ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **11** (2022), no.1, 25–34
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.11.1.3

COMMON FIXED POINT THEOREM FOR TWO MAPPINGS IN bi-b-METRIC SPACE

Varsha D. Borgaonkar[1], K.L. Bondar, and S.M. Jogdand

ABSTRACT. In this paper we have used the concept of bi-metric space and intoduced the concept of bi-b-metric space. our objective is to obtain the common fixed point theorems for two mappings on two different b-metric spaces induced on same set X. In this paper we prove that on the set X two b-metrics are defined to form two different b-metric spaces and the two mappings defined on X have unique common fixed point.

1. INTRODUCTION.

The Banach Contraction Mapping Principle is very useful theorem. Hence it is very popular tool in solving existence problems in many branches of mathematical analysis. Banach fixed point theorem has many applications inside and outside Mathematics. In 1989, an interesting concept of generalized b–metric spaces was introduced by Bakhtin [2]. In 1993 Czerwik [7] extended the results of b-metric spaces. Many researchers generalized the Banach fixed point theorem in b-metric space. Czerwik [8] 1998 presented the generalization of Banach fixed point theorem in b-metric spaces. The existence and uniqueness theorems in b-Metric Space

[1]*corresponding author*
2020 *Mathematics Subject Classification.* 47H10, 58C30.
Key words and phrases. Fixed Point of a mapping, b-Metric Space, Convergence in b-Metric Space, Cauchy Sequence, bi-metric space.
Submitted: 09.12.2021; *Accepted:* 27.12.2021; *Published:* 13.01.2022.

was presented by Agrawal [1] .in 1968 Maia generalized the result of well known Banach Contraction Principle by taking two metrics on a set X. Mishra [13] generalized the Maia's fixed point theorem in bi-metric spaces. Soni [12] gave the fixed point theorem for mappings in bi-metric spces.

We want to extend some fixed point theorems in bimetric spaces which are also valid in bi- b - metric spaces. Chopade [6] obtained common fixed point theorems for contractive type mappings in metric space. Roshan [10] gave the common fixed point of four maps in b–Metric space. Suzuki [11] obtained some basic inequalities on a b-Metric space and it's applications.

2. Some Basic Definitions and Preliminaries

Definition 2.1. *Let X be a non-empty set .A function $\delta : X \times X \longrightarrow R$ is called as a metric provided that for all $u, v, w \in X$,*

$i.$ $\delta(u, v) \geq 0$,
$ii.$ $\delta(u, v) = 0$ *if and only if* $u = v$,
$iii.$ $\delta(u, v) = \delta(v, u)$,
$iv.$ $\delta(u, v) = \delta(u, w) + \delta(w, v)$.

A pair (X, δ) is called a metric space.

Definition 2.2. *Let X be a non-empty set and $s \geq 1$ be a given real number. A function $\delta : X \times X \longrightarrow R$ is called as a b–metric provided that for all $u, v, w \in X$,*

$i.$ $\delta(u, v) \geq 0$,
$ii.$ $\delta(u, v) = 0$ *if and only if* $u = v$,
$iii.$ $\delta(u, v) = \delta(v, u)$,
$iv.$ $\delta(u, v) \leq s\{\delta(u, w) + \delta(w, v)\}$.

A pair (X, δ) is called a b–metric space. It is clear that definition of b–metric space is an extension of usual metric space.

Remark 2.1. *If $s = 1$, then the b–metric space is a usual metric space.*

Example 1. *Let (X, d) be a metric space and $\delta(u, v) = (d(u, v))^p$, where $p > 1$ is a real number. Clearly, $\delta(u, v)$ is b–metric with $s = 2^{p-1}$.*

Example 2. ***Example 2.2:*** *If $X = R$, be the set of real numbers and $d(u,v) = |u-v|$ a usual metric, then $\delta(u,v) = (u-v)^2$ is a b–metric on R with $s = 2$, but not a metric on R.*

Example 3. *By Boriceanu [2], Let $M = \{0,1,2\}$ and $\delta : M \times M \to R$ is defined as,*

$\delta(0,2) = \delta(2,0) = m \geq 2$,

$\delta(0,1) = \delta(1,0) = \delta(1,2) = \delta(2,1) = 1$,

$\delta(0,0) = \delta(1,1) = \delta(2,2) = 0$.

