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PROPERLY HEREDITARY PROPERTIES FOR SPECIAL BITOPOLOGICAL

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ABSTRACT. In this paper we discuss some pairwise properly hereditary properties concerning pairwise k- space, pairwise Baire space, and pairwise w-compact.

1. INTRODUCTION

The concept of bitopological spaces was initiated by Kelly [5]. Abitopological space is an order triple (X, τ_1, τ_2) where X is a non-empty set and (τ_1, τ_2) are two topologies on X. Since then several mathematicians studied various properties in bitopological spaces and bitopological spaces turned to be an important field in general topology, Fletcher, P., Hoyle, H. B. and Patty, C. W. (1969) [3], Kim, Y. W. (1968) [7], Fora, A. and Hdeib, H. (1983) [4], Kilicman, A. and Salleh, Z. (2007) [6]. Several results were obtained in the above studies that generalize topological properties in bitoplogical spaces. Still pairwise properly hereditary properties are not investigated. In this paper we discuss some pairwise properly hereditary properties and try to obtain various results concerning their properties.

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2. Properly Hereditary Properties For Special Bitopological

Definition 2.1. *[2] A topological property P is called properly (respectively, closed, open,F_σ, G_δ, etc.) hereditary property, if the following statement holds: if every proper (respectively, closed, open,F_σ, G_δ, etc.) subspace has the property P, then the whole space has the property P.*

Definition 2.2. *[9] A bitoplogical space (X, τ_1, τ_2) is said to be pairwise T_1 if for any pair of distinct points x and y in X , there exist τ_1-open set U and τ_2-open set V such that $x \in U, y \notin U$ and $x \notin V, y \in V$.*

Definition 2.3. *[5] A bitoplogical space (X, τ_1, τ_2) is said to be pairwise T_2 if for any pair of distinct points x and y in X, there exist τ_1-open set U and τ_2-open set V such that $x \in U, y \in V$ and $U \cap V = \emptyset$.*

Theorem 2.1. *[1] Let (X, τ_1, τ_2) be a bitoplogical space with more than two points. Then if every proper subspace of X is pairwise T_1, then X is pairwise T_1.*

Proof. Let $x \neq y$ in X and let $z \in X/\{x, y\}$. Let $A = X/\{z\}$. Since $(A, \tau_{1A}, \tau_{2A})$ is pairwise T_1, then there exist τ_{1A}-open set U_1 and τ_{1A}-open set V_1 such that $x \in U_1, y \notin U_1$ and $x \notin V_1, y \in V_1$, and hence there are τ_1-open set U and τ_2-open set V in X such that $U_1 = U \cap A$ and $V_1 = A \cap V$, $x \in U$, $y \notin U$ and $x \notin V, y \in V$. □

Theorem 2.2. *[1] Let (X, τ_1, τ_2) be a bitoplogical space with more than two points. Then if every proper subspace of X is pairwise T_2, then X is pairwise T_2.*

Proof. Let $x \neq y$ in X and let $z \in X/\{x, y\}$. Let $A = X/\{z\}$. Since (A, τ_{1A}, τ_{2A}) is pairwise T_2, then there exist τ_{1A}-open set U_1 and τ_{1A}-open set V_1 such that $x \in U_1, y \notin U_1$ and $x \notin V_1, y \in V_1$, and $U_1 \cap V_1 = \emptyset$. Thus there are τ_1-open set U and τ_2-open set V in X such that $U_1 = U \cap A$ and $V_1 = A \cap V, x \in U, y \notin U$ and $x \notin V, y \in V$ and $U \cap V = \emptyset$. □

Definition 2.4. *[8] Let (X, τ_1, τ_2) be a bitoplogical space then:*

(1) *G is called pairwise open if and only if G is τ_1-open and τ_2-open in X.*
(2) *F is called pairwise closed if and only if F is τ_1-open and τ_2-open in X.*

Definition 2.5. *[3] A cover $U = U_\alpha : \alpha \in \Delta$ is said to be pairwise open cover of X if and only if $U \subset \tau_1 \cap \tau_2$ and for each $i \in \{1, 2\}, U \cap \tau_i$ contains at least a non empty set.*

Definition 2.6. *[3] A bitoplogical space (X, τ_1, τ_2) is said to be pairwise compact if for every pairwise open cover of the space X has a finite subcover.*

Definition 2.7. *A subset U of bitopological space (X, τ_1,τ_2) is called pairwise k- open if for every pairwise compact subset K of$X, U \cap K$ is τ_1- open (τ_2 - open) in K.*

Definition 2.8. *A bitopological space (X, τ_1,τ_2) is said to be pairwise k- space if every τ_1- open (τ_2 - open) subset of X is τ_1- open (τ_2 - open) in X.*

