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SOME REMARKS ON FUZZY *P*-SPACES

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ABSTRACT. In this paper, a condition which ensures the existence of fuzzy simply open sets in a fuzzy P- space is obtained. It is established that disjoint fuzzy G_δ-sets in a fuzzy P-space are fuzzy cs dense sets. A condition for a fuzzy strongly irresolvable and fuzzy globally disconnected space to become a fuzzy P-space is obtained. It is established that fuzzy hyperconnected and fuzzy P-space is a fuzzy D-Baire space. It is obtained that fuzzy extremally disconnected, fuzzy strongly irresolvable and fuzzy P-space is a fuzzy semi-P space and fuzzy hyperconnected and fuzzy P-space does not have disjoint fuzzy G_δ-sets.

1. INTRODUCTION

Any application of mathematical concepts depends firmly and closely how one introduces basic ideas that may yield various theories in various directions. If the basic idea is suitably introduced, then not only the existing theories stand but also the possibility of emerging new theories increases. The concept of fuzzy set as a new approach for modelling uncertainties was introduced by L.A.Zadeh[31] in 1965. The concept of fuzzy topological spaces was introduced by C.L.Chang[3] in

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1968. Chang's works paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

In 1954, L.Gillman and M.Henriksen[6] defined and characterized the classes of P-spaces. A.K.Mishra[9] introduced the concepts of P-spaces as a generalization of ω_α-additive spaces of Sikorski[10] and L.W.Cohen and C.Goffman[4]. Almost P-spaces in classical topology was introduced by A.I.Veksler[30] and was also studied further by R.Levy[7]. The concept of P-spaces in fuzzy setting was introduced by G.Balasubramanian[11]. The concepts of fuzzy almost P-spaces is introduced and studied in [17].

In this paper, it is established that fuzzy σ-boundary sets are fuzzy closed sets, fuzzy co -σ-boundary sets are fuzzy open sets and fuzzy Baire sets are fuzzy somewhere dense sets in fuzzy P-spaces. The condition which ensures the existence of fuzzy simply open sets in fuzzy P-spaces is established by means of fuzzy boundary of fuzzy sets. It is obtained that disjoint fuzzy G_δ-sets in fuzzy P-spaces are fuzzy cs dense sets. It is established that fuzzy P-spaces are fuzzy basically disconnected and fuzzy almost P-spaces and fuzzy globally disconnected, fuzzy Baire and fuzzy P-spaces are fuzzy D-Baire spaces. A condition for a fuzzy strongly irresolvable and fuzzy globally disconnected space to become a fuzzy P-space is obtained. It is established that fuzzy extremally disconnected, fuzzy strongly irresolvable and fuzzy P-spaces are fuzzy semi-P spaces and fuzzy hyperconnected and fuzzy P-spaces are not having disjoint fuzzy G_δ-sets. It is obtained that fuzzy Baire sets in fuzzy hyperconnected and fuzzy P-spaces, are fuzzy semi-open and fuzzy simply open sets.

2. DEFINITION

Some basic notions and results used in the sequel, are given in order to make the exposition self - contained. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang(1968). Let X be a non-empty set and I the unit interval $[0, 1]$. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X\ (x) = 0$, for all $x \epsilon\ X$ and the fuzzy set 1_X is defined as $1_X\ (x) = 1$, for all $x \epsilon\ X$.

Definition 2.1. *[3] Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior, the closure and the complement of λ are defined respectively as follows:*

(1) $int(\lambda) = \vee \{\mu \,/\mu \leq \lambda,\ \mu \in T\}$;
(2) $cl(\lambda) = \wedge \{\mu \,/\lambda \leq \mu,\ 1-\mu \in T\}$
(3) $\lambda'(x) = 1 - \lambda(x)$, *for all* $x \in X$.

For a family $\{\lambda_i/i \in I\}$ of fuzzy sets in (X,T), the union $\psi = \vee_i\ (\lambda_i)$ and intersection $\delta = \wedge_i\ (\lambda_i)$, are defined respectively as

(4) $\psi(x) = sup_i\{\lambda_i(x)/x\ \epsilon X\}$
(5) $\delta(x) = inf_i\{\lambda_i(x)/x\ \epsilon X\}$

Lemma 2.1. *[1] For a fuzzy set λ of a fuzzy topological space X,*

(1) $1 - int(\lambda) = cl(1-\lambda)$*, and*
(2) $1 - cl(\lambda) = int(1-\lambda)$.

Definition 2.2. *A fuzzy set λ in a fuzzy topological space (X,T) is called a*

(1) *fuzzy regular-open if $\lambda = int\ cl(\lambda)$ and fuzzy regular-closed if $\lambda = cl\ int(\lambda)$ [1].*
(2) *fuzzy semi-open if $\lambda \leq cl\ int(\lambda)$ and fuzzy semi-closed if $int\ cl(\lambda) \leq \lambda$[1].*
(3) *fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i\ \epsilon\ T$ for $i\ \epsilon\ I$ [2].*
(4) *fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i\ \epsilon\ T$ for $i\ \epsilon\ I$ [2].*

Definition 2.3. *A fuzzy set λ in a fuzzy topological space (X,T) is called a*

(1) *fuzzy dense set if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X,T) [12] .*
(2) *fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\ cl(\lambda) = 0$, in (X,T) [12].*
(3) *fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category [12].*
(4) *fuzzy residual set if $1-\lambda$ is a fuzzy first category set in (X,T) [15].*
(5) *fuzzy somewhere dense set if there exists a non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\ cl(\lambda) \neq 0$, in (X,T) [13] and $1-\lambda$ is called a fuzzy complement of fuzzy somewhere dense set in (X,T) and is denoted as fuzzy cs dense set in (X,T) [23].*

(6) *fuzzy σ-boundary set if $\lambda = \vee_{i-1}^{\infty}(\mu_i)$, where $\mu_i = cl(\lambda_i) \wedge (1-\lambda_i)$ and $(\lambda_i)'$s are fuzzy regular open sets in (X,T) [20].*
(7) *fuzzy co-σ-boundary set if $1-\lambda$ is a fuzzy σ-boundary set in (X,T) [20].*
(8) *fuzzy Baire set if $\lambda = \mu \wedge \delta$, where μ is a fuzzy open set and δ is a fuzzy residual set in (X,T) [20].*

Definition 2.4. *A fuzzy topological space (X,T) is called a*

(1) *fuzzy P-space if each fuzzy G_δ-set in (X,T) is fuzzy open in (X,T) [11].*
(2) *fuzzy almost P-space if for every non-zero fuzzy G_δ-set λ in (X,T), $int(\lambda) \neq 0$ in (X,T) [17].*
(3) *fuzzy globally disconnected space if each fuzzy semi-open set is fuzzy open in (X,T) [24].*
(4) *fuzzy perfectly disconnected space if for any two non-zero fuzzy sets λ and μ defined on X with $\lambda \leq 1-\mu$, $cl(\lambda) \leq 1 - cl(\mu)$, in (X,T) [25].*
(5) *fuzzy Baire space if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'$s are fuzzy nowhere dense sets in (X,T) [15].*
(6) *fuzzy extremally disconnected space if the closure of every fuzzy open set of (X,T) is fuzzy open in (X,T) [5].*
(7) *fuzzy basically disconnected space if the closure of every fuzzy open F_σ-set of (X,T) is fuzzy open in (X,T) [11].*
(8) *fuzzy F-space if $\lambda \leq 1-\mu$, where λ and μ are fuzzy F_σ-sets, then there exist fuzzy G_δ-sets α and β in (X,T) such that $\lambda \leq \alpha,\ \mu \leq \beta$ and $\alpha \leq 1-\beta$, in (X,T) [27].*
(9) *fuzzy strongly irresolvable space if for each fuzzy set λ defined on X in (X,T), $cl\ [int(\lambda) \vee int(1-\lambda)] = 1$, in (X,T) [28].*
(10) *fuzzy GID-space if for each fuzzy dense and fuzzy G_δ-set λ in (X,T), $cl\ int(\lambda) = 1$, in (X,T) [18].*
(11) *fuzzy Moscow space if for each fuzzy open set λ in (X,T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where $(\delta_i)'$s are fuzzy G_δ-sets in (X,T) [29].*
(12) *fuzzy D-Baire space if every fuzzy first category set in (X,T) is a fuzzy nowhere dense set in (X,T) [16].*
(13) *Fuzzy hyperconnected space if every non-null fuzzy open subset of (X,T) is fuzzy dense in (X,T) [8].*

(14) *fuzzy submaximal space if for each fuzzy set* λ *in* (X, T) *such that* $cl(\lambda) = 1$, $\lambda \in T$ *[2].*

Theorem 2.1. *[28] If* $cl(\lambda) = 1$, *for a fuzzy set* λ *defined on* X *in a fuzzy strongly irresolvable space* (X, T), *then* $cl\ int(\lambda) = 1$, *in* (X, T).