Here, $\delta(u,v)$ is a b–metric on M with $s = \frac{m}{2}$.

Definition 2.3. *Let (X,δ) be a b–metric space then a sequence $\{u_n\}$ in X is called as convergent sequence if there exists $u \in X$ such that for all $\epsilon > 0$ there exist $n(\epsilon) \in N$ such that $n \geq n(\epsilon)$ we have, $\delta(u_n,u) < \epsilon$. In this case we write $\lim_{n\to\infty} u_n = u$.*

Definition 2.4. *Let (X,δ) be a b–metric space then a sequence $\{u_n\}$ in X is called as Cauchy sequence if for all $\epsilon > 0$ there exist $n(\epsilon) \in N$ such that $m,n \geq n(\epsilon)$ we have, $\delta(u_n,u_m) < \epsilon$.*

Definition 2.5. *Let (X,δ) be a b–metric space then X is said to be complete if every Cauchy sequence in X is convergent sequence in X.*

3. Main Result

We use the following Lemma to prove the main result.

Lemma 3.1. *[11] Let (X,δ) be a complete b–metric space and let $\{x_n\}$ be a sequence in X. Assume that there exist $r \in [0,1)$ satisfying*

$$\delta(x_{n+1},x_{n+2}) \leq r\delta(x_n,x_{n+1}) \text{ for any } n \in N.$$

Then $\{x_n\}$ is Cauchy sequence in X.

Theorem 3.1. *Let (X,δ_1,s) and (X,δ_2,t) be a bi - b -metric space ,where, $s \geq 1$ and $t \geq 1$ such that,*

i. $\delta_1(u,v) \leq \delta_2(u,v)$ *for all* $u,v \in X$.

ii. $S : X \to X$ *and* $T : X \to X$ *be any two selfmaps on* X *satisfying,*

$$\begin{aligned} \delta_2(Su, Tv) \leq \alpha & \frac{\delta_2(u, Su).\delta_2(v, Tv)}{\delta_2(v, Tv) + \delta_2(v, Su)} \\ & + \beta \frac{\delta_2(u, v)[1 + \delta_2(u, Su) + \delta_2(v, Su)]}{1 + \delta_2(u, v) + \delta_2(u, Su).\delta_2(u, Tv).\delta_2(v, Su).\delta_2(v, Tv)} \\ & + \gamma \frac{[\delta_2(u, Su)\delta_2(u, Tv)]}{\delta_2(u, v)}, \end{aligned} \tag{3.1.1}$$

where, $\alpha, \beta, \gamma \in [0, 1)$ *are such that* $\alpha + \beta + 2\gamma t < 1$

iii. *There exist a point* $u_0 \in X$ *such that the sequence* $\{u_n\}$ *of iterates defined as* $u_1 = Su_0, u_2 = Tu_1, ..., u_{2n} = Tu_{2n-1}, u_{2n+1} = Su_{2n}$ *for any* $n \in N$ *has a convergent subsequence* u_{n_k} *converging to* u^* *in* (X, δ_1)*.*

iv. *Both the mappings* T *and* S *are continuous in* (X, δ_1)*. Then,* T *and* S *have unique common fixed point in* X*.*

Proof.

