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MEASURING THE IMPACT OF COVID-19 PANDEMIC ON THE OIL PRICES VOLATILITY BY GARCH MODELING

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ABSTRACT. The aim of this work is the study of crude oil price volatility, particularly during the Covid-19 pandemic, which changed the world scene and inflicted serious crises not only in the global health system, but also in the international financial markets and economy. This economic situation has many policy makers, financiers and portfolio managers worried about avoiding the risk of potential damage. Hedging strategies are based on the correct estimation of price volatility. For this aim, we use the $GARCH$ model to measure the impact of volatility and shocks.

More precisely, the model used for predicting volatility associated with the price variable is the $GARCH(1,2)$ model, in through this analysis. Although the $EGARCH$ and $TGARCH$ models are better for their asymmetry property of volatility, but the $GARCH(1,2)$ model was adopted because it presents lower values of the forecasting criteria compared to the two other models. The forecast is made for the last thrée months of 2021. The result concludes that the predicted values and the current values are very close. The oil price series that will be examined here is WTI (West Texas Intermediate).

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1. INTRODUCTION

The global financial crises and their impacts on international markets have attracted great attention from academic researchers, investors and policy makers. The first decade of this millennium saw at least two crises, such as the dotcom bubble of 2002 and the subprime financial crisis of 2008. This decade too was not distinct from the previous one, the COVID-19 pandemic had hit the world hard and caused serious crises, and which still surprises the whole world until now because of its significant effects, Asset price fluctuations, in world markets in particular, are one of them. To reminding, the outbreak of COVID–19, which was started in the Wuhan city of China during November 2019 has been declared as a global pandemic by the World Health Organisation (WHO) on March 11, 2020. The outbreak of COVID–19 has shaken the global financial markets, commodity markets, economic activities, employment and GDP of the countries, until they have their fully magnitudes as indicated by Maher B.K and al. [16].

The reason for taking the crude oil because it's a vital commodity and has significance in both financial markets and economic growth.

The price of crude oil fell from 9 $ per barrel in December 1998 to 145 $ in July 2008. It then fell to 32 $ in December 2008, before rising again in 2009 and reaching the end of the year a level of 80 dollars (79.36 $ in 31 December 2009). To know other fluctuations at the end of 2014, falling to the threshold of 50 $ then to 30 dollard in mid 2016,

And it continued to fall until reaching a collapse that the history of American oil prices has not known at all, on Monday (but not a black) April 20, 2020, where the price per barrel reached (-37.63), or 306%, for deliveries in May, i.e., the sellers are now offering to pay the buyers for this contract. Tuesday, April 21 in the morning, just a day later, the price of WTI for delivery in May rebounded on the Asian markets to trade around 1.1 dollar. For its part, the barrel for delivery in June rose above 21 dollars. Of course, this happened because of the Covid-19 pandemic which was at its peak during this time.

But after signs of recovery emerged from the Covid-19 pandemic, even signs of recovery in U.S. crude prices where it returned to a wild rise which exceeds 90 dollard in recent months, especially with the recent strained geopolitical relations between the major powers. This conjunction of an upward trend and high volatility is likely to continue in the coming years. Of course, experience has shown how difficult it is to predict the evolution of the price of oil.

In fact, considerable interest in the literature has gone into examining the volatility of oil prices. Thus, in the same interest, we wanted to have a contribution in this study.

We will treat this study in its natural framework, which is the time series. The price data series is denoted WTI, It is the same universally recognized name (West Texas Intermediate: one of a type of crude oils used as a standard in pricing crude oil and as a commodity for oil futures contracts on the New York Mercantile Exchange (raw materials stock Exchange). The data sample frequency is daily oil closing prices and spans from January 01, 2015 to December 31, 2021 excluding holidays (1826 obsrvations). The data can be downloaded from the website: investing.com.

$GARCH$ models are the main tool in this analysis because they are a type of economic and monetary series that are characterized by irregular fluctuations over time.

