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ESTIMATION OF TAIL-RELATED RISK PREMIUMS FOR HEAVY TAILED LOSSES

Helal Nacera[1], Mami Tawfiq Fawzi, and Ouadjed Hakim

ABSTRACT. This paper addresses the problem of estimating the distortion risk premiums for heavy-tailed losses by different methods, The finite sample performance of our estimators is assessed on a simulation study and we showcase our techniques on two sets of real data.

1. INTRODUCTION

Insurance is a fundamental financial product that is particularly useful to hedge against risk of losses. The insured transfers the risk of losses to the insurer, usually insurance companies, by paying a certain amount of premium. After pooling risks in a large scale, the insurer reduces the average risk over a large number of the insured by diversification or other hedge methods. One possible option for the insurer (cedant) is to buy reinsurance to further transfer their risk to the reinsurer. Consequently both of the reinsurer and the insurer allocate a portion of the risk of losses as well as the premiums. With no doubt, the determination of the amount of premium is one of most crucial topics for both insurers and reinsurers. If the

[1]*corresponding author*

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premium is too high, the insurance companies lose their market. If the amount of premium is too small, the insurers or reinsurers expose themselves to risk of significant financial loss. Greater variability and a heavier right-tail necessitate a higher price. The strategy for obtaining the price gives rise to a risk measure, which is defined as a mapping from the set of all loss random variables to the non-negative real numbers.

The use of risk measures in actuarial science was in the development of the principle of calculating premiums. According to [1] a risk measure $\mathcal{R}(.)$ is said to be coherent when it satisfies the following four coherence properties:

- *(P1)-Positive homogeneity:*

$$\mathcal{R}(\zeta X) = \zeta \mathcal{R}(X),\ \zeta > 0.$$

  This property means that risk measure should be independent of the monetary units in which the risk is measured.

- *(P2)-Translation invariance:*

$$\mathcal{R}(X + \delta) = \mathcal{R}(X) + \delta,\ \delta \in \mathbf{R}.$$

  This property means the risk measure of combining a random loss and a fixed loss should be the risk measure of the random loss plus the fixed loss. The reserve to cover a fixed loss should be just the fixed loss.

- *(P3)-Sub-additivity:* For any random loss variables X, Y:

$$\mathcal{R}(X + Y) \leq \mathcal{R}(X) + \mathcal{R}(Y).$$

  This property means that diversification benefits exist if different risks are combined.

- *(P4)-Monotony:* For any random loss variables X, Y, with $X \leq Y$ in probability

$$\mathcal{R}(X) \leq \mathcal{R}(Y).$$

  This property means that higher losses essentially leads to a higher level of risk.

In order to quantify the risk premium properly, a variety of premium principles is developed. The VaR_{1-p} is the traditional premium principle widely used by financial institutions which is the quantile risk measure at the confidence level $0 < p < 1$, is then simply

$$VaR_{1-p} = Q(1-p) = F^{-1}(1-p).$$

It is not coherent in the sense of [1].

Another important risk measure is the CTE which is coherent and given by

$$CTE_{1-p} = E(X|X > VaR_{1-p}).$$

In this paper, we focus on a recent coherent principle proposed by Li Zhu and Haijun Li in [11] based on proportional hazards transform (see [23], [4], [21], [9], [16]). We call the premium computed according to this principle as proportional hazards premium, or in short PH-premium, as $p \to 0$ given by

$$\Pi_{1-p} = VaR_{1-p} + \frac{\int_{VaR_{1-p}}^{\infty} (1 - F(x))^{\rho} dx}{p^{\rho}}. \tag{1.1}$$

The motivation for using π is similar to that discussed in [23]: to obtain a risk-loaded premium.

The premium calculation principle given in (1.1) belongs to the family of distortion risk measure which has been studied by many authors such that [22] and [6], [13], [14], [15].

We assume that the tail $1-F(x)$ has regular variation function near infinity with index $-\alpha$, that is, for all $x > 0$,

$$\lim_{t\to\infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, \tag{1.2}$$

where $\alpha > 1$ is the tail index. It follows that the survival function can be expressed as

$$S(x) = 1 - F(x) = x^{-\alpha}\mathcal{L}(x), \ x > 0, \tag{1.3}$$

where $\mathcal{L}(x)$ is a slowly varying function at infinity:

$$\lim_{t\to\infty} \frac{\mathcal{L}(tx)}{\mathcal{L}(t)} = 1. \tag{1.4}$$

The assessment of risk for heavy-tailed distributions is a crucial question in various fields of application in finance and insurance. Many distributions are heavy tailed, including: Cauchy, Fréchet, LogNormal, Pareto.

