{22}$$

where k_1^2, k_2^2 are the characteristic equation roots having positive real parts,

$$c_T^2k^4\ -\ (s^2c_T^2+(1+\varepsilon)s^{\alpha+1})k^2+s^3s^\alpha=0. \tag{23}$$

Equation solution (22) may be expressed as follows:

$$\bar{\theta}\ =\ \bar{\theta}_1+\bar{\theta}_2, \tag{24}$$

where the homogeneous equation $(\nabla^2 \ - \ k_i^2)\ \bar{\theta}_i = 0$, solution is denoted by $\bar{\theta}_i$ is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\bar{\theta}_i}{\partial r}\right)+\frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\bar{\theta}_i}{\partial\vartheta}\right)-k_i^2\bar{\theta}_i=0. \tag{25}$$

The equation solution (25) may be expressed as follows:

$$\bar{\theta}_i=\frac{1}{\sqrt{r}}\sum_{i=1}^{2}\sum_{n=0}^{\infty}P_n(\xi)A_{ni}(k_i^2+s^2)I_{n+1/2}(k_ir). \tag{26}$$

Here $\xi \ = \ \cos\vartheta$, $A_{ni}(s)$, i=1, 2 are some boundary conditions parameters, and $P_n(\xi)$, $I_n(k_ir)$ is the Legendre polynomial of n order as well as the first-order n updated Bessel function.

Similarly, the solution for $\bar{e}$ is compatible with equations (17) and (18) it can be written as

$$\bar{e}=\frac{1}{\sqrt{r}}\sum_{i=1}^{2}\sum_{n=0}^{\infty}P_n(\xi)A_{ni}k_i^2I_{n+1/2}(k_ir). \tag{27}$$

Putting the values of Eq. (5) into Eq. (9), we get by using the Laplace transform

$$\nabla^2\bar{u}+\frac{2}{r}\frac{\partial\bar{u}}{\partial r}+\frac{2\bar{u}}{r^2}+\beta^2s^2\bar{u}=\beta^2\frac{\partial}{\partial r}(\bar{\theta}+\bar{e})+\frac{\partial\bar{e}}{\partial r}+\frac{2\bar{e}}{r}. \tag{28}$$

The equation solution (28) may be expressed as follows:

$$\begin{aligned}\bar{u}&=\frac{1}{r\sqrt{r}}\\&\left\{\sum_{n=0}^{\infty}P_n(\xi)\sum_{i=1}^{2}[k_irI_{n+3/2}(k_ir)-nI_{n+1/2}(k_ir)]+\sum_{n=0}^{\infty}c_nP_n(\xi)I_{n+1/2}(\beta\ s\ r)\right\}.\end{aligned} \tag{29}$$

Here c_n is a boundary condition determination parameter.

Similarly, from equations (5), (27) and (29), we obtain

$$\begin{aligned}\bar{\nu}&=\frac{1}{r\sqrt{r}}\sum_{n=1}^{\infty}Q_n(\vartheta)\\&\left\{\sum_{i=1}^{2}nA_{ni}I_{n+1/2}(k_ir)+c_n\left(\frac{\beta sr}{n+1}I_{n+3/2}(\beta\ s\ r)-I_{n+1/2}(\beta\ s\ r)\right)\right\},\end{aligned} \tag{30}$$

where

$$Q_n(\vartheta)=\frac{P_{n-1}(\xi)-\xi P_n(\xi)}{\sin\vartheta}.$$

Putting from (29) and (27) equations into (11) and (12) equations, we get

$$
\begin{aligned}
\overline{\sigma}_{rr} &= \frac{1}{r^2\sqrt{r}} \\
&\cdot \sum_{n=0}^{\infty} P_n(\xi) \sum_{i=1}^{2} A_{ni}[(\beta^2 s^2 r^2 - 2n(n-1))I_{n+1/2}(k_i r) - 4k_i r I_{n+3/2}(k_i r)] \\
&+ \frac{2}{r^2\sqrt{r}} \sum_{n=1}^{\infty} P_n(\xi) c_n((n-1)I_{n+1/2}(\beta\ s\ r) - (\beta\ s\ r)I_{n+3/2}(\beta\ s\ r))
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
\overline{\sigma}_{r\vartheta} &= -\frac{2}{r^{5/2}} \sum_{n=1}^{\infty} nQ_n(\vartheta) \sum_{i=1}^{2} A_{ni}[(1-n)I_{n+1/2}(k_i r) + k_i r I_{n+3/2}(k_i r)] \\
&- \frac{1}{r^{5/2}} \sum_{n=1}^{\infty} \frac{Q_n(\vartheta)}{n+1} c_n((-\beta^2 s^2 r^2 + 2n^2 - 2)I_{n+1/2}(\beta\ s\ r) + 2(\beta s r)I_{n+3/2}(\beta\ s\ r)).
\end{aligned} \tag{32}
$$