The objective of this work is to seek, in the family of $GARCH$ models, the adiquat model and generator of WTI data in order to perform, mainly, the forecast, an important step in the methodology of Box-Jencins (1979). It should be noted that to study the behavior of the WTI series, we will also need the yield series, which represent the residuals $(\varepsilon_t)_{t\in\mathbb{Z}}$ which help us to check whether the generating process of WTI is an $ARIMA$ or it is a model of the $GARCH$ family. Generally, WTI yields or residuals are often expressed as

$$\varepsilon_t = \ln \frac{WTI_t}{WTI_{t-1}} = \ln WTI_t - \ln WTI_{t-1},$$

such that, $\ln$ is the natural logarithm, WTI_t and WTI_{t-1} are the oil prices at date t and $t-1$, respectively.

The evolution of the financial markets has certain empirical characteristics, which any model seeks to reproduce. The calculation of the risks taken by the players must, in order to be as predictive as possible, take account of these characteristics, it is for example the aggregation of volatility and its asymmetry. A popular class for the among practitioners in the finance area is the $GARCH$ model proposed by Bollerslev T. [2], which is a generalization of the $ARCH$ model proposed by Engle R. [11] who quickly became one of the pioneers of financial econometrics. The model principle allows to consider an essential characteristic of the markets: volatility is not constant over time. In principle, taking this phenomenon into account increases the predictive potential of the models.

The remainder of the paper is organized as follows: Section 2 we will dedicate it to the review of the literature, section 3 is devoted to a brief theoretical presentation of $GARCH$ models and its extensions, while section 4 presents data and the empirical results. Section 5 concludes this paper.

2. LITERATURE REVIEW

Several GARCH extensions have been introduced in the finance and economic literature to improve some aspects of the GARCH model so that the models are more flexible and adequate in accommodating some characteristics and dynamics of a time series, for example, see Bollerslev T. and al. [3] and Higgins and Bera [15]. Ng and McAleer [20] used simple $GARCH(1,1)$ and $TARCH(1,1)$ models for testing estimation and forecasting the volatility of daily returns in S&P 500 Composite Index and the Nikkei 225 Index. They concluded that $TARCH(1,1)$ was the best performing model with S&P 500 data, whereas the $GARCH(1,1)$ model was better in some cases with Nikkei 225. Ramzan and al [24] modeled exchange rate dynamics in Pakistan, using the $GARCH$ family models, on the monthly data from July 1981 to May 2010. The study results showed that $GARCH(1,2)$ was better than $EGARCH(1,2)$ model. However, the $GARCH(1,2)$ model was used to remove the persistence in volatility while $EGARCH(1,2)$ successfully overcame the leverage effect in the exchange rate returns. Moreover they concluded that the $GARCH$ family of models captures the volatility and leverage effect in the exchange rate returns, giving fairly good forecasting performance for the model.

Also, there are many research works have been done in the area of the commodities market, natural gas, crude oil and price volatility. Some of the important works are mentioned here. A paper investigates the behaviour of crude oil and natural gas price volatility in the United States since 1990 using $GARCH$ model is by Pindyck [23]. Again, a study concerned with modelling of price volatility of crude oil market in Nigeria employing both symmetric and asymmetric models of $GARCH$ family appeared in Dum and Essi [8].

There was a study investigating the causal association between the stock market returns and crude oil price anomalies in the Indian stock market covering 10 companies of oil drilling and exploration sectors listed in the CNX NIFTY indexes and BSE Sensex from 2009 to 2018 (see Hawaldar and al [14]). Furthermore, the study focussed on comparing between realized $GARCH$ model with some conventional $GARCH$ models such as $GARCH$, $GJR-GARCH$ and $EGARCH$ by using the Gold 5 min intra-day data from April 2012 to April 2018. Nugroho and al. [21] used the $GARCH(1,1)$ for modeling of the volatility of returns,.This study proposed two new classes of $GARCH(1,1)$ model by applying the Tukey transformations to the returns and to the lagged variance and it recommends the use of Excel Solver for finance academics and practitioners working on volatility using $GARCH(1,1)$ models. The empirical findings conclude that $GARCH(1,1)$ models under Tukey transformations are more appropriate than standard for describing returns and volatility of financial time series and its stylized facts including fat tails and mean reverting.

In the paper of Merabet F. and al. [17], the behavior of the oil price series is examined. The study is based on a combination of the Box-Jenkins methodology with the $GARCH$ processes. Of the models identified this analysis, the model $ARIMA(3,1,1)-EGARCH(1,2)$ is retained and is the best forecast model.