Theorem 2.2. *[18]* (X, T) *be a fuzzy topological space. Then, the following are equivalent :*

(1) (X, T) *is a fuzzy* GID*-space.*
(2) *Each fuzzy dense and fuzzy* G_δ*-set in* (X, T) *is fuzzy semi-open in* (X, T).

Theorem 2.3. *[21] If* λ *is a fuzzy Baire set in a fuzzy topological space* (X, T), *then there exists a fuzzy* G_δ*-set* η *in* (X, T) *such that* $\mu \wedge \eta \leq \lambda$, *where* μ *is a fuzzy open set in* (X, T).

Theorem 2.4. *[20] If* λ *is a fuzzy* σ*-boundary set in a fuzzy topological space* (X, T), *then* λ *is a fuzzy* F_σ*-set in* (X, T).

Theorem 2.5. *[29] If* λ *is a fuzzy open set in the fuzzy Moscow space* (X, T), *then there exists a fuzzy* G_δ*-set* μ *in* (X, T) *such that* $\mu \leq cl(\lambda)$.

Theorem 2.6. *[29] If* η *is a fuzzy closed set in a fuzzy Moscow space* (X, T), *then there exists a fuzzy* F_σ*-set* δ *in* (X, T) *such that* $int(\eta) \leq \delta$

Theorem 2.7. *[20] If* γ *is a fuzzy co-*σ*-boundary set in* (X, T), *then* $1 - \gamma$ *is a fuzzy* σ*-boundary set in* (X, T).

Theorem 2.8. *[16] If* (X, T) *is a fuzzy* D*-Baire space, then* (X, T) *is a fuzzy Baire space.*

Theorem 2.9. *[25] If* (X, T) *is a fuzzy perfectly disconnected space, then* (X, T) *is a fuzzy extremally disconnected space.*

Theorem 2.10. *[24] If* λ *is a fuzzy residual set in a fuzzy globally disconnected space* (X, T), *then* λ *is a fuzzy* G_δ*-set in* (X, T).

Theorem 2.11. *[15] Let* (X, T) *be a fuzzy topological space. Then the following are equivalent:*

(1) (X, T) *is an fuzzy Baire space.*

(2) $int(\lambda) = 0,$ *for every fuzzy first category set* λ *in* (X, T)
(3) $cl(\mu) = 1,$ *for every fuzzy residual set* μ *in* (X, T).

Theorem 2.12. *[28] If* $cl(\lambda)$ *is a fuzzy open set for a fuzzy set* λ *defined on* X *in a fuzzy strongly irresolvable space* (X, T)*, then* λ *is a fuzzy semi-open set in* (X, T).

Theorem 2.13. *[26] If* (X, T) *is a fuzzy perfectly disconnected space, then* 0_X *and* 1_X *are the only two fuzzy simply open sets in* (X, T).

Theorem 2.14. *[19] If a fuzzy topological space* (X, T) *is a fuzzy* P*-space, then* (X, T) *is not a fuzzy* D*-Baire space.*

Theorem 2.15. *[17] If a fuzzy topological space* (X, T) *is a fuzzy almost* P*-space, then* (X, T) *is a fuzzy second category space.*

Theorem 2.16. *[23] If the fuzzy set* λ *is an fuzzy cs dense set in a fuzzy hyperconnected space* (X, T)*, then* $int(\lambda) = 0$*, in* (X, T).

Theorem 2.17. *[23] If* λ *is a fuzzy somewhere dense set in a fuzzy topological space* (X, T)*, then there exists a fuzzy regular closed set* η *in* (X, T) *such that* $\eta \leq cl(\lambda)$.

Theorem 2.18. *[22] If* λ *is a fuzzy semi-open set in a fuzzy topological space* (X, T)*, then* λ *is a fuzzy simply open set in* (X, T).

3. FUZZY P-SPACES

In this section several characterizations of fuzzy P-spaces are established.

Proposition 3.1. *If* λ *is a fuzzy* σ*-boundary set in a fuzzy* P*-space* (X, T)*, then* λ *is a fuzzy closed set in* (X, T).

Proof. Let λ be a fuzzy σ-boundary set in (X, T). Then, by Theorem 2.4, λ is a fuzzy F_σ-set in (X, T) and $1 - \lambda$ is a G_δ-set in (X, T). Since (X, T) is a fuzzy P-space, the fuzzy G_δ-set $1 - \lambda$ is a fuzzy open set in (X, T) and hence λ is a fuzzy closed set in (X, T). □

Proposition 3.2. *If* λ *is a fuzzy co -*σ*- boundary set in a fuzzy* P*-space* (X, T)*, then* λ *is a fuzzy open set in* (X, T).

Proof. Let λ be a fuzzy co-σ-boundary set in (X,T). Then, 1-λ is a fuzzy σ-boundary set in (X,T). Since (X,T) is a fuzzy P-space, by Proposition 3.1, 1-λ is a fuzzy closed set in (X,T) and thus λ is a fuzzy open set in (X,T). □

Remark 3.1. *In view of the above propositions, one will have the following result: "Fuzzy σ-boundary sets are fuzzy closed F_σ- sets and fuzzy co-σ-boundary sets are fuzzy open G_δ-sets in fuzzy P-spaces."*

Proposition 3.3. *If λ is a fuzzy Baire set in a fuzzy P-space (X,T), then*

(i) $int(\lambda) \neq 0$, *in* (X,T)*;*

(ii) λ *is a fuzzy somewhere dense set in* (X,T).

Proof.

(i) Let λ be a fuzzy Baire set in (X,T). Then, by Theorem 2.3, there exists a fuzzy G_δ-set η in (X,T) such that $\mu \wedge \eta \leq \lambda$, where μ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy G_δ-set η is a fuzzy open set in (X,T). Let $\gamma = \mu \wedge \eta$. Then, γ is a fuzzy open set in (X,T). Hence, for the fuzzy Baire set λ, there exists a fuzzy open set γ in (X,T) such that $\gamma \leq \lambda$. Thus, $int(\lambda) \neq 0$, in (X,T).