Existence: Given, $u_0 \in X$, and $\{u_n\}$ be a sequence of iterates in X defined as

$$S(u_{2n}) = u_{2n+1} \quad \text{and} \quad T(u_{2n-1}) = u_{2n}, n = 1, 2, \ldots. \tag{3.1.2}$$

Using equation (3.1.1) and (3.1.2) we obtain that,

$$\begin{aligned} & \delta_2(u_{2n+1}, u_{2n+2}) \\ & = \delta_2(Su_{2n}, Tu_{2n+1}) \\ & \leq \alpha \frac{\delta_2(u_{2n}, Su_{2n}).\delta_2(u_{2n+1}, Tu_{2n+1})}{\delta_2(u_{2n+1}, Tu_{2n+1}) + \delta_2(u_{2n+1}, Su_{2n})} \\ & + \beta \frac{\delta_2(u_{2n}, u_{2n+1})[1 + \delta_2(u_{2n}, Su_{2n}) + u_{2n+1}, Su_{2n}]}{1 + \delta_2(u_{2n}, u_{2n+1}) + \delta_2(u_{2n}, Su_{2n})\delta_2(u_{2n}, Tu_{2n+1})\delta_2(u_{2n+1}, Su_{2n})\delta_2(u_{2n+1}, Tu_{2n+1})} \\ & + \gamma \frac{\delta_2(u_{2n}, Su_{2n})\delta_2(u_{2n}, Tu_{2n+1})}{\delta_2(u_{2n}, u_{2n+1})} \\ & \leq \alpha \frac{\delta_2(u_{2n}, u_{2n+1}).\delta_2(u_{2n+1}, u_{2n+2})}{\delta_2(u_{2n+1}, u_{2n+2})} \\ & + \beta \frac{\delta_2(u_{2n}, u_{2n+1})[1 + \delta_2(u_{2n}, u_{2n+1})]}{1 + \delta_2(u_{2n}, u_{2n+1})} \\ & + \gamma \frac{\delta_2(u_{2n}, u_{2n+1})\delta_2(u_{2n}, u_{2n+2})}{\delta_2(u_{2n}, u_{2n+1})} \\ & \leq \alpha\delta_2(u_{2n}, u_{2n+1}) + \beta\delta_2(u_{2n}, u_{2n+1}) + \gamma\delta_2(u_{2n}, u_{2n+2}) \\ & \leq (\alpha + \beta)\delta_2(u_{2n}, u_{2n+1}) + \gamma t\delta_2(u_{2n}, u_{2n+1}) + \gamma t\delta_2(u_{2n+1}, u_{2n+2}) \end{aligned}$$

$$(1-\gamma t)\delta_2(u_{2n+1},u_{2n+2}) \leq (\alpha+\beta+\gamma t)\delta_2(u_{2n},u_{2n+1})$$
$$\delta_2(u_{2n+1},u_{2n+2}) \leq \frac{(\alpha+\beta+\gamma t)}{(1-\gamma t)}\delta_2(u_{2n},u_{2n+1})$$
$$\delta_2(u_{2n+1},u_{2n+2}) \leq r.\delta_2(u_{2n},u_{2n+1}),$$

where $r = \frac{(\alpha+\beta+\gamma t)}{(1-\gamma t)} < 1$. In general, for all $n \in N$,

(3.1.3) $$\delta(u_{n+1},u_{n+2}) \leq r\delta(u_n,u_{n+1}),$$

where $r = \frac{(\alpha+\beta+\gamma t)}{(1-\gamma t)} < 1$.

Therefore by Lemma 3.1 the sequence $\{u_n\}$ is Cauchy Sequence in X. Since the cauchy sequence $\{u_n\}$ defined by (3.1.2) has convergent subsequence $\{u_{n_k}\}$ in (X,δ_1) converging to u^* in (X,δ_1) , the sequence $\{u_n\}$ also converges to u^* in (X,δ_1). Hence,

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} u_{2n} = \lim_{n\to\infty} u_{2n-1} = \lim_{n\to\infty} u_{2n+1} = u^*.$$

Now we show that u^* is fixed point of both the mappings S and T. As S and T are continuous in (X,δ_1), therefore,

$$S(u^*) = S[\lim_{n\to\infty} u_{2n}] = \lim_{n\to\infty}[Su_{2n}] = u^*.$$

Similarly,

$$T(u^*) = T[\lim_{n\to\infty} u_{2n-1}] = \lim_{n\to\infty}[Tu_{2n-1}] = u^*.$$

Thus u^* is common fixed point of the mappings S and T.

