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# MARKET CAPITALIZATION OF THE STOCK IN BIRTH - DEATH PROCESSES

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ABSTRACT. In this article the inability to predict the future growth rates and earnings of growth stocks leads to the high volatility of share prices and difficulty in applying the traditional valuation methods. To demonstrate that the high volatility of share prices can nevertheless be used in building a model that leads to a particular cross-sectional size distribution. The model focuses on both transient and steady-state behavior of the market capitalization of the stock, which in turn is modeled as a birth-death process.

## 1. INTRODUCTION

1.1. **Growth Stocks.** Issuing stocks is arguably the most important way for growth companies to finance their projects and in turn helps transfer new ideas into products and services for the society. Although the content of growth stocks may change over time. Studying the general properties of growth stocks is essential for understanding financial markets and economic growth. However uncertainty is manifest for growth stocks. For example, as demonstrated in the recent market Capitalization from 1978 to 2020 for 42 years stock growth rate with 21 projects Performance in the firm.

(1) Growth stocks tend to have low or even negative earnings.

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(2) The volatility of growth stocks is high both their daily appreciation and depreciation rate are high.

(3) It is difficult to predict the future growth rates and earnings.

Consequently, it poses a great challenge to derive a meaningful mathematical model within the classical valuation framework. such as the net-present-value method. Which relies on current earnings and the prediction of future earnings.

Indeed, since it appears that the only thing that we are sure about growth stocks is their uncertainty, we may wonder whether there is much more to say about them. The current paper attempts to illustrate that a mathematical model for growth stocks can, nevertheless, be built via birth-death processes. mainly by utilizing the high volatility of their share prices.

One motivation of the current study comes from a report on internet stocks in the wall street journal 27 December 1999 researchers at Credit Suisse First Boston observed that there is literally a mathematical relationship between the ranking of the internet stock and its capitalization. It is suggested that a linear downward pattern emerges when the market capitalizations of internet stocks are plotted against their associated ranks on a log-log scale, with rank one being the largest market capitalization. The same article, more interestingly also reported that this phenomenon does not seem to hold for nongrowth stocks. The article challenges people to investigate whether such a phenomenon happens simple by chance or if there is certain mechanism behind it.

## 2. CROSS-SECTIONAL SIZE DISTRIBUTIONS

Studying the stochastic relationships between some values of interest and their relative ranks within a group, termed the (cross-sectional) ‘Size distribution’ has a long history in probability [13, 20, 21, 22, 4]. Staring from Simon (1955) economists began to use various stochastic processes to model cross-sectional size distributions in economics, for example the size of business firms [15, 17, 10, 16, 1]. However most of the theory developed so far focuses on the steady state size distribution and pays little attention to the transient behavior of size distributions [14, 11, 18, 3, 8, 9, 6].

A detailed analysis of the transient behavior of the size distribution, which is not well addressed in the size-distribution literature. The analysis of the transient behavior is crucial to our study as it explains why the size-distribution theory can be applied to growth stocks but not to nongrowth stocks.The theory of the size distribution may have an interesting application in studying growth stocks which is difficult for traditional methods. such as the net-present-value approach.

## 3. STEADY- STATE DISTRIBUTION

The steady-state measure of a birth-death process is given by

$$\pi_0 = 1, \quad \pi_n = \frac{\lambda_0, \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n}, n = 1, 2, \ldots.$$

Normalizing $\{\pi_n\}$ provide the steady –state distribution of the birth-death process:

$$\lim_{t \to \infty} P(X(t) = n) = \frac{\pi_n}{s}, \quad s := \sum_{n=0}^{n=\infty} \pi_n.$$

In our case,

$$\pi_n = (\frac{\lambda}{\mu})^n (\frac{g}{\lambda})(1 + \frac{g}{\lambda})(2 + \frac{g}{\lambda}) \ldots, n \geq 1.$$

Using the gamma function, it can be succinctly expressed as

$$\pi_n = \frac{\Gamma(1 + \frac{h}{\mu})}{\Gamma(\frac{g}{\lambda})} (\frac{\lambda}{\mu})^n \frac{\Gamma(n + \frac{g}{\lambda})}{\Gamma(n + 1 + \frac{h}{\mu})}, n \geq 0.$$

## 4. TRANSIENT BEHAVIOR OF THE MODEL

As mentioned above, most of the literature on size distributions focuses on the steady-state properties, and except for some numerical example demonstrated through numerical calculation that, if the convergence rate is not large enough, it may take 42 years with 21 Projects for some birth-death processes to reach steady state, the theoretical properties of the transient behavior are hardly addressed in the literature. In this sense, this section constitutes the main technical contribution of the current paper to the size –distribution literature.