Several estimators of α have been proposed. One of the famous estimators was introduced by [10] and defined by

$$\widehat{\alpha}^H = \left(\frac{1}{k} \sum_{i=1}^{k} \log X_{n-i,n} - \log X_{n-k+1,n} \right)^{-1}, \tag{1.5}$$

where $X_{1,n} \leq X_{2,n} \leq \ldots \leq X_{n,n}$ are the order statistics and $k = k_n$ is an intermediate sequence such that

(1.6) $$k \to \infty \;\; k/n \to 0, \;\; n \to \infty.$$

In [25] authors proposed the following semi-parametric estimator of a high quantile

(1.7) $$\widehat{VaR}^{H}_{1-p} = \widehat{Q}^{H}(1-p) = X_{n-k,n}\left(\frac{np}{k}\right)^{-1/\widehat{\alpha}^{H}}.$$

2. Peak over threshold (POT) method

The method of excess beyond a threshold (or Peak Over Threshold, POT) is based on the behavior of the values observed above a given threshold. In other words, it consists in observing not the maximum or the greatest values but all the values of the realizations which exceed a certain high threshold. The basic idea of this approach is to choose a sufficiently high threshold and to study the excesses beyond this threshold. This method initially developed by [18] and extensively studied by various authors such as [20], [5], [19]. For this method we will discuss about the parametric approach based on the generalized Pareto distribution (GPD) which is an important distribution in extreme value modeling and given by

$$G_{\xi,\sigma}(x) = \begin{cases} 1 - \left(1 + \dfrac{\xi x}{\sigma}\right)^{-1/\xi} & \text{if} \quad \xi \neq 0, \\ 1 - \exp\left(-\dfrac{x}{\sigma}\right) & \text{if} \quad \xi = 0, \end{cases}$$

where $\xi = \dfrac{1}{\alpha}$ (called the extreme value index) and

$$\begin{cases} x \geq 0 & \text{if} \quad \xi \geq 0, \\ 0 \leq x < -\dfrac{\sigma}{\xi} & \text{if} \quad \xi < 0. \end{cases}$$

Let the right-end-point $x_F = \sup\{x \in \mathbf{R}, F(x) < 1\}$. For any high threshold $u < x_F$ define the excess distribution function

$$F_u(x) = \mathbf{P}[X - u \leq x | X > u], 0 \leq x < x_F - u$$
$$= \frac{F(u+x) - F(u)}{1 - F(u)}. \tag{2.1}$$

In [2] authors showed that the distribution F_u can be approximated by the generalized Pareto distribution $G_{\xi,\sigma}$. The convergence can be described by the following expression

$$\lim_{u \to u_F} \sup_{0 < x_F < x_F - u} |F_u(x) - G_{\xi,\sigma}(x)| = 0. \tag{2.2}$$

The approximation (2.2) motivates us to take an estimator for $\widehat{S}_u(x)$ as follow

$$\widehat{S}_u(x) = \widehat{G}_{\widehat{\xi},\widehat{\sigma}}(x), \ x > 0. \tag{2.3}$$

For $x > u$, the relation in (2.1) can also be written as

$$S(x) = S(u) S_u(x - u). \tag{2.4}$$

Using (2.3) and (2.4), we can get the estimator of the tail for $x > u$, as

$$\widehat{S}(x) = \frac{N}{n} \left(1 + \widehat{\xi} \frac{x - u}{\widehat{\sigma}}\right)^{-1/\widehat{\xi}}, \tag{2.5}$$

where $N = \sum_{i=1}^{n} \mathbf{1}_{\{X_i > u\}}$.

We invert the formula (2.5) to obtain the estimator of high quantile of F as

$$\widehat{VaR}_{1-p}^{POT} = u + \frac{\widehat{\sigma}}{\widehat{\xi}} \left(\left(\frac{np}{N} \right)^{-\widehat{\xi}} - 1 \right). \tag{2.6}$$

The parameters of the GPD can be estimated in various ways. Maximum likelihood is the most popular.

3. Estimation of Π_{1-p}

From [12] we have the empirical estimator given by

$$\widehat{\Pi}_{1-p}^{emp} = \frac{1}{p^{\rho}} \sum_{i=[n(1-p)]}^{n} \left[\left(\frac{n-i+1}{n} \right)^{\rho} - \left(\frac{n-i}{n} \right)^{\rho} \right] X_{i,n} \tag{3.1}$$

and the semi parametric estimator using Hill estimator based on extreme theory approach given by

$$\widehat{\Pi}^{H}_{1-p} = \frac{\widehat{\alpha}^{H}\rho}{\widehat{\alpha}^{H}\rho - 1}\widehat{VaR}^{H}_{1-p} = \frac{\widehat{\alpha}^{H}\rho}{\widehat{\alpha}^{H}\rho - 1}X_{n-k,n}\left(\frac{np}{k}\right)^{-1/\widehat{\alpha}^{H}}. \tag{3.2}$$

The parameter ρ is called also the risk-aversion index. It controls the amount of risk loading in the Π.