Uniqueness:

Suppose, u^* and v^* be two common fixed points of the mappings T and S. Therefore, $Su^* = Tu^* = u^*$ and $Sv^* = Tv^* = v^*$. Consider:

$$\delta_2(u^*,v^*) = \delta_2(Su^*,Tv^*),$$
$$\delta_2(u^*,v^*) \leq \alpha\frac{\delta_2(u^*,Su^*).\delta_2(v^*,Tv^*)}{\delta_2(v^*,Tv^*)+\delta_2(v^*,Su^*)}$$
$$+\beta\frac{\delta_2(u^*,v^*)[1+\delta_2(u^*,Su^*)+\delta_2(v^*,Su^*)]}{1+\delta_2(u^*,v)+\delta_2(u^*,Su^*).\delta_2(u^*,Tv^*).\delta_2(v^*,Su^*).\delta_2(v^*,Tv^*)}$$
$$+\gamma\frac{[\delta_2(u^*,Su^*)\delta_2(u^*,Tv^*)]}{\delta_2(u^*,v^*)}$$

$$
\begin{aligned}
&\leq \alpha \frac{\delta_2(u^*,u^*).\delta_2(v^*,v^*)}{\delta_2(v^*,v^*)+\delta_2(v^*,u^*)} \\
&+ \beta \frac{\delta_2(u^*,v^*)[1+\delta_2(u^*,u^*)+\delta_2(v^*,u^*)]}{1+\delta_2(u^*,v^*)+\delta_2(u^*,u^*).\delta_2(u^*,v^*).\delta_2(v^*,u^*).\delta_2(v^*,v^*)} \\
&+ \gamma \frac{[\delta_2(u^*,u^*)\delta_2(u^*,v^*)]}{\delta_2(u^*,v^*)} \\
&\leq \beta \frac{\delta_2(u^*,v^*)[1+\delta_2(u^*,v^*)]}{[1+\delta_2(u^*,v^*)]}
\end{aligned}
$$

$$\delta_2(u^*,v^*) \leq \beta\delta_2(u^*,v^*).$$

But, $\beta < 1$. Therefore we have, $\delta(u^*,v^*) < \delta(u^*,v^*)$. This is contradiction, Hence, T and S have unique common fixed point in X.

This completes the proof. □

Corollary 3.1. *If Conditions (i), (ii) and (iv) of theorem (3.1) holds and moreover* (X,δ_1) *is complete b–metric space then* $T : X \to X$ *and* $S : X \to X$ *have unique common fixed point in* X.

Theorem 3.2. *Let* (X,δ_1,s) *and* (X,δ_2,t) *be a bi - b -metric space ,where,* $s \geq 1$ *and* $t \geq 1$. *Let* $P = \{T_i : i \in I$*, the set of positive integers* $\}$ *be a family of mappings on* X *such that the following conditions holds*

$i.$ $\delta_1(u,v) \leq \delta_2(u,v)$ *for all* $u,v \in X$.

$ii.$ X *is complete with respect to* δ_1.