The speed of convergence of a birth-death process can be measured by the decay parameter, which is defined by

$$\gamma = \sup\{\alpha \geq 0 : p_{ij}(t) - \frac{\pi_j}{s} = 0(e^{-at}) \text{ for all } i, j \geq 1\}.$$

Here, as before, $p_{ij}(t)$ is the transition probability at time $t$ and $\frac{\pi_j}{s}$ is the steady –state probability. For further background on the speed of convergence and rate of exponential ergodicity.

The decay parameter $\gamma$ affects the convergence in an exponential way. In other words, a small difference in $\gamma$ can have a remarkable effect on the speed of convergence, which in turn suggests that the steady-state analysis of the size distribution in our model based on the birth-death process is only relevant when the decay parameter is large. In addition, since the infinitesimal generator of the birth-death process is an infinite-state matrix, the analysis of the convergence rate is different from that for finite-state Markov chains.

Notations:

$X(t)$ = due to market price of the stocks

$\lambda_i$ = birth rate (appreciation)

$\mu_i$ = death rate (Depreciation)

$g$ = rate of increasing

$h$ = attempts to capture the rate of decreasing.

## 5. MATHEMATICAL MODEL

In modeling growth stocks, instead of working on the price of a growth stock, it makes more sense to study the market capitalization, defined as the product of the total number of outstanding shares and the market price of the stock, because growth stocks tend to have frequent stock splits which immediately makes the price drop significantly but has little effect on the market capitalization.

Consider at time $t$ a growth stock with a total market capitalization $M(t)$. We postulate that

$$M(t) = \Theta(t)X(t).$$

Here $\Theta(t)$ represents the overall economic and sector trend and $X(t)$ represents each individual variation within the sector. Hence, $\Theta(t)$ is the same for all firms with the same industry sector and the individual variation term $X(t)$ varies for different firms within the sector.

The individual variation term $X(t)$ is modeled as a birth –death process given that X(t) is in state I the instantaneous changes are

$i \to i + 1$ birth rate $i\lambda + g$ for $i \geq 0$, and

$i \to i-1$. death rate $i\mu + h$ for $i \geq 1$,
where the parameters are such that $\lambda, \mu > 0, g > 0, h \geq 0, \lambda < \mu$. The unit of $X(t)$ could be, for example, millions or billions of dollars.

Under the standard notation, $X(t)$ is a birth-death process with the birth rate $\lambda_i$ (upward price) and the death rate $\mu_i$ ( downward price ) satisfying

$$\lambda_i = i\lambda + g, \mu = i\mu + h, i \geq 1, \qquad \lambda_i = g, \mu_0 = 0,$$

and the infinitesimal generator of $X(t)$ is given by the infinite matrix

$$\begin{bmatrix} -g & g & 0 & 0 & \cdots \\ \mu + h & -\lambda - \mu - g - h & \lambda + g & 0 & \cdots \\ 0 & 2\mu - h & -2\lambda - 2\mu - g - h & 2\lambda + g & \cdots \end{bmatrix}$$

In this model, the state 0 only signifies that the size of $X(t)$ is below a certain minimal level. It does not imply, for example, that the company goes bankrupt.

The two parameters $\lambda$ and $\mu$ represent the instantaneous appreciation and depreciation rates of $X(t)$ due to market fluctuation, the model assumes that they influence $X(t)$ proportionally to the current value. In general because of the difficulty of predicting the instantaneous upward and downward price movements for both growth stocks and nongrowth stocks $\lambda$ and $\mu$ must be quit close $\frac{\lambda}{\mu} \approx 1$, In addition, for growth stocks, both $\lambda$ and $\mu$ must be large, because of their high volatility. The requirement that $\lambda < \mu$ is postulated here to ensure that the birth - death $X(t)$ has a steady-state distribution.

The parameter $g > 0$ models the rate of increase in $X(t)$ due to nonmarket factors. Such as the effect of additional shares being issued through public offerings or the effect of warranties on the stock being exercised. The parameter $h$ attempts to capture the rate of decrease in $X(t)$ due to nonmarket factors, such as the effect of divided payments, for most growth stocks, $h \approx 0$, as no dividends are paid.

The parameters of the value in exponential distribution is $\mu = 0.2, \lambda = 0.3$.