(ii) Now $int(\lambda) \leq int\ cl(\lambda)$, implies that $int\ cl(\lambda) \neq 0$, in (X,T). Hence λ is a fuzzy somewhere dense set in (X,T). □

Proposition 3.4. *If $(\lambda_i)'s$ $(i = 1\ to\ \infty)$ are fuzzy sets defined on X in a fuzzy P-space (X,T), then $\vee_{i=1}^{\infty}\ [bd(\lambda_i)]$ is a fuzzy closed set in (X,T).*

Proof. Let $(\lambda_i)'$s $(i = 1\ to\ \infty)$ be fuzzy sets defined on X in (X,T). Then, $bd\ (\lambda_i) = cl(\lambda_i)\ \wedge\ cl(1-\lambda_i)$ and $bd(\lambda_i)$ is a fuzzy closed set in (X,T). Now $\vee_{i=1}^{\infty}[bd(\lambda_i)]$ is a fuzzy F_σ-set in (X,T). Since (X,T) is a fuzzy P-space, $\vee_{i=1}^{\infty}[bd\ (\lambda_i)]$ is a fuzzy closed set in (X,T). □

Proposition 3.5. *If $int(\vee_{i=1}^{\infty}\ [bd(\lambda_i)]) = 0$, where $(\lambda_i)'s$ $(i = 1\ to\ \infty)$ are fuzzy sets defined on X in a fuzzy P-space (X,T), then $(\lambda_i)'s$ are fuzzy simply open sets in (X,T).*

Proof. Let $(\lambda_i)'$s $(i = 1\ to\ \infty)$ be fuzzy sets defined on X in (X,T). Since (X,T) is a fuzzy P-space, by proposition 3.4, $\vee_{i=1}^{\infty}\ [bd\ (\lambda_i)]$ is a fuzzy closed set in (X,T). Then, $cl\ (\vee_{i=1}^{\infty}\ [bd\ (\lambda_i)]) = \vee_{i=1}^{\infty}\ [bd\ (\lambda_i)]$ in (X,T) and this implies that

$int\ cl\ (\vee_{i=1}^{\infty}\ [bd\ (\lambda_i)]) = int\ (\vee_{i=1}^{\infty}\ [bd\ (\lambda_i)])$. By hypothesis, $int\ (\vee_{i=1}^{\infty}\ [bd\ (\lambda_i)]) = 0$ and then $int\ cl\ (\vee_{i=1}^{\infty}\ [bd\ (\lambda_i)]) = 0$. Now $\vee_{i=1}^{\infty} int\ cl\ [bd\ (\lambda_i)] \leq int\ cl\ (\vee_{i=1}^{\infty}[bd\ (\lambda_i)])$, implies that $\vee_{i=1}^{\infty} int\ cl\ [bd\ (\lambda_i)] \leq 0$. That is, $\vee_{i=1}^{\infty} int\ cl\ [bd\ (\lambda_i)] = 0$ and this implies that $int\ cl[bd\ (\lambda_i)] = 0$, in (X, T). Hence $(\lambda_i)'$s are fuzzy simply open sets in (X, T). □

Proposition 3.6. *If λ and μ are fuzzy G_δ-sets in a fuzzy P-space (X, T) such that $\lambda \wedge \mu = 0$, then*

(1) $cl(\lambda) \neq 1$ *and* $cl(\mu) \neq 1$, *in* (X, T).

(2) $1-\lambda$ *and* $1-\mu$ *are fuzzy somewhere dense sets in* (X, T) *and* $(1-\lambda) \wedge (1-\mu) = 1$ *in* (X, T).

Proof.

(i) Let λ and μ be fuzzy G_δ-sets in (X, T) such that $\lambda \wedge \mu = 0$. Since (X, T) is a fuzzy P-space, λ and μ are fuzzy open sets in (X, T). Now $\lambda \wedge \mu = 0$, implies that $\lambda \leq 1 - \mu$ and then $int(\lambda) \leq int(1 - \mu)$. By Lemma 2.1, $int(1 - \mu) = 1 - cl(\mu)$, in (X, T). Then, $\lambda \leq 1 - cl(\mu)$ and $cl(\mu) \leq 1 - \lambda$. Since $1 - \lambda$ is a fuzzy closed set in (X, T), $cl(\mu) \neq 1$, in (X, T). Also $\lambda \leq 1 - \mu$, implies that $cl(\lambda) \leq cl(1 - \mu)$ and $cl(\lambda) \leq 1 - int(\mu) = 1 - \mu$, in (X, T). Since $1 - \mu$ is a fuzzy closed set in (X, T), $cl(\lambda) \neq 1$ in (X, T).

(ii) By(i), $cl(\lambda) \neq 1$ and then $1 - cl(\lambda) \neq 0$. This implies that $int(1 - \lambda) \neq 0$. Since $int(1 - \lambda) \leq int\ cl(1 - \lambda), int\ cl(1 - \lambda) \neq 0$ and thus $1 - \lambda$ is a fuzzy somewhere dense set in (X, T). Now $cl(\mu) \neq 1$ and then $1 - cl(\mu) \neq 0$. This implies, by Lemma 2.1, that $int(1 - \mu) \neq 0$. Since $int(1 - \mu) \leq int\ cl(1 - \mu)$, $int\ cl(1 - \mu) \neq 0$ and thus $1 - \mu$ is a fuzzy somewhere dense set in (X, T). Hence $1 - \lambda$ and $1 - \mu$ are fuzzy somewhere dense sets in (X, T). Now $(1 - \lambda) \vee (1 - \mu) = 1 - (\lambda \wedge \mu) = 1 - 0 = 1$. Hence $(1 - \lambda) \vee (1 - \mu) = 1$, where $1 - \lambda$ and $1 - \mu$ are fuzzy somewhere dense sets in (X, T). □

Proposition 3.7. *If λ and μ are fuzzy G_δ-sets in a fuzzy P-space (X, T), such that $\lambda \wedge \mu = 0$, then λ and μ are fuzzy cs dense sets in (X, T).*

Proof. Let λ and μ be fuzzy G_δ-sets in (X, T) such that $\lambda \wedge \mu = 0$. Since (X, T) is a fuzzy P-space, by proposition 3.6 (ii), $1 - \lambda$ and $1 - \mu$ are fuzzy somewhere dense sets in (X, T) and then λ and μ are fuzzy cs dense sets in (X, T). □

Proposition 3.8. *If λ is a fuzzy G_δ-set in a fuzzy almost P-space (X,T), then $int(\lambda)$ is a fuzzy G_δ-set in (X,T).*

Proof. Let λ be a fuzzy G_δ-set in (X,T). Then, $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$. Since (X,T) is a fuzzy almost P-space, $int(\lambda) \neq 0$, in (X,T) and $int(\lambda)$ is a fuzzy open set in (X,T). Now $int(\lambda) \leq \lambda$, $int(\lambda) = int(\lambda) \wedge \lambda = int(\lambda) \wedge [\wedge_{i=1}^{\delta}(\lambda_i)]$ and thus $int(\lambda)$ is a fuzzy G_δ-set in (X,T). □

Proposition 3.9. *If λ is a fuzzy Baire set in a fuzzy P-space (X,T), then there exists a fuzzy regular closed set η in (X,T) such that $\eta \leq cl(\lambda)$.*

Proof. Let λ be a fuzzy Baire set in (X,T). Since (X,T) is a fuzzy P-space, by Proposition 3.3 (ii), λ is a fuzzy somewhere dense set in (X,T). Then, by Theorem 2.17, there exists a fuzzy regular closed set η in (X,T) such that $\eta \leq cl(\lambda)$. □

4. FUZZY P-SPACES AND OTHER TOPOLOGICAL SPACES

Proposition 4.1. *If a fuzzy topological space (X,T) is a fuzzy P-space, then (X,T) is a fuzzy basically disconnected space.*

Proof. Let λ be a fuzzy open F_σ-set in (X,T). Then, $1-\lambda$ is a fuzzy closed G_δ-set in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy G_δ-set $1-\lambda$ is a fuzzy open set and thus $int(1-\lambda) = 1-\lambda$, in (X,T). By Lemma 2.1, $int(1-\lambda) = 1 - cl(\lambda)$ and then $1 - cl(\lambda) = 1 - \lambda$. This implies that $cl(\lambda) = \lambda$ and $\lambda \in T$, implies that $cl(\lambda)$ is fuzzy open in (X,T). Hence (X,T) is a fuzzy basically disconnected space. □

Example 1. *Let $X = \{a,b,c\}$. Let $I = [0,1]$. The fuzzy sets α, β, and γ, are defined on X, as follows:*