$iii.$ *for each* $T_j : X \to X \in P$ *there exist* $T_i : X \to X \in P$ *such that*

$$
\begin{aligned}
\delta_2({T_i}^m u, {T_j}^n v) \leq\ & \alpha \frac{\delta_2(u,v).\delta_2(v,{T_j}^n v)}{\delta_2(u,{T_i}^m u)+\delta_2(v,{T_i}^m u)} \\
&+ \beta \frac{\delta_2(u,v)[1+\delta_2(u,{T_i}^m u)+\delta_2(v,{T_i}^m u)]}{1+\delta_2(u,v)} \\
&+ \gamma \frac{\delta_2(u,{T_i}^m u).\delta_2(u,{T_j}^n v)}{\delta_2(u,v)},
\end{aligned}
\tag{3.2.1}
$$

where, m,n *are positive integers and* $\alpha,\beta,\gamma \in [0,1)$ *are such that* $\alpha+\beta+2\gamma t < 1$,

$iv.$ *mapping* T_i *is continuous in* (X,δ_1) *for all* $i \in I$*, then* P *has a unique common fixed point.*

Proof.
Existence: Given, $u_0 \in X$. We define a sequence of iterates $\{u_n\}$ in X as

$$u_{2n-1} = T_i{}^m(u_{2n-2}) \qquad and \qquad u_{2n} = T_j{}^n(u_{2n-1}), n = 1, 2, \ldots. \tag{3.2.2}$$

Using equation (3.2.1) and (3.2.2) we obtain that,

$$\begin{aligned}
&\delta_2(u_{2n+1}, u_{2n+2}) \\
&= \delta_2(T_i{}^m u_{2n}, T_j{}^n u_{2n+1}) \\
&\leq \alpha \frac{\delta_2(u_{2n}, u_{2n+1}).\delta_2(u_{2n+1}, T_j{}^n u_{2n+1})}{\delta_2(u_{2n}, T_i{}^m u_{2n}) + \delta_2(u_{2n+1}, T_i{}^m u_{2n})} \\
&+ \beta \frac{\delta_2(u_{2n}, u_{2n+1})[1 + \delta_2(u_{2n}, T_i{}^m u_{2n}) + \delta_2(u_{2n+1}, T_i{}^m u_{2n})]}{1 + \delta_2(u_{2n}, u_{2n+1})} \\
&+ \gamma \frac{\delta_2(u_{2n}, T_i{}^m u_{2n}).\delta_2(u_{2n}, T_j{}^n u_{2n+1})}{\delta_2(u_{2n}, u_{2n+1})} \\
&\leq \alpha \frac{\delta_2(u_{2n}, u_{2n+1}).\delta_2(u_{2n+1}, u_{2n+2})}{\delta_2(u_{2n}, u_{2n+1}) + \delta_2(u_{2n+1}, u_{2n+1})} \\
&+ \beta \frac{\delta_2(u_{2n}, u_{2n+1})[1 + \delta_2(u_{2n}, u_{2n+1}) + \delta_2(u_{2n+1}, u_{2n+1})]}{1 + \delta_2(u_{2n}, u_{2n+1})} \\
&+ \gamma \frac{\delta_2(u_{2n}, u_{2n+1}).\delta_2(u_{2n}, u_{2n+2})}{\delta_2(u_{2n}, u_{2n+1})} \\
&\leq \alpha.\delta_2(u_{2n+1}, u_{2n+2}) + \beta \frac{\delta_2(u_{2n}, u_{2n+1})[1 + \delta_2(u_{2n}, u_{2n+1})]}{1 + \delta_2(u_{2n}, u_{2n+1})} + \gamma \delta_2(u_{2n}, u_{2n+2}) \\
&\leq \alpha \delta_2(u_{2n+1}, u_{2n+2}) + \beta \delta_2(u_{2n}, u_{2n+1}) + \gamma \delta_2(u_{2n}, u_{2n+2}) \\
&\leq \alpha \delta_2(u_{2n+1}, u_{2n+2}) + \beta \delta_2(u_{2n}, u_{2n+1}) + \gamma t \delta_2(u_{2n}, u_{2n+1}) + \gamma t \delta_2(u_{2n+1}, u_{2n+2})
\end{aligned}$$