Apply the boundary conditions (13) and (14), we get

$$
\frac{1}{\sqrt{a}} \sum_{n=0}^{\infty} P_n(\xi) \sum_{i=1}^{2} A_{ni}(k_i^2 - s^2)I_{n+1/2}(k_i a) = \bar{f}(\vartheta, s) \tag{33}
$$

$$
\begin{aligned}
&\sum_{n=0}^{\infty} P_n(\xi) \sum_{i=i}^{2} A_{ni}[(\beta^2 s^2 a^2 - 2n(n-1))I_{n+1/2}(k_i a) - 4k_i a I_{n+3/2}(k_i a)] \\
&\quad + 2\sum_{n=1}^{\infty} P_n(\xi) c_n((n-1)I_{n+1/2}(\beta\ s\ a) - (\beta\ s\ a)I_{n+3/2}(\beta\ s\ a)) = 0
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
&2\sum_{n=1}^{\infty} nQ_n(\vartheta) \sum_{i=1}^{2} A_{ni}[(1-n)I_{n+1/2}(k_i a) + k_i a I_{n+3/2}(k_i a)] \\
&\quad + \sum_{n=1}^{\infty} Q_n(\vartheta) c_n((-\beta^2 s^2 a^2 + 2n^2 - 2)I_{n+1/2}(\beta\ s\ a) \\
&\quad + 2(\beta\ s\ a)I_{n+3/2}(\beta\ s\ a)) = 0.
\end{aligned} \tag{35}
$$

Equations (33–35) is a linear equations system with $A_{ni}(s), c_n$ as an unidentified parameter. When we solve such equations, we have a perfect solution to the transform domain issue.

4. NUMERICAL LAPLACE TRANSFORMS INVERSION

Laplace transformation of the continuous $f(t)$ function is presented

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t) dt \tag{36}$$

for $t > 0$ and $s = x + iy$.

The inversion integral is utilized to identify the actual function $f(t)$ when the solution is provided in the Laplace domain,

$$f(t) = \int_{\gamma - i\infty}^{\gamma + i\infty} e^{-st} \bar{f}(s) ds. \tag{37}$$

Here, contour should be placed to the right of all $\bar{f}(s)$ singularities. The direct Equation (37) integration is usually challenging and sometimes not feasible analytically. We use a numerical inverse approach based on the Stehfest for ultimate solution of the stress distribution, displacement temperature in the time domain [41].In the given approach, the inverse $f(t)$ of Laplace $\bar{f}(s)$ is estimated by the relationship.

$$f(t) = \frac{\ln 2}{t} \sum_{j=1}^{N} V_j \, F\left(\frac{\ln 2}{t} j\right), \tag{38}$$

where the following equation is presented V_j:

$$V_j = (-1)^{((N/2)+1)} \sum_{k=(i+1)/2}^{\min(i,N/2)} \frac{k^{((N/2)+1)} (2k)!}{(N/2 - k)! \; k! \; (i-k)! \; (2k-1)!}. \tag{39}$$

The N parameter is the summation number (39) of terms and must be maximized by trial and error. Rising N improves the result accuracy to a point and subsequently decreases accuracy due to increased round-off errors. All parameters' solutions in the space time domain are therefore provided with

$$\theta(r, \vartheta, t) = \frac{\ln 2}{t} \sum_{j=1}^{N} V_j \, \bar{\theta}\left(r, \vartheta, \frac{\ln 2}{t} j\right) \tag{40}$$

$$u(r, \vartheta, t) = \frac{\ln 2}{t} \sum_{j=1}^{N} V_j \, \bar{u}\left(r, \vartheta, \frac{\ln 2}{t} j\right) \tag{41}$$

$$\sigma_{rr}(r,\vartheta,t) = \frac{\ln 2}{t}\sum_{j=1}^{N} V_j\, \bar{\sigma}_{rr}\left(r,\vartheta,\frac{\ln 2}{t}j\right) \tag{42}$$

$$\sigma_{r\vartheta}(r,\vartheta,t) = \frac{\ln 2}{t}\sum_{j=1}^{N} V_j\, \bar{\sigma}_{r\vartheta}\left(r,\vartheta,\frac{\ln 2}{t}j\right) \tag{43}$$

5. Numerical Results and Discussion

We consider $f(\vartheta,t) = \cos^2\vartheta H(t)$ in numerical computations.