$\alpha : X \to I$ *is defined by* $\alpha(a) = 0.6;\ \ \alpha(b) = 0.5;\ \ \alpha(c) = 0.4$;
$\beta : X \to I$ *is defined by* $\beta(a) = 0.5;\ \ \beta(b) = 0.5,\ \ \beta(c) = 0.5$;
$\gamma : X \to I$ *is defined by* $\gamma(a) = 0.4;\ \ \gamma(b) = 0.5;\ \ \gamma(c) = 0.6$;
$\theta : X \to I$ *is defined by* $\theta(a) = 0.4;\ \ \theta(b) = 0.5;\ \ \gamma(c) = 0.6$;

Then, $T = \{0, \alpha,\ \beta,\ \gamma,\ \alpha\vee\beta,\ \alpha\vee\gamma,\ \beta\vee\gamma,\ \alpha\wedge\beta,\ \alpha\wedge\gamma,\ \beta\wedge\gamma, 1\}$ is a fuzzy topology on X. By computation $\alpha\wedge\gamma = \alpha \wedge\beta \wedge\gamma \wedge (\alpha\vee\beta) \wedge (\alpha\vee\gamma) \wedge (\beta\vee\gamma) \wedge (\alpha\wedge\beta) \wedge (\beta\wedge\gamma)$ and $\alpha \wedge \gamma$ are fuzzy G_δ-sets in (X,T) and $\alpha \wedge \gamma \in T$, implies that (X,T) is a fuzzy P-spaces. By computation one can find that

$1-(\alpha\wedge\gamma)=(1-\alpha)\ \vee\ (1-\gamma)\ \vee\ (1-[\alpha\wedge\beta])\ \vee\ (1-[\beta\wedge\gamma])$;
$1-\beta=(1-[\alpha\vee\beta])\vee(1-[\alpha\vee\gamma])\ \vee\ (1-[\beta\vee\gamma])\vee(1-[\alpha\wedge\beta])$;
$1-(\alpha\wedge\beta)=(1-\alpha)\ \vee\ (1-\beta)\vee(1-[\beta\vee\gamma])$, *and* $1-(\alpha\wedge\gamma),\ 1-\beta$ *and* $1-(\alpha\wedge\beta)$ *are fuzzy F_σ-sets in* (X,T) *and* $1-(\alpha\wedge\gamma)(=\alpha\vee\gamma), 1-\beta(=\beta), 1\check{}(\alpha\wedge\beta)\ (=\beta\vee\gamma)$ *are fuzzy open sets in* (X,T).

Also $cl(1-\alpha\wedge\gamma))=1-(\alpha\wedge\gamma)=\alpha\ \vee\ \gamma\,\epsilon\,T$.
$cl(1-\beta)=1-\beta=\beta\,\epsilon\,T$.
$cl(1-(\alpha\wedge\beta))=1-(\alpha\wedge\beta)=\beta\vee\gamma\,\epsilon\,T$.
Hence (X,T) *is a fuzzy basically disconnected space.*

Remark 4.1. *The converse of the above proposition need not be true. That is, a fuzzy basically disconnected space need not be a fuzzy P-space. For, consider the following:*

Example 2. *Let* $X=\{a,b,c\}$. *Let* $I=[0,1]$. *The fuzzy sets* α, μ, γ, *and* θ *are defined on* X, *as follows:*

$\alpha: X\to I$ *is defined by* $\alpha(a)=0.5;\ \alpha(b)=0.6;\ \alpha(c)=0.4$;
$\beta: X\to I$ *is defined by* $\beta(a)=0.4;\ \beta(b)=0.5,\ \beta(c)=0.6$;
$\gamma: X\to I$ *is defined by* $\gamma(a)=0.6;\ \gamma(b)=0.4;\ \gamma(c)=0.5$;
$\theta: X\to I$ *is defined by* $\theta(a)=0.5;\ \theta(b)=0.5;\ \theta(c)=0.5$;

Then, $T=\{0,\alpha,\ \beta,\ \gamma,\ \alpha\vee\beta,\ \alpha\vee\gamma,\ \beta\vee\gamma,\ \alpha\wedge\beta,\ \alpha\wedge\gamma,\ \beta\wedge\gamma,\ (\alpha\vee[\beta\wedge\gamma]),(\beta\vee[\alpha\wedge\gamma]),\ (\gamma\vee[\alpha\wedge\beta]),\ (\alpha\wedge[\beta\vee\gamma]),\ (\beta\wedge[\alpha\vee\gamma]),\ (\gamma\wedge[\alpha\vee\beta]),\ [\alpha\vee\beta\vee\gamma],\ [\alpha\wedge\beta\wedge\gamma],\ 1\}$ *is a fuzzy topology on* X. *By computation, one can find that*

$(\alpha\vee\gamma),\ (\beta\vee\gamma),\ \beta\wedge(\alpha\vee\gamma)$ *and* $\alpha\ \vee\beta\ \vee\ \gamma$ *are fuzzy F_σ-sets in* (X,T)
$cl(\alpha\vee\gamma)=1-(\beta\wedge\gamma)=\alpha\vee\gamma\,\epsilon\,T$
$cl(\beta\vee\gamma)=1-(\alpha\wedge\beta)=\beta\vee\gamma\,\epsilon\,T$
$cl[\beta\ \wedge(\alpha\vee\gamma)]=1-\gamma\vee(\alpha\wedge\beta)=\beta\wedge(\alpha\vee\gamma)\,\epsilon\,T$
$cl(\alpha\ \vee\ \beta\ \vee\gamma)=1-(\alpha\ \wedge\ \beta\ \wedge\ \gamma)=\alpha\ \vee\ \beta\ \vee\ \gamma\,\epsilon\,T$.

Hence (X,T) *is a fuzzy basically disconnected space. Since* θ *is a fuzzy G_δ-set but not a fuzzy open set in* (X,T), (X,T) *is a fuzzy P-spaces*

The following proposition gives a condition for a fuzzy strongly irresolvable and fuzzy globally disconnected space to become a fuzzy P-space.

Proposition 4.2. *If each fuzzy G_δ-set is a fuzzy dense set in a fuzzy strongly irresolvable and fuzzy globally disconnected space* (X,T), *then* (X,T) *is a fuzzy P-space.*

Proof. Let λ be a fuzzy G_δ-set in (X, T). By hypothesis, λ is a fuzzy dense set in (X, T). Since (X, T) is a fuzzy strongly irresolvable space, by Theorem 2.1, $cl(\lambda) = 1$ implies that $cl\ int(\lambda) = 1$, in (X, T). Then, $\lambda \leq cl\ int(\lambda)$ and thus λ is a fuzzy semi-open set in (X, T). Since (X, T) is a fuzzy globally disconnected space, the fuzzy semi-open set λ is fuzzy open in (X, T). Thus, the fuzzy G_δ-set is fuzzy open in (X, T), implies that (X, T) is a fuzzy P-space. □

The following proposition gives a condition for a fuzzy globally disconnected and fuzzy GID-space to become a fuzzy P-space.