For $\rho = 1$ we find the estimator of CTE as

$$\widehat{CTE}^{H}_{1-p} = \frac{\widehat{\alpha}^{H}}{\widehat{\alpha}^{H} - 1}X_{n-k,n}\left(\frac{np}{k}\right)^{-1/\widehat{\alpha}^{H}}. \tag{3.3}$$

An estimator of Π_{1-p} using the POT method is given by

$$\widehat{\Pi}^{POT}_{1-p} = \widehat{VaR}^{POT}_{1-p} - \frac{\int_0^p s^{\rho} d\widehat{VaR}^{POT}(1-s)}{p^{\rho}}.$$

After integration we find the following estimator

$$\widehat{\Pi}^{POT}_{1-p} = \frac{\rho\widehat{VaR}^{POT}_{1-p}}{\rho - \widehat{\xi}} + \frac{\widehat{\sigma} - \widehat{\xi}u}{\rho - \widehat{\xi}}.$$

If we fix the number of data in the tail to be $N = k$. This effectively gives us a random threshold as $u = X_{n-k,n}$, then for $p < k/n$, we have

$$\widehat{\Pi}^{POT}_{1-p} = \frac{\rho\widehat{VaR}^{POT}_{1-p}}{\rho - \widehat{\xi}} + \frac{\widehat{\sigma} - \widehat{\xi}X_{n-k,n}}{\rho - \widehat{\xi}}. \tag{3.4}$$

4. Simulation study

For selecting the optimal sample fraction k_{opt} for the Hill estimator, we use an implementation of the heuristic algorithm proposed in Caeiro and Gomes [3]. Consider now the stationary solution of the MA(1) equation

$$X_t = \lambda Z_{t-1} + Z_t,\ 1 \leq t \leq n, \tag{4.1}$$

where $0 < \lambda < 1$ and $\{Z_t\}$ i.i.d. innovations such that

$$F_Z(x) = (1 - x^{-\alpha})\mathbf{1}_{\{x \geq 1\}},\ 1 < \alpha < 2.$$

To illustrate the performance of our estimator with $p = 0.05$, we fix the distortion parameter $\rho = 0.96$ and $\rho = 0.98$, then we generate 100 replications of the time series $(X_1, \ldots, X_n)$ for different sample sizes $(18000, 19000)$, where X_t is an MA(1) process satisfying (4.1), where $\lambda = 0.5$, and we use two tail indices $\alpha = 1.7$ and $\alpha = 1.8$. The simulation results are presented in the Table 1, where (abias) is the absolute bias and (RMSE) is the root mean squared error. We remark that:

(1) The Π decreases when ρ increases, because

$$\frac{\partial \Pi_{1-p}}{\partial \rho} = -\frac{\alpha}{(\alpha\rho - 1)^2} VaR_{1-p} < 0. \tag{4.2}$$

(2) The abias and RMSE of our estimator decrease when the sample size increases, which indicates that the estimator is consistent.

(3) For the same ρ and different values of α the $DTVaR$ increase when α decrease, this is caused by the tail of the distribution that becomes heavier.

TABLE 1. Performance of $\widehat{\Pi}_{0.95}$

α	1.7		1.8	
ρ	0.96	0.98	0.96	0.98
	$n = 18000$			
$\Pi_{0.95}$	17.61432	17.06332	14.42502	14.03167
$\widehat{\Pi}^H_{0.95}$	17.33421	17.0026	14.6725	14.41983
abias	0.2801044	0.0607203	0.2474728	0.3881551
RMSE	0.718439	0.6770245	0.5417785	0.6266744
	$n = 19000$			
$\Pi_{0.95}$	17.61432	17.06332	14.42502	14.03167
$\widehat{\Pi}^H_{0.95}$	17.40061	17.05756	14.5617	14.3415
abias	0.2137127	0.005764552	0.1366807	0.3098274
RMSE	0.6927158	0.6717101	0.5283651	0.5083966

5. Application for automobile claims data

Let X_t=" Secura Belgian Re dataset divided by 10^6 " contains 371 automobile claims from 1988 till 2001 gathered from several European insurance companies, which are at least as large as 1,200,000 Euro. The data are available in the package "ReIns" of the statistical software R. The time plot of all the claim values is given in the left of Figure 6.