By applying Laplace transform of the above mentioned functions, we deduce:

$$\bar{f}(\vartheta,s) = \frac{1}{s}\left[\frac{1}{3}P_0(\cos\vartheta) + \frac{2}{3}P_2(\cos\vartheta)\right].$$

The copper material has been selected for numerical assessment, and the problem constants are determined as follows The numerical calculation and the graphs

Table 1. Material constants

k = 386 W/(m. K)	ρ = 8954 kg/m^3	η = 8886.73
λ = 7.76 $\cdot 10^{10}$ kg/(m. s^2)	α_t = 1.78 $\cdot$ 10^{-5}K^{-1}	T_0 = 293 K
c_E = 383.1 J/(kg $\cdot$ K)	μ = 3.86 $\cdot$ 10^{10} kg/(m$\cdot$ s^2)	ε = 0.0168
c_T=7.0 m/s	β = 2, $\vartheta = \pi/N$	a = 1.5

are done using the PTC Mathcad Prime-3.1 computational mathematical software.

We analyzed the axisymmetric 2D thermoelastic problem of a sphere without energy dissipation in terms of fractional-order thermoelasticity theory. As an example, we performed numeric calculations on a copper material sphere and analyzed the thermoelastic behavior in the condition for radial temperature, thermal stress as well as the displacement and at varying t time = 0.1, 0.3, 0.5, 0.7, and fractional order parameter α = 0, 0.5, 0.75, 1.

Figures 1–4 show the space temperature variation at intervals $\alpha = 0.25$ for varied t values. Figure 1 demonstrates radial direction variance in temperature with varying time parameters. This is apparent that the original temperature at the center is zero while increases as r rises for different t values. Figure 2 reveals the displacement variation in radial direction, it is noted that the value of the displacement decreases within region $0 \le r \le 0.6$ where as rises in the $0.6 \le r \le 1.5$ region for distinct t values. Figure 3 reveals the radial stress distribution in a radial direction with different time parameters. Initially, radial stress rises to a maximum close to $r \approx 0.62$ and then to zero at $r = 1.5$.

It is obvious that radial tension produces radial tensile strains for different t values. The change in the axial stress distribution in the radial direction is exhibited in Figure 4, it is apparent that the axial stress generates radial compressive stresses at varying t values.

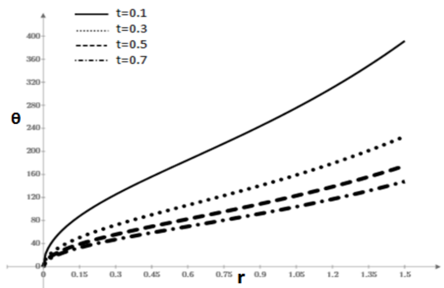

Figure 1. Temperature distribution at $\alpha = 0.25$ and different values of t.

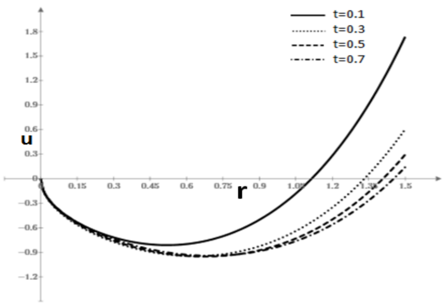

Figure 2. Displacement distribution at $\alpha = 0.25$ and different values of t.

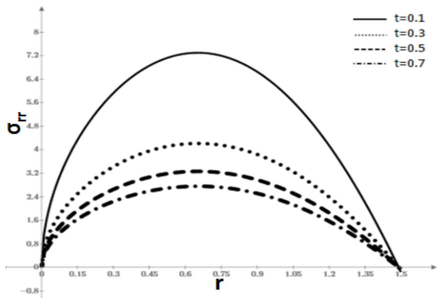

Figure 3. Radial stress distribution at $\alpha = 0.25$ and different values of t.

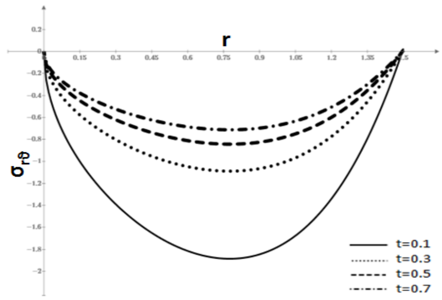

Figure 4. Axial stress distribution at $\alpha = 0.25$ and different values of t.