In Mia M.S. and Rahman M.S. [18] authors have built a model to forecast the exchange rate of Bangladesh. A study on Monthly average exchange rates of Bangladesh for the period from August, 2004 to April, 2019. They have selected $ARIMA(1,1,1)$ as a main model for this study. Then they tried to model the volatility of exchange rate using $ARCH$, $GARCH$, $EGARCH$, IGARCH and TARCH models. $ARIMA(1,1,1)-GARCH(1,1)$ is selected as a best model compared to others since it has the lower values of RMSE, MAE, MAPE and Theil than other models. However, to the authors´ best knowledge, very few publications can be found on volatility of the oil price by symmetric and assymetric models that capture most common stylized facts about oil price such as volatility clustering and leverage effect.

Not to mention the family of multivariate $GARCH$ models which are in this regard very crucial due to their efficiency and diversity, we will certainly discuss them in detail in the next articles, where we intend to use the multivariate model of $GARCH-BEKK$ as studied by El Ghini and Saidi [9], where they had studied the extent of the transmission of the 2008 financial crisis from America to other countries of the world. Olstad A. and al. [22] investigates the time-varying correlation between the volatilities of two oil benchmarks (Brent and WTI) and six currencies of the major oilimporters and oil-exporters, for the period from February 1, 1999 to May 30, 2016, using a multivariate $Diag-BEKK-GARCH$ model. The analysis reveals that oil and currency volatilities exhibit positive correlations during major global or region-specific economic events (such as the Global Financial Crisis of 2007–2009 and the EU debt crisis period). Yaya and al. [25] use the $CCC-MGARCH$ model to study the return transmission between oil and gold markets before and after the global financial crisis. They find a bidirectional return spillover before the crisis, and unidirectional spillover from gold to the oil market after the crisis. Guesmi and al. [13] apply the $VARMA-DCC-GJR-GARCH$ model and provide evidence of significant volatility spill-over between Bitcoin and other financial markets, including the oil market. Moreover, Bitcoin provides hedging and diversification benefits against oil, gold, and emerging stock markets.

The shocks affect, caused by the current Ukrainian crisis, in the world economy, in particular on the ambivalent fluctuations of the indices of the world stock exchanges and the prices of strategic raw materials, will mark many publications and scientific works using the technique of multivariate $GARCH$ models.

There are a lot of papers hwo made a studes of impact of Covid-19 on both oil prices volatility and auther stocks volatility prices using the $GARCH$ models familly. Endri E. and al. [10] has examined the response of stock prices on the Indonesia Stock Exchange (IDX) to COVID-19 using an event study approach and the $GARCH$ model. While Meher B. K. and al. [16] have focused on measuring the impact and leverage of corona virus disease 2019 on price volatility of crude oil and natural gas quoted on MCX, India, using the $EGARCH$ model with general error distribution.

3. RESEARCH METHODOLOGY

3.1. **Non-Linear Models and heteroskedasticity.** Financial time series returns are notoriously unstable and characterized by heteroscedasticity and in particular asymmetric, the constraints that $ARMA(p,q)$ models suffer from taking them into account, and sometimes involve the use of nonlinear models that may make inadequate the $ARMA$ specification Eq. (3.1).

$$r_t = \mu_t + \varepsilon_t, \tag{3.1}$$

where r_t is the series of price returns which is written as the sum of μ_t: the conditional expectation of returns (which is modeled by ARMA) and the residuals ε_t (which are assumed to be a white noise in the $ARMA$ specification).

When ε_t is not white noise in the (3.1), ε_t, an Autoregressive Conditionally Heteroscedastic ($ARCH$) models ([11]) are introduced by integrating volatility into the ARMA specification (3.1).

3.1.1. *Conventional models (Symetric).*

i) ARCH Models: In 1982 Robert Engle diveloped the Autoregressive Conditional Heteroskedasticity (ARCH) models to model the time-varying volatility often observed in economical time series data that $ARMA$ linear models are no longer able to take them into account. For this contribution, he won the 2003 Nobel Prize in Economics with the co-integration of Cliv Granger.

$ARCH$ models assume the variance of the current error term or innovation to be a function of actual sizes of the previous time periods' error terms: often the variance is related to the squares of the previous innovations, i. e., the conditional varaince to the information available at time $t-1$ varies over time, and that is given as follows:

$$h_t = \mathbb{V}ar(\varepsilon_t \mid I_{t-1}) = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2,$$

where

$$\mathbb{E}(\varepsilon_t \mid I_{t-1}) = 0, \ \ w > 0, \ \alpha_i \geq 0 \text{ for } i = 1, \ldots, q,$$

and

$$I_{t-1} = \{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}\},$$

is the information set available at time $t-1$.