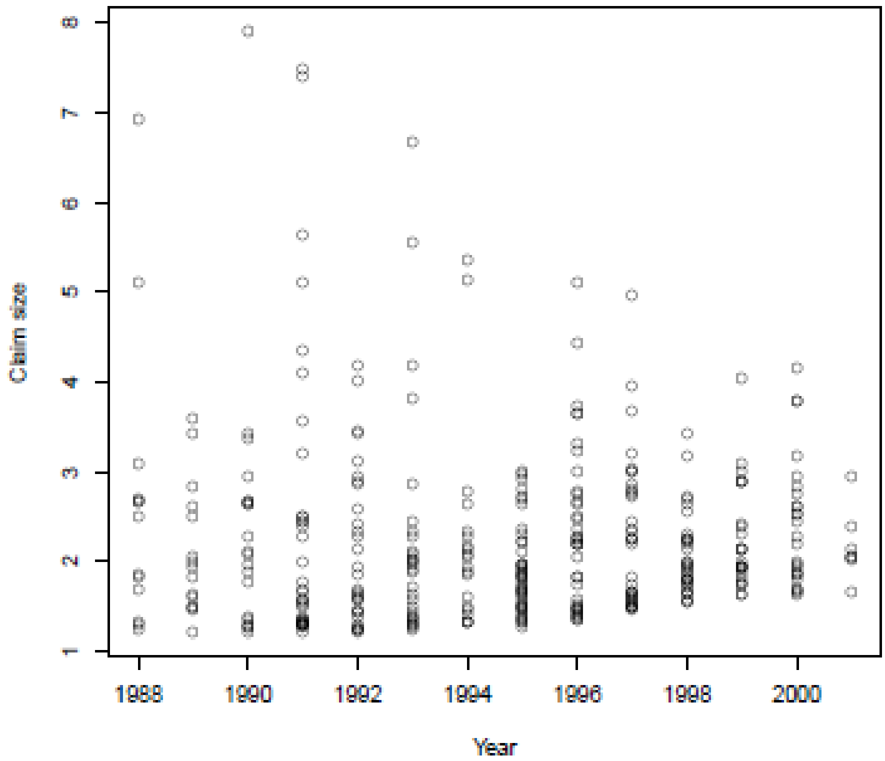

Figure 1. Time plot for the Secura data.

The distribution of the claims are displayed in left of the Figure 8, it is clear that this distribution is highly skewed to the right even. The normal Q-Q plot in the right of the Figure 8 of the claims shows the divergence from a normal distribution at the right tail. On the other hand, the $p-value$ of the Shapiro test is $< 2.2\times10^{-16}$, thus confirming the rejection of the assumption that the claims would normally be distributed.

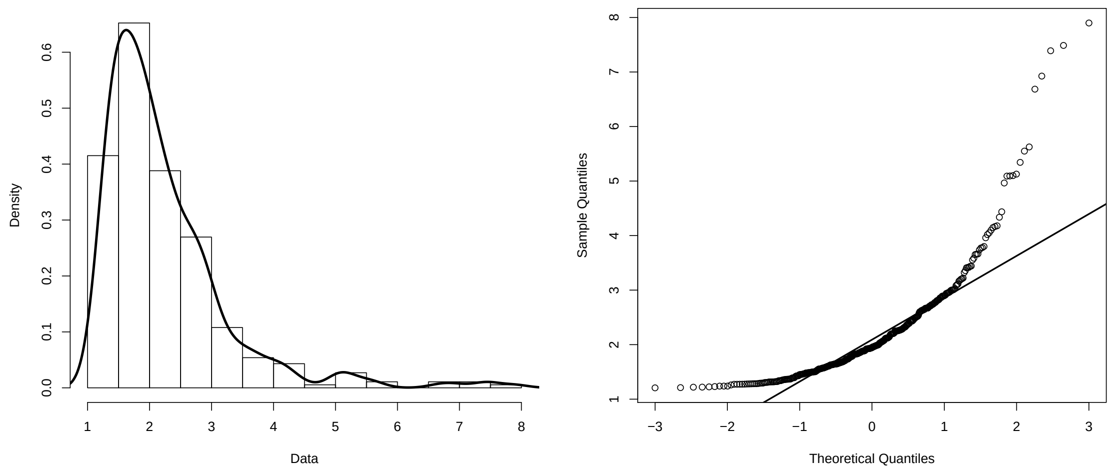

FIGURE 2. Distribution of the claims (left) and normal Q-Q plot (right).

The Hill plot in Figure 5 shows that the data are heavy tailed. We see that there is a region of stability between $k = 70$ and $k = 80$.

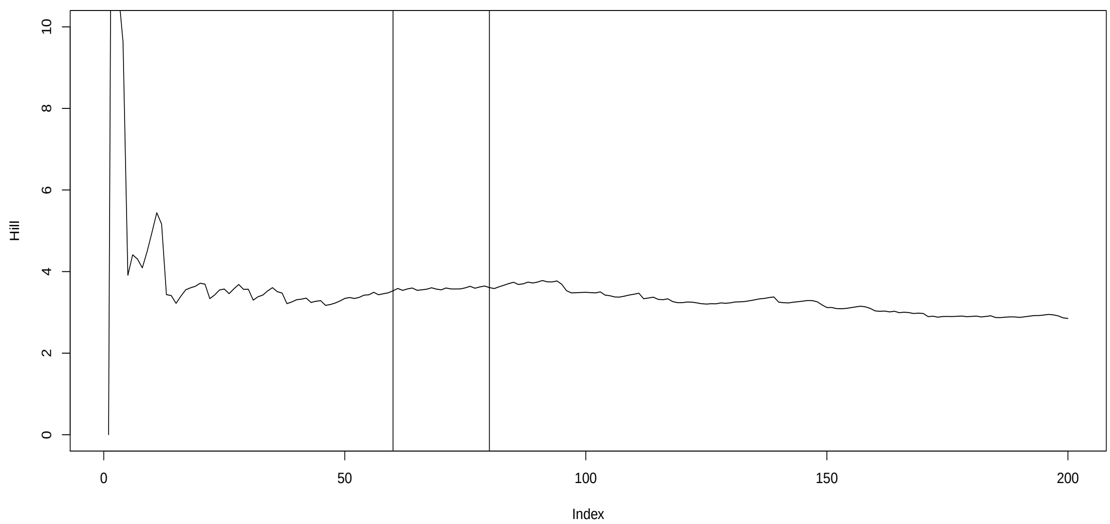

FIGURE 3. Hill plot of the data.