Theorem 2.3. *Pairwise k-space is properly hereditary property.*

Proof. Let (X, τ_1,τ_2) be a bitopological space. Since proper subspace of X is pairwise T_2 so X is pairwise T_2. Let U be a nonempty pairwise proper subset of X such that $U \cap K$ is pairwise open in K. For each pairwise compact subset K of X. Let $x|inX/U$ and $A = X/\{x\}$, Then $U \cap K_A$ is pairwise open set in K_A for each pairwise compact subset K_A of A, so U is pairwise open set in A, therefore, U is pairwise open set in X. □

Definition 2.9. *A subset E of a bitopological space (X, τ_1, τ_2) is called pairwise dense if it $cl\tau_i(E) = X$for each i =1, 2.*

Definition 2.10. *A bitopological space X is a pairwise Baire space if and only if the intersection of each pairwise countable family of pairwise dense pairwise open sets in X is pairwise dense.*

Theorem 2.4. *If every proper subspace of X is a pairwise Baire subspace, the. X is a pairwise Baire space.*

Proof. Let $(F_m : m \in M)$ be a pairwise countable family of pairwise dense pairwise open set in X. Let $x \in X/\cap_{m \in MF_m}$, then $F_m/\{x\}$ is a pairwise dense pairwise open set in $A = X/\{x\}$ for all $m \in M$. Because if not for some $m_o \in M$, then there exist an pairwise open set V in A such that $V \cap (F_{mo}/\{x\}) = \emptyset$, so $V \cup \{x\}$ is pairwise open in X and hence $(V \cup \{x\}) \cap F_{mo} = \{x\}$, is pairwise open in X, which is contradiction. So, $\cap m \in MF_m/\{x\}$ is a pairwise dense subset of A. Let U be an pairwise open set in X, since $(\cap m \in MF_m/\{x\}) \cap (U/\{x\}) \neq \emptyset$, then $(\cap m \in MF_m) \cap U \neq \emptyset$. Therefore, $\cap m \in MF_m$ is a pairwise dense subset of X. □

Definition 2.11. *A bitoplogical space (X, τ_1, τ_2) is said to be pairwise w-compact if for every pairwise open cover of the space X has a finite subcover $\{U_{\alpha 0}, U_{\alpha 1}, U_{\alpha 2}, \cdots, U_{\alpha n}\}$ such that $cl\tau_i(U_{\alpha 0} \cup U_{\alpha 1} \cup U_{\alpha 2}, \cdots, U_{\alpha n}) = X$ for each i =1, 2.*

Definition 2.12. *Let (X, τ_1, τ_2) be a bitopological space. If f is a real valued function on X that is τ_1-lsc and τ_2-usc then $\{x \in X : f(x) \leq 0\}$ is a τ_1 zero-set with respect to τ_2, and $\{x \in X : f(x) \geq 0\}$ is a τ_2 zero-set with respect to τ_1. A subset is called a τ_i cozero-set if its complement is a τ_i zero-set, i=1, 2.*

Theorem 2.5. *The pairwise w-compact is a closed hereditary property.*

Proof. Let $U = \{U_\alpha : \alpha \in \Delta\}$ be a pairwise open cover of a bitoplogical space (X, τ_1, τ_2). Pick $U_{\alpha 0} \in U$ with $U_{\alpha 0} \neq \emptyset$. Let $A = X/U_{\alpha 0}$, therefore (A, τ_{1A}, τ_{2A}) is pairwise c w-compact. Then $U = \{U_\alpha \cap A : \alpha \in \Delta\}$ is a pairwise open cover of A, hence it has finite subcover $\{U_{\alpha 1} \cap A, U_{\alpha 2} \cap A, \ldots, U_{\alpha n} \cap A\}$ such that $cl\tau_i((U_{\alpha 1} \cap A) \cup (U_{\alpha 2} \cap A), \cdots, (U_{\alpha n} \cap A)) = A$ for each i =1, 2. Let $x \in X$ and B any pairwise cozero-set neighbourhood of x in X, if $U_{\alpha 0} \cap B \neq \emptyset$, then $cl\tau_i((U_{\alpha 0} \cap A) \cup (U_{\alpha 1} \cap A) \cup (U_{\alpha 2} \cap A), \cdots, \cup(U_{\alpha n} \cap A)) = X$ for each i =1,2. If $U_{\alpha 0} \cup B = \emptyset$, then B is pairwise cozero-set neighbourhood of x in A, so $((U_{\alpha 0} \cap A) \cup (U_{\alpha 1} \cap A) \cup (U_{\alpha 2} \cap A), \cdots, \cup(U_{\alpha n} \cap A) \neq \emptyset)$ and $x \in cl\tau_i(U_{\alpha 0} \cup U_{\alpha 1} \cup U_{\alpha 2}, \ldots, \cup U_{\alpha n})$, then $cl\tau_i(U_{\alpha 0} \cup U_{\alpha 1} \cup U_{\alpha 2}, \cdots, \cup U_{\alpha n}) = X$ for each $i = 1, 2$. □

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