The condition of stationnarity of the $ARCH\,(q)$ is

$$\sum_{i=1}^{q} \alpha_i < 1.$$

ii) Generalized ARCH models: The model is generalized by Robert Engle's doctoral student, Bollerslev Tim in 1986. The conditional variance not only depends on the previous innovations but also on its previous conditional variances, and that is given as follows:

$$h_t = \mathbb{V}ar\left(\varepsilon_t \mid I_{t-1}\right) = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \tag{3.2}$$

such that the following conditions are necessary:

$$\mathbb{E}(\varepsilon_t \mid I_{t-1}) = 0,$$

$$w > 0,\ \alpha_i \geq 0 \text{ and } \beta_j \geq 0 \text{ for } i = 1, \ldots, q \text{ and } \ j = 1, \ldots, p,$$

where I_{t-1} is the information set available at time $t-1$.

The stationarity condition of the model is as follows:

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1. \tag{3.3}$$

The term $\sum\limits_{i=1}^{q} \alpha_i + \sum\limits_{j=1}^{p} \beta_j$, is a measure of the degree of volatility persistence.

The $GARCH(1,1)$ model is presented in the form

$$h_t = \mathbb{V}ar\left(\varepsilon_t \mid I_{t-1}\right) = w + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1},$$

with the same conditions of general case.

3.1.2. *Asymmetric models.* The $ARCH$ and $GARCH$ models are based on a hypothesis of symmetry of the impact of shocks on volatility. They assign the same weight to positive shocks (good news) as to negative shocks (bad news), which is contrary to empirical reality. The asymmetry phenomena, can easily be observed through the graphs of the raw series and the volatility together: when the prices of a financial asset are in a bullish phase, low volatility is observed, on the other hand when they are in a bearish phase, high volatility is observed. Therefore, extensions of the $GARCH$ models that take into account the asymmetric behavior of volatility have been introduced:

- $EGARCH$ (Nelson [19]).
- $TGARCH$ or $GJR-GARCH$ (Glosten and al. [12]).
- $TARCH$ (Zakoian [26]).

1. EGARCH model

The standard $GARCH$ model is unable to capture the skewness or asymmetric nature caused by the negative correlation between returns and volatility which is referred to as the leverage effect. Therefore, if the conditions allow, the $EGARCH$ model, is the best framwork we will rely on in our analysis to measure the leverage effect of COVID-19 on the price volatility of the crude oil. That is pepresented by

$$\ln\left(h_t\right) = w + \sum_{i=1}^{q}\left[\alpha_i\left|z_{t-i}\right| + \gamma_i z_{t-i}\right] + \sum_{j=1}^{p}\beta_j \ln\left(h_{t-j}\right), \tag{3.4}$$

while, h_t the actual volatilite, $z_t = \frac{\varepsilon_t}{\sqrt{h_t}} \backsim i.i.d.\ \mathcal{N}(0,1)$ (standardized noise white) who ε_t the residuals of the estimation of a process $ARMA(p,q)$, $\ln\left(h_t\right) = \ln$ of variance or $\ln$ returns, w a constant, α_i the $ARCH$ effects, γ_i is asymmetric effects and β_j is a $GARCH$ effects.

If $\gamma_i = 0$ the model is the standard $GARCH$.

If $\gamma_i < 0$ it implises that bad news (negative shocks) generate large volatility that good news (positive shocks).

The $EGARCH(1,1)$ is given as bellow,

$$\ln\left(h_t\right) = w + \alpha\left|z_{t-i}\right| + \gamma z_{t-i} + \beta \ln\left(h_{t-j}\right),$$

where $z_t = \frac{\varepsilon_t}{\sqrt{h_t}}$.

2. Threshold-GARCH model

$Threshold-GARCH$ $(TGARCH)$ model one of the ways taking into account the asymmetry of volatility which is a reality of financial series. To do that it simply adds into the variance equation a multiplicative dummy variable to check whether there is statistically significant difference whene shocks are negative.