$$\begin{aligned}
&(1 - \alpha - \gamma t)\delta_2(u_{2n+1}, u_{2n+2}) \leq (\beta + \gamma t)\delta_2(u_{2n}, u_{2n+1}) \\
&\delta_2(u_{2n+1}, u_{2n+2}) \leq \frac{(\beta + \gamma t)}{(1 - \alpha - \gamma t)} \delta_2(u_{2n}, u_{2n+1}) \\
&\delta_2(u_{2n+1}, u_{2n+2}) \leq r.\delta_2(u_{2n}, u_{2n+1}),
\end{aligned}$$

where $r = \frac{(\beta+\gamma t)}{(1-\alpha-\gamma t)} < 1$. In general, for all $n \in N$,

$$\delta(u_{n+1}, u_{n+2}) \leq r\delta(u_n, u_{n+1}), \tag{3.2.3}$$

where $r = \frac{(\beta+\gamma t)}{(1-\alpha-\gamma t)} < 1$. Therefore, by Lemma 3.1 the sequence $\{u_n\}$ is Cauchy Sequence in X. Since, the sequence $\{u_n\}$ defined by (3.2.2) is a cauchy sequence in

X and (X, δ_1) is complete, therefore, sequence u_n is convergent in (X, δ_1). Hence,

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} u_{2n} = \lim_{n\to\infty} u_{2n-1} = \lim_{n\to\infty} u_{2n+1} = u^*.$$

Now we show that u^* is fixed point of both the mappings ${T_i}^m$ and ${T_j}^n$.

As ${T_i}^m$ and ${T_j}^n$ are continuous in (X, δ_1), therefore,

$${T_i}^m(u^*) = {T_i}^m[\lim_{n\to\infty} u_{2n}] = \lim_{n\to\infty}[{T_i}^m u_{2n}] = u^*.$$

Similarly,

$${T_j}^n(u^*) = {T_j}^n[\lim_{n\to\infty} u_{2n-1}] = \lim_{n\to\infty}[{T_j}^n u_{2n-1}] = u^*.$$

Thus, u^* is common fixed point of the mappings ${T_i}^m$ and ${T_j}^n$.

Uniqueness:

Suppose, u^* and v^* be two common fixed points of the mappings ${T_i}^m$ and ${T_j}^n$. Therefore, ${T_i}^m u^* = {T_j}^n u^* = u^*$ and ${T_i}^m v^* = {T_j}^n v^* = v^*$. Consider

$$\delta_2(u^*, v^*) = \delta_2({T_i}^m u^*, {T_j}^n v^*) = \delta_2({T_i}^m u^*, {T_j}^n v^*)$$

$$\begin{aligned}
\delta_2(u^*, v^*) \le\ & \alpha \frac{\delta_2(u^*, v^*).\delta_2(v^*, {T_j}^n v^*)}{\delta_2(u^*, {T_i}^m u^*) + \delta_2(v^*, {T_i}^m u^*)} \\
& + \beta \frac{\delta_2(u^*, v^*)[1 + \delta_2(u^*, {T_i}^m u^*) + \delta_2(v^*, {T_i}^m u^*)]}{1 + \delta_2(u^*, v^*)} \\
& + \gamma \frac{\delta_2(u^*, {T_i}^m u^*).\delta_2(u^*, {T_j}^n v^*)}{\delta_2(u^* v^*)}
\end{aligned}$$

$$\begin{aligned}
\delta_2(u^*, v^*) \le\ & \alpha \frac{\delta_2(u^*, v^*).\delta_2(v^*, v^*)}{\delta_2(u^*, u^*) + \delta_2(v^*, u^*)} \\
& + \beta \frac{\delta_2(u^*, v^*)[1 + \delta_2(u^*, u^*) + \delta_2(v^*, u^*)]}{1 + \delta_2(u^*, v^*)} \\
& + \gamma \frac{[\delta_2(u^*, u^*)\delta_2(u^*, v^*)]}{\delta_2(u^*, v^*)} \\
& + \beta \frac{\delta_2(u^*, v^*)[1 + \delta_2(v^*, u^*)]}{1 + \delta_2(u^*, v^*)}
\end{aligned}$$

$$\delta_2(u^*, v^*) \le \beta \delta_2(u^*, v^*)$$

$$\delta_2(u^*, v^*) < \delta_2(u^*, v^*),$$

since, $\beta < 1$. Therefore we have, $\delta(u^*, v^*) < \delta(u^*, v^*)$. This is contradiction. Hence, u^* is unique common fixed point of ${T_i}^m$ and ${T_j}^n$.