Using the algorithm based on sample path stability proposed in [3], we find an optimal value $k_{opt} = 74$, then we have $\widehat{\alpha}^{H,kopt} = 3.57551$.

To evaluate how well the estimated tail index fit the data, we use equation (1.2) to estimate the underlying distribution function F. Let $\widehat{\alpha}^{H,kopt}$ be the estimator of the tail index α and $u > 0$ be a high threshold. An estimate of $F(x)$, can be defined as

$$F_n(x) = 1 - (1 - F_n(u)) \left(\frac{x}{u}\right)^{-\widehat{\alpha}^{H,kopt}}, \; x > u.$$

Now we choose $k = 74, \; u = X_{n-k,n} = 2.736901$, we can estimate $F(x)$ by

$$F_{n1}(x) = 1 - \left(1 - \frac{296}{371}\right) \left(\frac{x}{2.736901}\right)^{-3.57551}, \; x > 2.736901.$$

We plot the two distribution functions $F_n(x)$: the empirical distribution for Secura data and the estimated distribution $F_{n1}(x)$ for $x > 2.7369018$ in Figure 7, and conclude that F_{n1} fit the empirical distribution F_n equally well.

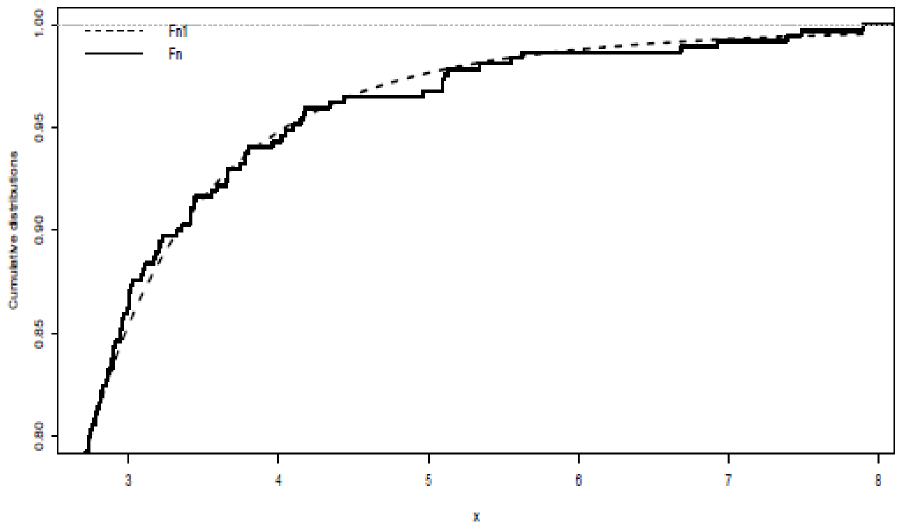

FIGURE 4. Plot F_{n1} and F_n.

’ In Table 1-2 we list some estimations of high Π. It can be seen that the estimated Π using the POT method is closer to the empirical Π than estimated using the Hill estimator, because the GPD law depends on two parameters (ξ and σ (scale parameter)) which provide greater estimation flexibility.

ρ	0.8	0.9
$\widehat{\Pi}_{0.90}^{emp}$	4.920752	4.77631
$\widehat{\Pi}_{0.90}^{H}$	5.104383	4.816709
$\widehat{\Pi}_{0.90}^{POT}$	5.037848	4.7745

TABLE 2. Estimation of $\Pi_{0.90}$ for Secura data.

ρ	0.8	0.9
$\widehat{\Pi}_{0.95}^{emp}$	5.816162	5.693705
$\widehat{\Pi}_{0.95}^{H}$	6.196342	5.847128
$\widehat{\Pi}_{0.95}^{POT}$	6.072267	5.75874

TABLE 3. Estimation of $\Pi_{0.95}$ for Secura data.

6. APPLICATION FOR NORWEGIAN FIRE INSURANCE DATA

Let X ="Norwegian fire insurance data" concerning a Norwegian fire insurance portfolio from 1972 to 1992, with $n = 9181$ occurrence of the claims in thousands of Norwegian Kroner. The data are available in the package "ReIns" of the statistical software R. The time plot of all the claim values is given in the left of Figure 5.