Proposition 4.3. *If each fuzzy G_δ-set is a fuzzy dense set in a fuzzy globally disconnected and fuzzy GID-space (X, T), then (X, T) is a fuzzy P-space.*

Proof. Let λ be a fuzzy G_δ-set in (X, T). By hypothesis, λ is a fuzzy dense set in (X, T). Thus, λ is a fuzzy dense and fuzzy G_δ-set in (X, T). Since (X, T) is the fuzzy GID-space, by theorem 2.2, λ is a fuzzy semi-open set in (X, T). Also since (X, T) is a fuzzy globally disconnected space, the fuzzy semi-open set λ is fuzzy open in (X, T). Thus, the fuzzy G_δ-set λ is fuzzy open in (X, T), implies that (X, T) is a fuzzy P-space. □

Proposition 4.4. *If λ is a fuzzy σ- boundary set in a fuzzy Moscow and fuzzy P-space (X, T), then there exists a fuzzy F_σ-set δ in (X, T) such that $int(\lambda) \leq \delta$.*

Proof. Let λ be a fuzzy σ-boundary set in (X, T). Since (X, T) is a fuzzy P-space, by Proposition 3.1, λ is a fuzzy closed set in (X, T). Then,$1 - \lambda$ is a fuzzy open set in (X, T). Since(X, T) is the fuzzy Moscow space, by Theorem 2.5, there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq cl(1-\lambda)$. By Lemma 2.1, $cl(1-\lambda) = 1-int(\lambda)$ and thus $\mu \leq 1 - int(\lambda)$. This implies that $int(\lambda) \leq 1 - \mu$. Let $\delta = 1 - \mu$. Then, δ is the fuzzy F_σ-set in (X, T). Thus, for the fuzzy σ-boundary set λ in (X, T), there exists a fuzzy F_σ-set δ in (X, T) such that $int(\lambda) \leq \delta$. □

Proposition 4.5. *If λ is a fuzzy σ- boundary set in a fuzzy Moscow and fuzzy P-space (X, T), then $cl\ int(\lambda) \neq 1$, in (X, T).*

Proof. Let λ be a fuzzy σ-boundary set in (X, T). Since (X, T) is a fuzzy P-space, by Proposition 3.1, λ is a fuzzy closed set in (X, T). Then, $1 - \lambda$ is a fuzzy open set in (X, T). Since (X, T) is a fuzzy Moscow space, by Theorem 2.5, there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq cl(1 - \lambda)$. Since (X, T) is a fuzzy P-space, the fuzzy G_δ-set μ is fuzzy open in (X, T) and thus $int\ cl(1 - \lambda) \neq 0$ and then $1 - cl\ int(\lambda) \neq 0$, in (X, T). This implies that $cl\ int(\lambda) \neq 1$, in (X, T). □

Proposition 4.6. *If λ is a fuzzy co-σ-boundary set in a fuzzy Moscow and fuzzy P-space (X, T), then λ is a fuzzy somewhere dense set in (X, T).*

Proof. Let λ be a fuzzy co-σ-boundary set in (X, T). Then, $1 - \lambda$ is a fuzzy σ-boundary set in (X, T). Since the fuzzy topological space (X, T) is a fuzzy Moscow and fuzzy P-space, by Proposition 4.5, $cl\ int(1 - \lambda) \neq 1$, in (X, T). By Lemma 2.1, $cl\ int(1 - \lambda) = 1 - int\ cl(\lambda)$, in (X, T). This implies that $1 - int\ cl(\lambda) \neq 1$, in (X, T) and then $int\ cl(\lambda) \neq 0$. Hence λ is a fuzzy somewhere dense set in (X, T). □

Proposition 4.7. *If the fuzzy topological space (X, T) is a fuzzy F-space and fuzzy P-space and α and β are fuzzy closed sets such that $\alpha \wedge \beta = 0$, in (X, T), then there exist fuzzy open sets γ and η in (X, T) such that $\alpha \leq \gamma$ and $\beta \leq \eta$ and $\gamma \leq 1 - \eta$.*

Proof. Suppose that α and β are fuzzy closed sets such that $\alpha \wedge \beta = 0$, in (X, T). Now $\alpha \wedge \beta = 0$, implies that $\alpha \leq 1 - \beta$ in (X, T). Consider the fuzzy sets λ and μ defined as follows:

$\lambda = \alpha \vee (\vee_{i=1}^{\infty}(\theta_i))$,

$\mu = \beta \vee (\vee_{i=1}^{\infty}(\theta_i)$,

where $(\theta_i))'$s are fuzzy closed sets in (X, T). Then, λ and μ are fuzzy F_σ-sets in (X, T) and $\alpha \leq \lambda$ and $\beta \leq \mu$. This implies that $\alpha \leq \lambda$ and $1 - \mu \leq 1 - \beta$. If $\lambda \leq 1 - \mu$, in the fuzzy F-space (X, T), then there exist fuzzy G_δ-sets γ and η in (X, T) such that $\lambda \leq \gamma$, $\mu \leq \eta$ and $\gamma \leq 1 - \eta$. Since (X, T) is a fuzzy P-space, the fuzzy G_δ-sets γ and η are fuzzy open sets in (X, T). Then, $\alpha \leq \lambda \leq \gamma$ and $\beta \leq \mu \leq \eta$ and $\gamma \leq 1 - \eta$. Thus, for the fuzzy closed sets α and β in (X, T) such that $\alpha \wedge \beta = 0$, there exist fuzzy open sets γ and η in (X, T) such that $\alpha \leq \gamma$ and $\beta \leq \eta$ and $\gamma \leq 1 - \eta$. □

Proposition 4.8. *If λ is a fuzzy G_δ-set in a fuzzy extremally disconnected and fuzzy P-space (X, T), then $cl(\lambda)$ is a fuzzy open set in (X, T).*

Proof. Let λ be a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy P-space, the fuzzy G_δ-set λ is a fuzzy open set in (X, T). Also since (X, T) is a fuzzy extremally disconnected space, $cl(\lambda)$ is a fuzzy open set in (X, T). □

Proposition 4.9. *If λ is a fuzzy G_δ-set in a fuzzy perfectly disconnected and fuzzy P-space (X,T), then $cl(\lambda)$ is a fuzzy open set in (X,T).*

Proof. Let λ be a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy G_δ- set λ is a fuzzy open set in (X,T). Also since (X,T) is a fuzzy perfectly disconnected space, by Theorem 2.9,(X,T) is a fuzzy extremally disconnected space. and thus (X,T) is a fuzzy extremally disconnected and fuzzy P-space. Then, by Proposition 4.8, $cl(\lambda)$ is a fuzzy open set in (X,T). □

The following propositions show that fuzzy residual sets are fuzzy open sets and fuzzy first category sets are fuzzy closed sets in fuzzy globally disconnected and fuzzy P-spaces.

Proposition 4.10. *If λ is a fuzzy residual set in a fuzzy globally disconnected and fuzzy P-space (X,T), then λ is a fuzzy open set in (X,T).*

Proof. Let λ be a fuzzy residual set in (X,T). Since (X,T) is a fuzzy globally disconnected space, by Theorem 2.10, λ is a fuzzy G_δ-set in (X,T). Also since (X,T) is a fuzzy P-space, the fuzzy G_δ-set λ is a fuzzy open set in (X,T). Thus, the fuzzy residual set in a fuzzy globally disconnected and fuzzy P-space (X,T) is a fuzzy open set in (X,T). □

Proposition 4.11. *If λ is a fuzzy first category set in a fuzzy globally disconnected and fuzzy P-space (X,T), then λ is a fuzzy closed set in (X,T).*

Proof. Let λ be a fuzzy first category set in (X,T). Then, $1-\lambda$ is a fuzzy residual set in (X,T). Since (X,T) is a fuzzy globally disconnected and fuzzy P-space, by Proposition 4.10, $1-\lambda$ is a fuzzy open set in (X,T) and hence λ is a fuzzy closed set in (X,T). □

Proposition 4.12. *If λ is a fuzzy first category set in a fuzzy globally disconnected, fuzzy Baire and fuzzy P-space (X,T), then λ is a fuzzy nowhere dense set in (X,T).*

Proof. Let λ be a fuzzy first category set in (X,T). Since (X,T) is a fuzzy globally disconnected and fuzzy P-space, by Proposition 4.11, λ is a fuzzy closed set in (X,T). Also since (X,T) is a fuzzy Baire space, by Theorem 2.11, $int(\lambda)=0$, in (X,T). This implies that $int\ cl(\lambda)=int(\lambda)=0$, in (X,T). Hence λ is a fuzzy nowhere dense set in (X,T). □