TABLE 1.

| X (No. of Years) | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Y (Market capitalization in millions) | 100 | 110 | 130 | 150 | 170 | 200 | 210 | 140 | 145 | 270 | 300 |
| $\mu$ (growth rate increasing level) | 0 | 1.2 | 2.2 | 3.6 | 4.8 | 6.0 | 7.2 | 8.4 | 9.6 | 10.8 | 12.0 |

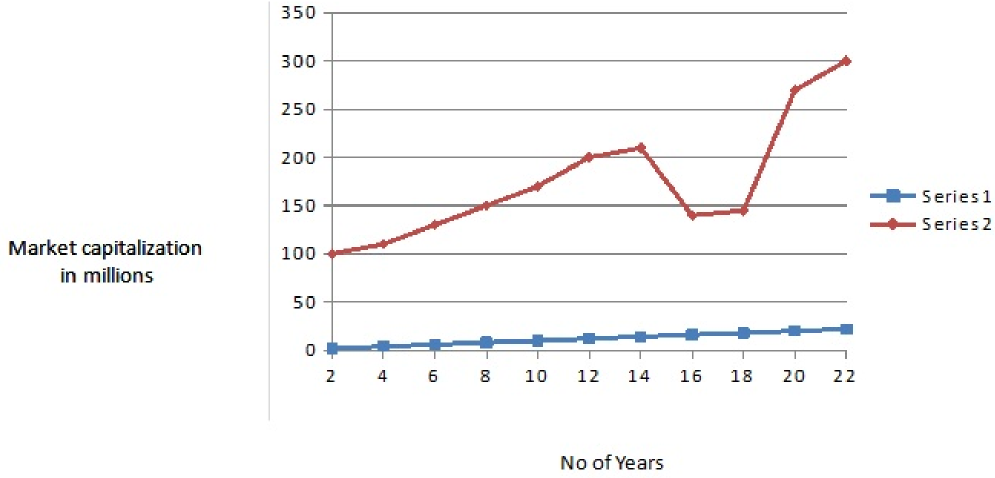


FIGURE 1.

TABLE 2.

| X (No. of Years) | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Y (Market capitalization in millions) | 300 | 260 | 240 | 230 | 210 | 200 | 170 | 150 | 120 | 70 | 50 |
| $\mu$ (growth rate decreasing level) | 4.6 | 5.9 | 7.2 | 8.5 | 9.8 | 11.1 | 12.4 | 13.7 | 15 | 16.3 | 17.6 |

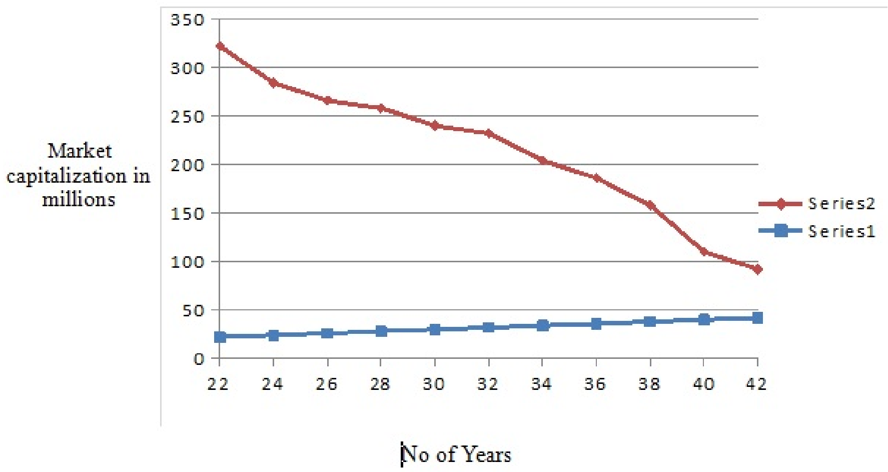


FIGURE 2.

## 6. APPLICATION

(1) Stochastic differential equation and stochastic control. Application of queuing theory in traffic engineering. Application of Markov process in communication theory engineering. Applications to risk theory, insurance, actuarial science and system risk engineering.
(2) Companies list shares of their stock on an exchange through a process called an initial public offering (IPO). Investors purchase those shares, which allows the company to raise money to grow its business. Nowadays, the stock market works electronically, through the internet and online stockbrokers.
(3) The broker passes on our buy order for shares to the stock exchange. The stock exchange searches for a sell order for the same share. Once a seller and a buyer are found and fixed, a price is agreed to finalize the transaction.

## 7. CONCLUSION

Hence, by utilizing the high volatility of growth stocks, based on both the transient and steady-state behavior of birth-death processes, a model for growth stocks, which are otherwise quite difficult to analyze using traditional valuation methods. The main contribution of the current paper is that it provides an understanding of the size distribution for growth stocks, by building a stochastic model. The model leads to a cross-sectional equation for growth stocks, including both biotechnology and internet stocks. The cross-sectional model only uses regression and relative ranks, and is, thus, easy to implement. The cross-sectional model remains valid irrespective to the market ups and downs, mainly because the model competes the relative value of a stock against the other stocks within its peer group.

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