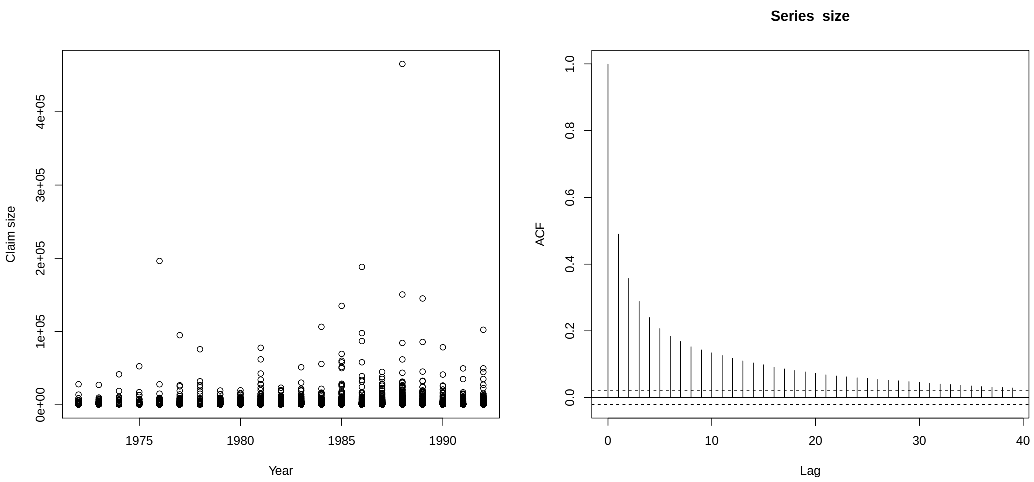

FIGURE 5. Time plot for the Norwegian fire insurance data (left) and the ACF (right).

From the autocorrelation function (ACF) in the right of Figure 5, it seems reasonable to assume that these data are correlated. In addition the p-value of the Box-Pierce test is $< 2.2 \times 10^{-16}$, showing that these data are correlated. To verify the stationarity of the data we perform a Phillips-Perron test. We get a $p - value$ of 0.01, so the data are stationary.

The distribution of the claims are displayed in left of the Figure 6, it is clear that this distribution is highly skewed to the right even on a log scale. The normal Q-Q plot in the right of the Figure 6 of the claims shows the divergence from a normal distribution at the right tail. On the other hand, the $p - value$ of the Kolmogorov-Smirnov test statistic is $< 2.2 \times 10^{-16}$, thus confirming the rejection of the assumption that the claims would normally be distributed.

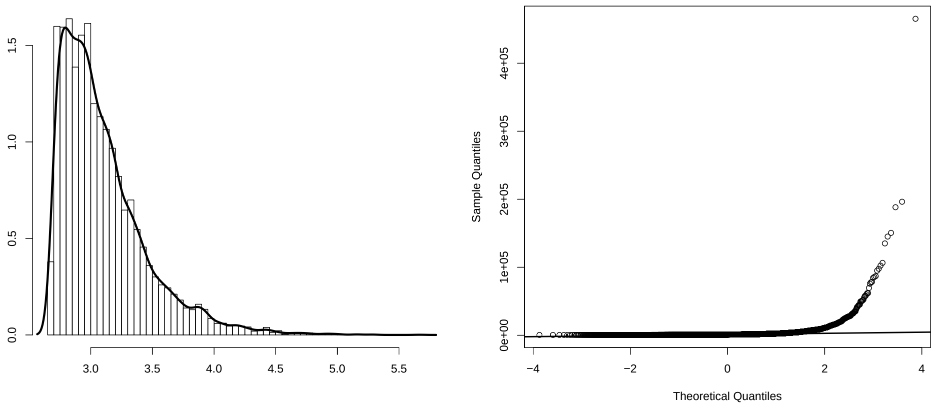

FIGURE 6. Distribution of the claims (left) and normal Q-Q plot (right).

The Hill plot in Figure 7 shows that the data are heavy tailed. For selecting the optimal sample fraction k_{opt} and estimate the tail index α we use the heuristic algorithm proposed in Caeiro and Gomes [3]. Then we find $k_{opt} = 2453$, $\widehat{\alpha} = 1.308801$.

Using the two estimators (1.7) and (3.3) for $p = 0.05$, then we obtain $\widehat{VaR}^{H}_{0.95} = 6100.69$ and $\widehat{CTE}^{H}_{0.95} = 25856.75$.

In Figure 8 we plot the percentage of risk loading $\dfrac{\widehat{\Pi}^{H}_{0.95} - \widehat{CTE}^{H}_{0.95}}{\widehat{CTE}^{H}_{0.95}}$. We remark that this percentage increases as the distortion index decreases.