Proposition 4.13. *If (X, T) is a fuzzy globally disconnected, fuzzy Baire and fuzzy P-space, then (X, T) is a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy first category set in (X, T). Since (X, T) is a fuzzy globally disconnected, fuzzy Baire and fuzzy P-space, by Proposition 4.12, λ is a fuzzy nowhere dense set in (X, T) and hence (X, T) is a fuzzy D-Baire space. □

Remark 4.2. *The converse of the above proposition need not be true. That is, a fuzzy D-Baire space need not be a fuzzy globally disconnected space. For, consider the following example:*

Example 3. *Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets λ, μ, γ, δ, η, α, β, and σ are defined on X, as follows:*

$\lambda : X \to [0, 1]$ *is defined by* $\lambda(a) = 0.7;\ \lambda(b) = 0.6;\ \lambda(c) = 0.2$;
$\mu : X \to [0, 1]$ *is defined by* $\mu(a) = 0.5;\ \mu(b) = 0.4;\ \mu(c) = 0.8$;
$\gamma : X \to [0, 1]$ *is defined by* $\gamma(a) = 0.5;\ \gamma(b) = 0.3;\ \gamma(c) = 0.6$;
$\delta : X \to [0, 1]$ *is defined by* $\delta(a) = 0.2;\ \delta(b) = 0.3;\ \delta(c) = 0.7$;
$\eta : X \to [0, 1]$ *is defined by* $\eta(a) = 0.3;\ \eta(b) = 0.4;\ \eta(c) = 0.1$;
$\alpha : X \to [0, 1]$ *is defined by* $\alpha(a) = 0.2;\ \alpha(b) = 0.4;\ \alpha(c) = 0.3$;
$\beta : X \to [0, 1]$ *is defined by* $\beta(a) = 0.3;\ \beta(b) = 0.4,\ \beta(c) = 0.7$;
$\sigma : X \to [0, 1]$ *is defined by* $\sigma(a) = 0.5;\ \sigma(b) = 0.5;\ \sigma(c) = 0.8$;

Then, $T = \{0, \lambda,\ \mu,\ \gamma,\ \lambda \vee \mu,\ \lambda \vee \gamma,\ \lambda \wedge \mu,\ \lambda \wedge \gamma,\ \gamma \vee [\lambda \wedge \mu],\ 1\}$ *is a fuzzy topology on X. By computation, one can find that* $cl(\lambda) = 1$; $int(1 - \lambda) = 0$

$cl(\mu) = 1 - (\lambda \wedge \mu)$; $\quad int(1 - \mu) = (\lambda \wedge \mu)$.
$cl(\gamma) = 1 - (\lambda \wedge \mu)$; $\quad int(1 - \gamma) = (\lambda \wedge \mu)$.
$cl(\lambda \vee \mu) = 1$; $\quad int(1 - [\lambda \vee \mu]) = 0$.
$cl(\lambda \vee \gamma) = 1$; $\quad int(1 - [\lambda \vee \gamma]) = 0$.
$cl(\lambda \wedge \mu) = 1 - \mu$; $\quad int(1 - [\lambda \wedge \mu]) = \mu$.
$cl(\lambda \vee \gamma) = 1 - \mu$; $\quad int(1 - [\lambda \wedge \gamma]) = \mu$.
$cl(\gamma \vee [\lambda \wedge \mu]) = 1 - (\lambda \wedge \mu)$; $\quad int(1 - [\gamma \vee [\lambda \wedge \mu]]) = \lambda \wedge \mu$.

The fuzzy nowhere dense sets in (X, T) are $1 - \lambda$, $1 - (\lambda \wedge \mu)$, $1 - (\lambda \vee \gamma)$, δ, η and α. Also $\beta = \delta \vee \eta \vee \alpha$ and $1 - \lambda = (1 - \lambda) \vee [1 - (\lambda \wedge \mu)] \vee [1 - (\lambda \vee \gamma)]$. Then, β and $1 - \lambda$ are fuzzy first category sets in (X, T). Now $int\ cl(\beta) = int(1 - \lambda) = 0$ and $int\ cl(1 - \lambda) = int(1 - \lambda) = 0$ and thus the fuzzy first category sets β and $1 - \lambda$ are fuzzy nowhere dense sets in (X, T). Hence (X, T) is a fuzzy D- Baire space.

Also $cl\ int(\sigma) = cl(\mu) = 1 - [\lambda \wedge \mu]$ *and* $\sigma \leq cl\ int(\sigma)$ *implies that* σ *is a fuzzy semi-open set in* (X, T)*. But* σ *is not a fuzzy open set in* (X, T)*, and this implies that* (X, T) *is not a fuzzy globally disconnected space.*

Proposition 4.14. *If the fuzzy topological space* (X, T) *is a fuzzy hyperconnected space, then* (X, T) *is not a fuzzy* D*-Baire space.*

Proof. Let $(\lambda_i)'$s $(i = 1\ to\ \infty)$ be fuzzy open sets in (X, T). Since (X, T) is a fuzzy hyperconnected space, $(\lambda_i)'$s are fuzzy dense sets in (X, T). Then, $cl(\lambda_i) = 1$, in (X, T). Now $int\ cl(1 - \lambda_i) = 1 - cl\ int(\lambda_i) = 1 - cl(\lambda_i) = 1 - 1 = 0$ and thus $(1 - \lambda_i)'$s are fuzzy nowhere dense sets in (X, T). Let $\mu = \vee_{i=1}^{\infty}(1 - \lambda_i)$ and then μ is a fuzzy first category set in (X, T). Now $int\ cl(\mu) = int\ cl(\vee_{i=1}^{\infty}(1 - \lambda_i)) = int\ cl(1 - \wedge_{i=1}^{\infty}(\lambda_i)) = 1 - cl\ int(\wedge_{i=1}^{\infty}(\lambda_i)) \geq 1 - cl(\wedge_{i=1}^{\infty}(\lambda_i)) \geq 1 - cl(\lambda_i) = 1 - 1 = 0$ and $int\ cl(\mu) \geq 0$. Thus μ is not a fuzzy nowhere dense set in (X, T). Hence (X, T) is not a fuzzy D-Baire space. □

Remark 4.3. *It is to noted that from Proposition 4.14, a fuzzy hyperconnected space is not a fuzzy* D*-Baire space and by Theorem 2.14, a fuzzy* P*-space is not a fuzzy* D*-Baire space.*

The following proposition shows that a fuzzy hyperconnected and fuzzy *P*-space is a fuzzy *D*-Baire space.

Proposition 4.15. *If the fuzzy topological space* (X, T) *is a fuzzy hyperconnected and fuzzy* P*-space, then* (X, T) *is a fuzzy* D*-Baire space.*

Proof. Let λ be a fuzzy first category set in (X, T). Then, $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'$s are fuzzy nowhere dense sets in (X, T). Now $1 - cl(\lambda_i)$ is a fuzzy open set in (X, T). Let $\mu = \wedge_{i=1}^{\infty}[1 - cl(\lambda_i)]$ is a fuzzy G_δ-set in (X, T). Now $\mu = \wedge_{i=1}^{\infty}[1 - cl(\lambda_i)] = 1 - [\vee_{i=1}^{\infty} cl(\lambda_i)] \leq 1 - \vee_{i=1}^{\infty}(\lambda_i) = 1 - \lambda$. Thus $\mu \leq 1 - \lambda$. Since (X, T) is a fuzzy P-space, the fuzzy G_δ-set μ is a fuzzy open set in (X, T) and then $int\ cl(\lambda) \leq int\ cl(1 - \mu) = 1 - cl\ int(\mu) = 1 - cl(\mu)$. Since (X, T) is a fuzzy hyperconnected space, for the fuzzy open set μ, $cl(\mu) = 1$ and then $int\ cl(\lambda) \leq 1 - 1 = 0$. That is, $int\ cl(\lambda) = 0$. Thus, λ is a fuzzy nowhere dense set in (X, T) and hence (X, T) is a fuzzy D-Baire space. □

Remark 4.4. *The converse of the above proposition need not be true. That is, a fuzzy D-Baire space need not be a fuzzy hyperconnected space. For, consider the following example:*