The fixed point of ${T_i}^m$ is a fixed point of T_i and the fixed point of ${T_j}^n$ is fixed point of T_j. Therefore, u^* is unique common fixed point of T_i and T_j. Hence, u^* is unique common fixed point of P.

This completes the proof. □

4. Discussion and the concluding Remarks

In this paper, we have proved the existence and uniqueness of common fixed points for two contractive type mappings in bi-b-metri space.

References

[1] Agrawal Swati, K. Qureshi, Jyoti Nema: *A fixed Point Theorem for b-–Metric Space*, International Journal of Pure and Applied Mathematical Sciences, **9** (2016), 45–50.

[2] I.A. Bakhtin: *The Contraction Mapping Principle in almost Metric Spaces*, Funct. Anal. .30 Unisco, Gauss. Ped. Inst., (1989), 26–37.

[3] D. Borgaonkar Varsha, K.L. Bondar: *Fixed Point Theorems for a Family of Self-Map on Rings*, IJSIMR, **2** (2010), 750–756.

[4] M. Boriceanu: *Fixed Point theory for multivalued generalized contraction on a set with two b-Metric*, studia, univ Babes, Bolya: Math, Liv, **3** (2009), 1–14.

[5] V.C. Borkar, et. al.: *Common Fixed Point for nonaxpansive type mappings with application*, Acta Cinecia India, **4** (2010), 674–682.

[6] P.U. Chopade, et. al.: *Common Fixed Point Theorem for Some New Generalized Contractive mappings*, Int. Journal of Math. Analisis, **4** (2010), 1881–1890.

[7] S. Czerwik: *Contraction Mappings In b–Metric Spaces*, Acta, Mathematica ,et. Informatica Universities Ostraviensis, **1** (1993), 5–11.

[8] S. Czerwik: *Non-linear Set Valued Contraction Mappings in b–Metric Spaces*, Atti sem Maths, FIQ Univ. Modena., **46** (1998), 263–276.

[9] M. Kir, H. Kiziltune: *On Some Well known Fixed Point Theorems in b–Metric Space*, Turkshi Journal of Analysis and Number Theory, **1** (2013), 13–16.

[10] J.R. Roshan, et. al.: *Common Fixed Point of Four Maps in b–Metric Spaces*, Hacettep Journal of Mathematics and Statistics, **43** (2014), 613–624.

[11] S. Tomonari: *Basic Inequality on a b-Metric Space and It's Applications*, Journal of Inequalities and Applications, (2017), 2–11.

[12] G.K. Soni: *Fixed Point Theorem in Bimetric Space*, Research Journal of Mathematical and Statistical Sciences, **27** (2015), 13–14.

[13] S.N. MISHRA: *Remarks on Some Fixed Point Theorems in Bimetric Spaces*, Indian Journal Pure and Applied Mathematics, (1979), 1271–1274.

P.G. DEPARTMENT OF MATHEMATICS,
N.E.S. SCIENCE COLLEGE,
NANDED, M.S., INDIA.
Email address: borgaonkarvarsha@gmail.com

P.G. DEPARTMENT OF MATHEMATICS,
G. V. INSTITUTE OF SCIENCE AND HUMANITIES,
AMRAVATI, M.S., INDIA.
Email address: klbondar_75@rediffmail.com

DEPARTMENT OF MATHEMATICS,
S.S.G.M. MAHAVIDYALAYA,
LOHA, NANDED, M.S., INDIA.
Email address: smjog12@gmail.com