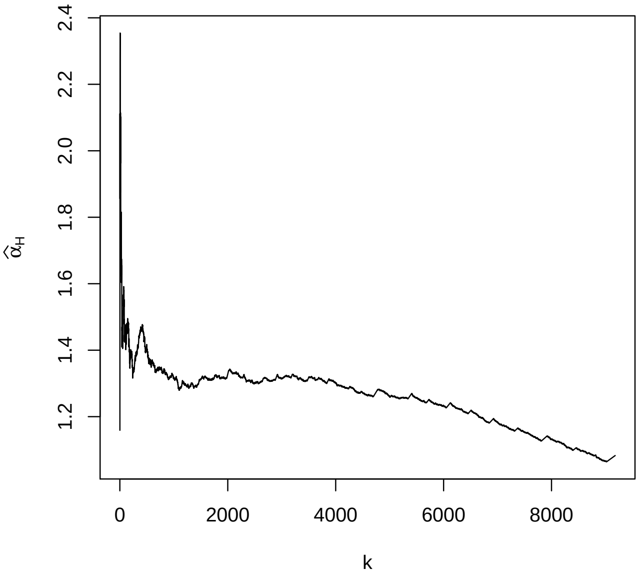

FIGURE 7. Hill plot of the data.

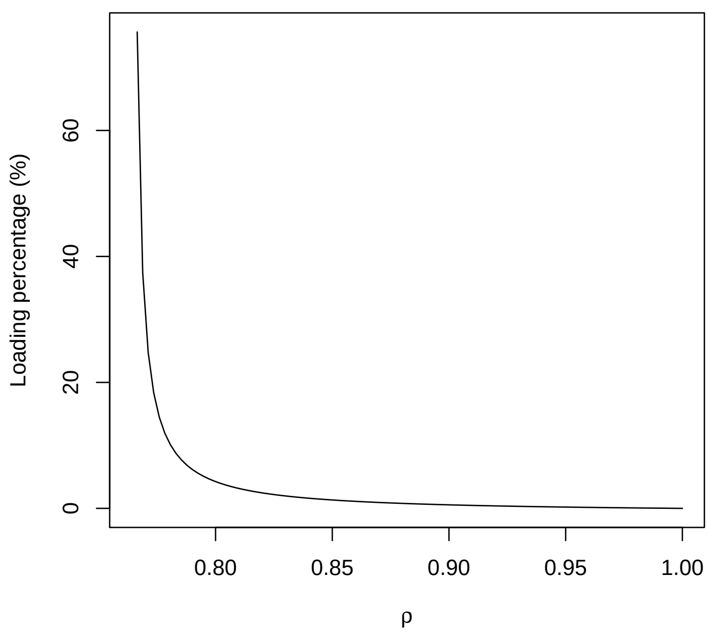

FIGURE 8. Percentage of risk loading.

Let X denote the total claim incurred for one insurance portfolio, having a regularly varying loss distribution with tail index α. The pure risk premium is the mean $E(X)$ of loss X, and the risk loading depends on the excess of a risk measurement over the mean loss. If an insurance company receives the excess $\Pi_{1-p} - E(X)$ from investors and invests the capital at the risk-free rate r_0, the company needs to pay the investors at a higher rate $r > r_0$, because their investment is exposed to risk. Thus Li Zhu and Haijun Li in [11] proposed that the premium paid by the policyholder in this simple pricing model is the sum of the pure risk premium and the risk loading:

$$\begin{aligned}\Delta_{1-p} &= E(X) + (r - r_0)(\Pi_{1-p} - E(X)) \\ &= (1 - r + r_0)E(X) + (r - r_0)\Pi_{1-p}\end{aligned}$$

Since $p \to 0$ we estimate Δ_{1-p} by

$$\widehat{\Delta}_{1-p} = (1 - r + r_0)\widehat{E(X)} + (r - r_0)\widehat{\Pi}^H_{1-p},$$

where $\widehat{E(X)} = \dfrac{1}{n}\sum_{i=1}^{n} X_i$ for $\alpha \geq 2$, and for $1 < \alpha < 2$ we use the Peng estimator(see [17])

$$\widehat{E(X)} = \int_0^{X_{n,n-k}} (1 - F_n(x))dx + \frac{k}{n}\frac{X_{n,n-k}}{\widehat{\alpha}^H - 1},$$

where $F_n(x) = \dfrac{1}{n}\sum_{i=1}^{n} I(X_i \leq x)$.

7. Conclusion

The risk measure Π which generalized the CTE tells us much more about the tail of the distribution than does VaR alone and it is a coherent risk measure. In this paper, we have presented and estimated the new risk measure Π by different approaches.

The POT method produces more efficient estimators and this approach has shown its importance and success in a number of statistical analysis problems relating to finance, insurance, hydrology, geographical phenomena, and other domains, but choosing a suitable threshold value u in the POT method is the main problem. The threshold must be chosen so as to make a trade-off, between bias and variance.

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REFERENCES

[1] P. ARTZNER, F. DELBAEN, J.-M. EBER AND D. HEATH: *Coherent measures of risk*. Mathematical finance, **9** (3) (1999), 203–228.

[2] A. A. BALKEMA, AND L. DE HAAN: *Residual life time at great age*. The Annals of probability, (1974), 792–804.