Example 4. *Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets λ, μ, γ, δ, ρ, and θ are defined on X, as follows:*

$\lambda : X \to [0, 1]$ *is defined by* $\lambda(a) = 0.5;\ \lambda(b) = 0.3;\ \lambda(c) = 0.5$*;*
$\mu : X \to [0, 1]$ *is defined by* $\mu(a) = 0.6;\ \mu(b) = 0.5;\ \mu(c) = 0.7$*;*
$\gamma : X \to [0, 1]$ *is defined by* $\gamma(a) = 0.5;\ \gamma(b) = 0.4;\ \gamma(c) = 0.6$*;*
$\delta : X \to [0, 1]$ *is defined by* $\delta(a) = 0.5;\ \delta(b) = 0.6;\ \delta(c) = 0.7$*;*
$\rho : X \to [0, 1]$ *is defined by* $\rho(a) = 0.7;\ \rho(b) = 0.8;\ \rho(c) = 0.6$*;*
$\theta : X \to [0, 1]$ *is defined by* $\theta(a) = 0.5;\ \theta(b) = 0.5;\ \theta(c) = 0.4$*;*

Then, $T = \{0, \lambda,\ \mu,\ \gamma,\ 1\}$ is a fuzzy topology on X. By computation, it follows that $cl(\lambda) = 1 - \lambda$, $cl(\mu) = 1$, $cl(\gamma) = 1$, $cl(\delta) = 1$ and $cl(1 - \delta) = 1 - \gamma$, $cl(\rho) = 1, cl(1 - \rho) = 1 - \gamma$, in (X, T). Also, $int(1 - \lambda) = \lambda$, $int(1 - \mu) = 0$, $int(1 - \gamma) = 0$, $int(1-\delta) = 0$, $int(1-\rho) = 0$, in (X, T). By computation $int\, cl(1-\mu) = int(1-\mu) = 0$; $int\, cl(1-\gamma) = int(1-\gamma) = 0$; $int\, cl(1-\delta) = int(1-\gamma) = 0$; $int\, cl(1-\rho) = int(1-\gamma) = 0$, in (X, T). Thus, $1-\mu, 1-\gamma, 1-\delta$ and $1-\rho$ are fuzzy nowhere dense sets in (X, T) and $1 - \gamma = (1 - \mu) \vee (1 - \gamma) \vee (1 - \delta) \vee (1 - \rho)$ and $\theta = (1 - \mu) \vee (1 - \delta) \vee (1 - \rho)$, implies that $1 - \gamma$ and θ are fuzzy first category sets in (X, T). Also, by computation, $int\ cl(\theta) = 0$ and θ is a fuzzy nowhere dense set in (X, T). Hence (X, T) is a fuzzy D-Baire space. Since $cl(\lambda) = 1 - \lambda,\ \lambda$ is not a fuzzy dense set in (X, T) and thus (X, T) is not a fuzzy hyperconnected space.

Also in example 3, (X, T) is a fuzzy D- Baire space but not a fuzzy hyperconnected space, since $cl(\gamma) = 1-(\lambda \wedge \mu)$, the fuzzy open set γ is not a fuzzy dense set in (X, T).

Proposition 4.16. *If λ is a fuzzy G_δ-set in a fuzzy extremally disconnected, fuzzy strongly irresolvable and fuzzy P-space (X, T), then*

(1) *λ is a fuzzy semi-open set in (X, T).*
(2) *$cl(\lambda)$ is a fuzzy regular closed set in (X, T).*

Proof.

(i) Let λ be a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy extremally disconnected and fuzzy P-space, by proposition 4.8, $cl(\lambda)$ is a fuzzy open set in (X, T).

Since (X, T) is a fuzzy strongly irresolvable space, by theorem 2.12, λ is a fuzzy semi-open set in (X, T).

(ii) By (i), $\lambda \leq cl\ int(\lambda)$ in (X, T). Then, $cl(\lambda) \leq cl[cl\ int(\lambda)] = cl\ int(\lambda) \leq cl\ int\ cl(\lambda)$. Also $cl\ int\ cl(\lambda) \leq cl\ cl(\lambda) = cl(\lambda)$. Thus, $cl\ int\ cl(\lambda) = cl(\lambda)$ and hence $cl(\lambda)$ is a fuzzy regular closed set in (X, T). □

Definition 4.1. *A fuzzy topological space (X, T) is called a fuzzy semi P-space if each fuzzy G_δ-set in (X, T) is a fuzzy semi-open set in (X, T).*

Proposition 4.17. *If the fuzzy topological space (X, T) is a fuzzy extremally disconnected, fuzzy strongly irresolvable and fuzzy P-space, then (X, T) is a fuzzy semi-P-space.*

Proof. Let λ be a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy extremally disconnected fuzzy strongly irresolvable and fuzzy P-space, by Proposition 4.16(i), λ is a fuzzy semi-open set in (X, T) and hence (X, T) is a fuzzy semi P-space. □

Proposition 4.18. *If (X, T) is a fuzzy perfectly disconnected space and fuzzy P-space, then $int(\vee_{i=1}^{\infty}[bd\ (\lambda_i)]) \neq 0$, where $(\lambda_i)'s$ $(i = 1\ to\ \infty)$ are fuzzy sets defined on X, in (X, T).*

Proof. Let (X, T) be a fuzzy perfectly disconnected and fuzzy P-space. Suppose that $int(\vee_{i=1}^{\infty}[bd\ (\lambda_i)]) = 0$, where $(\lambda_i)'$s $(i = 1\ to\ \infty)$ are fuzzy sets defined on X, in (X, T). Then, by proposition 3.5, $(\lambda_i)'$s are fuzzy simply open sets in (X, T). But this is a contradiction, since (X, T) is a fuzzy perfectly disconnected space in which 0_X and 1_X are the only fuzzy simply open sets in (X, T). Hence $int(\vee_{i=1}^{\infty}[bd\ (\lambda_i)]) \neq 0$, in (X, T). □

Proposition 4.19. *If a fuzzy topological space (X, T) is a fuzzy P-space, then (X, T) is a fuzzy basically disconnected and fuzzy almost P-space.*

Proof. Let λ be a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy P-space, λ is a fuzzy open set and hence $int(\lambda) = \lambda \neq 0$, in (X, T). This implies that (X, T) is a fuzzy almost P-space. By proposition 4.1,(X, T) is a fuzzy basically disconnected space. Hence (X, T) is a fuzzy basically disconnected and fuzzy almost P-space. □

Proposition 4.20. *If a fuzzy topological space (X, T) is a fuzzy P-space, then (X, T) is a fuzzy basically disconnected and fuzzy second category space.*

Proof. The proof follows from Proposition 4.19 and Theorem 2.15. □

Proposition 4.21. *If there are disjoint fuzzy G_δ-sets in a fuzzy P-space (X,T), then (X,T) is not a fuzzy hyperconnected space.*

Proof. Let λ and μ be fuzzy G_δ-sets in (X,T) such that $\lambda \wedge \mu = 0$. Since (X,T) is a fuzzy P-space, by proposition 3.7, the fuzzy G_δ-sets λ and μ are fuzzy cs dense sets in (X,T). If (X,T) is a fuzzy hyperconnected space, then by Theorem 2.16, for the fuzzy cs dense sets λ and μ, one will have $int(\lambda) = 0$ and $int(\mu) = 0$, in (X,T), a contradiction to $int(\lambda) = \lambda$ and $int(\mu) = \mu$ in the fuzzy P-space (X,T). Hence (X,T) is not a fuzzy hyperconnected space. □

Remark 4.5. *In view of the above proposition, one will have the following result: "If the fuzzy topological space (X,T) is a fuzzy hyperconnected and fuzzy P-space, then (X,T) does not have disjoint fuzzy G_δ-sets."*