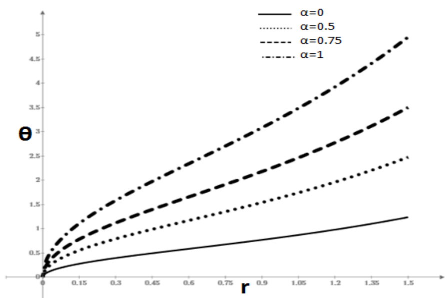

Figure 5. Temperature Distribution at $t = 0.5$ and different values of α.

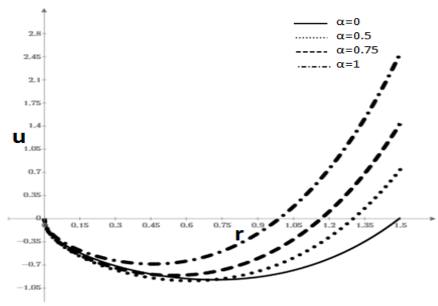

Figure 6. Displacement distribution at $t = 0.5$ and different values of α.

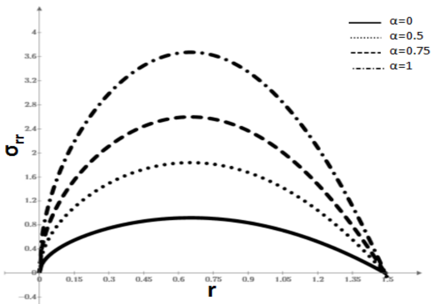

Figure 7. Radial stress distribution at $t = 0.5$ and different values of α.

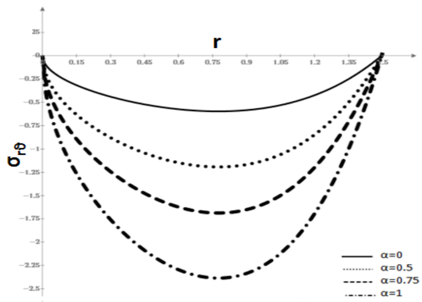

Figure 8. Axial stress distribution at $t = 0.5$ and different values of α.

Figures 5-8 show the temperature space variation at instants $t = 0.5$ for varying α values. Figure 5 illustrates temperature variations in varying the fractional-order parameter α values for $t = 0.5$ along with radial directions. The illustration indicates that the fractional-order parameter value rises, the magnitude of the rising temperatures as the radial thickness increases and the center becomes zero. Figure 6 illustrates the variance in a radial direction of displacement. The initial displacement to the outer circular edge $r = 1.5$ is zero and maximal. In Figures 7 and 8, axial and radial stress is shown radial direction, radial stress is obvious in the tensile nature and axial stress is compressive in nature for fractional order parameters.

6. Conclusion

A 2D axisymmetric thermoelastic issue of a solid sphere has been explored in this study in terms of the fractional thermoelasticity theory. The sphere surface is deemed to be free from traction and subject to an axisymmetric temperature distribution of heat supply. The Laplace transform approach was utilized to achieve the solution for thermal stress analytically, displacement, and temperature. The technique employed in this research offers an effective technique to the solution of thermoelastic issues. The numerical results are compared with varying time $t =$ =0.1, 0.3, 0.5, 0.7 and fractional order parameter $\alpha =$ 0, 0.5, 0.75, 1. The evaluation of the findings allows for some final remarks.

(1) In Figures 1 and 2, the body displacement of temperature and heat flow direction and displacement is directly proportional to each other.

(2) Figure 3 shows radial tensile stress, whereas Figure 4 exhibits the axial stress that increases with time to produce radial compressive stresses.
(3) Physical quantities are altered by the fractional-order parameter. The existence of parameter of fractional-order of the present model is thus important.
(4) The temperature, thermal stresses as well as displacement rely heavily on the parameter of fractional order.
(5) The wave velocity varies for the varying the fractional-order parameter values.
(6) The material thermal conductivity is exactly proportionate to the parameter of the fractional order.
(7) When $\alpha = 1$ and $1 + \alpha = 2$, we have the wave equations for temperature. In this case, we have the finite velocity of propagation of the disturbance.
(8) The technique described in this paper is applicable to a broad variety of thermoelasticity physical engineering issues.

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