[3] F. CAEIRO AND M. I. GOMES: *Threshold selection in extreme value analysis, Extreme value modeling and risk analysis: Methods and applications* (2015), 69–82.

[4] J. DHAENE, L. HENRARD, Z. LANDSMAN, A. VANDENDORPE AND S. VANDUFFEL: *Some results on the CTE-based capital allocation rule*. Insurance: Mathematics and Economics, **42** (2) (2008), 855–863.

[5] A. C. DAVISON AND R. L. SMITH: *Models for exceedances over high thresholds*. Journal of the Royal Statistical Society: Series B (Methodological) **52** (3) (1990), 393–425.

[6] J. EL METHNI AND G. STUPFLER: *Extreme versions of Wang risk measures and their estimation for heavy-tailed distributions*. Statistica Sinica (2017), 907–930.

[7] L. DE HAAN, A. FERREIRA AND A. FERREIRA : *Extreme value theory: an introduction*, **21** Springer (2006).

[8] L. DE HAAN, C. MERCADIER AND C. ZHOU: *Adapting extreme value statistics to financial time series: dealing with bias and serial dependence* . Finance and Stochastics, **20** (2) (2016), 321–354.

[9] O. HAKIM: *OT approach for estimation of extreme risk measures of EUR/USD returns*. Statistics, Optimization & Information Computing, **6** (2) (2018), 240–247.

[10] B. HILL: *A simple general approach to inference about the tail of a distribution*. The annals of statistics, JSTOR, (1975), 1163–1174.

[11] L. ZHU AND H. LI: *Tail distortion risk and its asymptotic analysis*. Insurance: Mathematics and Economics, **51** (1) (2012), 115–121.

[12] T.F MAMI, H. OUADJED AND N. HELAL : *Extreme distorted tail value at risk for Secura insurance data*. Nonlinear Studies **28** (4) (2021).

[13] H. OUADJED AND A. YOUSFATE : *Estimation of riensurance PHT premium for AR (1) process with infinite variance*, 36 (2013), 181–190.

[14] H. OUADJED AND A. YOUSFATE: *Estimation of Reinsurance Premium for Positive Strictly Stationary Sequence with Heavy-Tailed Marginals*. Journal of Statistics Applications & Probability, **3** (1) (2014), 93.

[15] H. OUADJED: *Estimation of the distortion risk premium for heavy-tailed losses under serial dependence* . Opuscula Mathematica, **38** (6) (2018).

[16] H. OUADJED AND T.F. MAMI : *Estimating the conditional tail expectation of Walmart stock data*. Croatian Operational Research Review, (2020), 95–106.

[17] L. PENG: *Estimating the mean of a heavy tailed distribution*.Statistics & Probability Letters, **52** (3) (2001), 255–264.

[18] III.J. PICKANDS : *Statistical inference using extreme order statistics*. Annals of statistics, **3** (1) (1975), 119–131.

[19] R. REISS, M. THOMASL AND R.D. REISS: *Statistical analysis of extreme values*, Springer, 2 (1997).

[20] R.L. SMITH: *Estimating tails of probability distributions*. The annals of Statistics, **15** (3) (1987), 1174–1207.

[21] K.S. TAN, C. WENG AND Y. ZHANG : *Optimality of general reinsurance contracts under CTE risk measure*. Insurance: Mathematics and Economics, **49** (2) (2011), 175–187.

[22] B. VANDEWALLE AND J. BEIRLANT : *On univariate extreme value statistics and the estimation of reinsurance premiums*. Insurance: Mathematics and Economics, **38** (3) (2006), 441–459.

[23] S. WANG: *Insurance pricing and increased limits ratemaking by proportional hazards transforms* . Insurance: Mathematics and Economics, **17** (1) (1995), 43–54.

[24] S. WANG: *Premium calculation by transforming the layer premium density* . ASTIN Bulletin: The Journal of the IAA, **26** (1) (1996), 71–92.

[25] I. WEISSMAN: *Estimation of parameters and large quantiles based on the k largest observations* . Journal of the American Statistical Association, **73** (364) (1978), 812–815.

ECOLE NATIONALE SUPÉRIEURE D'INFORMATIQUE
SIDI BEL ABBES, ALGERIA,
BIOMATHEMATICS LABORATORY, UNIV. SIDI BEL ABBES,
P.B. 89, 22000, ALGERIA.
Email address: n.helal@esi-sba.dz

SCIENCE INSTITUTE
UNIVERSITY OF BELHADJ BOUCHAIB
AI TEMOUCHENT, ALGERIA.
Email address: mami_math_sba@yahoo.fr

FACULTY OF ECONOMICS, BUSINESS AND MANAGEMENT SCIENCES
UNIVERSITY OF MUSTAPHA STAMBOULI
MASCARA, ALGERIA.
BIOMATHEMATICS LABORATORY, UNIV. SIDI BEL ABBES,
P.B. 89, 22000, ALGERIA.
Email address: oujda.hakim@univ-mascara.dz