Proposition 4.22. *If λ is a fuzzy Baire set in a fuzzy hyperconnected and fuzzy P-space, then λ is a fuzzy semi-open set in (X,T).*

Proof. Let λ be a fuzzy Baire set in (X,T). Since (X,T) is a fuzzy P-space, by Proposition 3.3(i), $int(\lambda) \neq 0$, in (X,T). Also since (X,T) is a fuzzy hyperconnected space, for the fuzzy open set $int(\lambda)$, $cl\ int(\lambda) = 1$, and then $\lambda \leq cl\ int(\lambda)$ in (X,T). Hence λ is a fuzzy semi-open set in (X,T). □

Proposition 4.23. *If λ is a fuzzy Baire set in a fuzzy hyperconnected and fuzzy P-space, then λ is a fuzzy simply open set in (X,T).*

Proof. Let λ be a fuzzy Baire set in (X,T). Since (X,T) is a fuzzy hyperconnected and fuzzy P-space, by Proposition 4.22 , λ is a fuzzy semi-open set in (X,T). By Theorem 2.18, the fuzzy semi-open set λ is a fuzzy simply open set in (X,T). □

CONCLUSION

In this paper, it is established that fuzzy σ-boundary sets are fuzzy closed sets, fuzzy co-σ-boundary sets are fuzzy open sets and fuzzy Baire sets are fuzzy somewhere dense sets in fuzzy P-spaces. The condition which ensures the existence of fuzzy simply open sets in fuzzy P-spaces is established by means of fuzzy boundary of fuzzy sets. It is obtained that disjoint fuzzy G_δ-sets in fuzzy P-spaces are fuzzy cs dense sets. It is established that fuzzy P-spaces are fuzzy basically disconnected, fuzzy almost P- spaces and fuzzy second category spaces. It is established that fuzzy globally disconnected, fuzzy Baire and fuzzy P-spaces are fuzzy D-Baire spaces and fuzzy hyperconnected and fuzzy P-spaces are fuzzy D-Baire spaces. A condition for fuzzy strongly irresolvable and fuzzy globally disconnected spaces to become fuzzy P-spaces is obtained. It is established that fuzzy extremally disconnected, fuzzy strongly irresolvable and fuzzy P-spaces are fuzzy semi-P spaces and fuzzy hyperconnected and fuzzy P-spaces are not having disjoint fuzzy G_δ-sets. It is obtained that fuzzy Baire sets in fuzzy hyperconnected and fuzzy P-spaces, are fuzzy semi-open and fuzzy simply open sets.

REFERENCES

[1] K.K. AZAD: *On Fuzzy Semi Continuity, Fuzzy Almost Continuity And Fuzzy Weakly Contiunity*,J. Math. Anal. Appl, **82** (1981), 14–32.

[2] G. BALASUBRAMANIAN: *Maximal fuzzy topologies*, Kybernetika., **31**(15) (1995), 459–464.

[3] C.L. CHANG: *Fuzzy Topological Spaces*, J.Math. Anal, **24** (1968), 182–190.

[4] L.W. COHEN, C. GOFFMAN: *A theory of transfinite convergence*, Trans. Amer. Math. Soc., **66** (1949), 65–74.

[5] B. GHOSH: *Fuzzy extremally disconnected spaces, Fuzzy Sets and Sys*, Fuzzy Sets and Sys. **46** (1992), 245–250.

[6] L. GILLMAN, M. HENRIKSEN: *Concerning rings of continuous functions*, Trans. Amer. Math. Soc. **77** (1954), 340 –362.

[7] R. LEVY: *Almost P-spaces*, Canand. J. Math. **XXIX**(5) (1977), 284–288.

[8] M. CALDAS, G. NAVALAGI, R. SARAF: *On fuzzy weakly semi-open functions*, Proyecciones, Universidad Catolica del Norte, Antofagasta–Chile. **21**(1) (2002), 51–63.

[9] A.K. MISHRA: *A topological view of P-spaces*, Gen. Topology Appl. **2**(4) (1972), 349–362.

[10] R.SIKORSKI: *Remarks on spaces of high power*, Fund. Math. **37** (1950), 125–136.

[11] G. THANGARAJ, G. BALASUBRAMANIAN: *On Fuzzy Basically Disconnected Spaces*, J.Fuzzy Math. **9**(1) (2001), 103–110.

[12] G. THANGARAJ, G. BALASUBRAMANIAN: *On Somewhat Fuzzy Continuous Functions*, J.Fuzzy Math. **11**(2) (2003), 725–736.

[13] G. THANGARAJ: *Resolvability and irresolvability in fuzzy topological spaces*, News Bull. Cal. Math. Soc. **31**(4–6) (2008), 11–14.

[14] G. THANGARAJ, G. BALASUBRAMANIAN: *On Fuzzy Resolvable and Fuzzy Irresolvable Spaces*, Fuzzy Sets, Rough Sets and Multivalued Operations and Appl. **1**(2) (2009), 173–180.

[15] G. THANGARAJ, S. ANJALMOSE: *On fuzzy Baire spaces*, J. Math. **21**(3) (2013), 667–676.

[16] G. THANGARAJ, S. ANJALMOSE: *On fuzzy D-Baire spaces*, Ann. Fuzzy. Math. Inform. **7**(1) (2014), 99–108.

[17] G. THANGARAJ, C. ANBAZHAGAN: *On Fuzzy Almost P-spaces*, Inter. J. Innov. Sci. Engin. & Tech. **2**(4) (2015), 389 –407.

[18] G. THANGARAJ, C. ANBAZHAGAN: *On fuzzy GID-Spaces*, Ann. Fuzzy. Math. Inform. **10**(4) (2015), 571–581.

[19] G. THANGARAJ, C. ANBAZHAGAN: *Some Remarks on fuzzy P-spaces*, Gen. Math. Notes, **26**(1) (2015), 8 –16.

[20] G. THANGARAJ, R. PALANI: *On fuzzy weakly Baire Spaces*, Bull. Inter. Math. Virtual Institute. **7** (2017), 479–489.

[21] G. THANGARAJ, R. PALANI: *On Fuzzy Baire Sets*, Jour. Mgt. Sci. & Humanities. **4**(2) (2017), 151–158.

[22] G. THANGARAJ, K. DINAKARAN: *On Fuzzy Simply Continuous Functions*, J. Fuzzy. Math. **25**(1) (2017), 99–124.

[23] G. THANGARAJ, S. SENTHIL: *Some Remarks on Somewhere Fuzzy Continuous Functions*, (communicated).

[24] G. THANGARAJ, S. MURUGANANTHAM: *On Fuzzy Globally Disconnected Spaces*, Jour. Tri. Math. Soc. **21** (2019), 37–46.

[25] G. THANGARAJ, S. MURUGANANTHAM: *On Fuzzy perfectly Disconnected Spaces*, Inter. J. Adv. Math. **17**(5) (2017), 12–21.

[26] G. THANGARAJ, S. MURUGANANTHAM: *A Note on Fuzzy perfectly Disconnected Spaces*, Adv. Fuzzy Math. **13**(1) (2018), 59–70.

[27] G. THANGARAJ, S. MURUGANANTHAM: *On fuzzy F- Spaces*, (communicated).

[28] G. THANGARAJ, S. LOGESHWARI: *Fuzzy Strongly Irresolvable Spaces and Fuzzy Simply Open Sets*, Adv. Fuzzy Sets & Sys. **27**(1) (2022), 1–34.

[29] G. THANGARAJ, A. VINOTHKUMAR: *On Fuzzy Moscow Spaces*, (accepted in Annl. Fuzzy Math. Inform).

[30] A.I. VEKSLER: P'-points, P'-sets, P'-spaces *A new class of order–continuous measure and functionals*, Soviet Math. Dokl. **4**(5) (1973) 1445–1450.

[31] L.A. ZADEH: *Fuzzy Sets*, Inform. and Control. **8** (1965), 338–353.

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