The model was introduced by Glosten, Jagannathan and Runkle in 1993.

The conditional variance for a TGARCH(1,1) is represented by:

$$h_t = \sigma_t^2 = w + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 + \beta h_{t-j}$$

where I_{t-1} takes the 1 (bad news) for $\varepsilon_t < 0$, and 0 otherwise. So "good news" and "bad news" have a different impact on the volatilité. Good news (positive shock) has an impact of α, while bad news (negative shock) has an impact of $\alpha + \gamma$. Such as $\alpha \geq 0$, $\beta \geq 0$, $\gamma \geq 0$ and $w > 0$.

γ is known as the asymmetry or levrage term. $\gamma > 0$ is asymmetry, while $\gamma = 0$ is symmetry (model callapses to the standard GARCH). If γ is a significant and positive, negative shocks will have larger effects on the volatilité h_t than positive shocks.

$TGARCH$ models can be extended to higher order specifications by including more lagged terms and noted TGARCH(p,q), as follows:

$$h_t = \sigma_t^2 = w + \sum_{i=1}^{q} (\alpha_i + \gamma_i I_{t-1})\, \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \tag{3.5}$$

with specific conditions.

4. ANALYSIS OF THE RESULTS AND DISCUSSION

The study is based on daily WTI crude oil prices covering the period from January 01, 2015 to December 31, 2021, the total of observations is 1827. The data was downloaded from the investing.com website. Our interest is to formulate models and forecasting price volatility of crude oil. E-Views 10 has been used.

4.1. **Statistical analysis of WTI raw serie's behavior.**

The WTI raw series in Figure 1 presents an unstable trend over time i. e, it is affected by an upward trend, which means that the series is not stationary in variance and in mean. the series shows a high variability throughout the period studied. This allows to say that a GARCH type process could be adapted to the modelling of the oil price series.

The preliminary analysis of stationnarity of the prices series, also give a sign of no-stationarity of the WTI, it would be much more in variance: the collerogram (Table 1) shows that, the all values of $Prob$ of the all lags (36) are less than 0.05

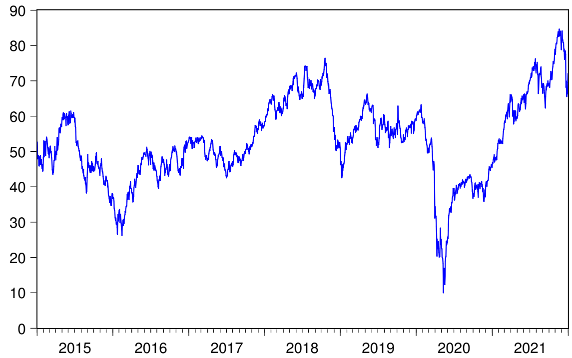

FIGURE 1. Crude oil prices (WTI) "1/01/2015-12/31/2021".

TABLE 1. Correlogram of WTI (One significant peak).

Date: 05/16/22 Time: 02:04
Sample: 1/01/2015 12/31/2021
Included observations: 1827

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.994	0.994	1806.8	0.000
		2	0.988	0.053	3594.2	0.000
		3	0.983	0.025	5363.2	0.000
		4	0.978	0.016	7114.8	0.000
		5	0.972	-0.017	8848.3	0.000
		6	0.966	-0.038	10562.	0.000
		7	0.961	-0.009	12257.	0.000
		8	0.954	-0.038	13930.	0.000
		9	0.947	-0.096	15579.	0.000
		10	0.939	-0.052	17202.	0.000
		11	0.932	-0.008	18799.	0.000
		12	0.924	0.017	20372.	0.000
		13	0.917	0.018	21922.	0.000
		14	0.910	-0.019	23447.	0.000
		15	0.902	-0.023	24947.	0.000

and the slow decrease of the ACF terms. So then, the WTI raw series would follow an $AR(1)$ but conventional process (Cfr correlogram).

4.2. **Stationarity study.** To formulate the appropriate model the logarithmic transformation of the prices oil data have been calculated. This has made the data of crude oil, less fluctuated but not stationary and keep the same shape and characteristic as presented in Figure 2 bellow.

In first, we determine the type of non-statonnarity of generating process of the raw time series, we perform the unit root test at level 5% using the Augmented-Dickey-Fuller test (ADF) introduced by David A. Fuller and Wayne A. Fuller (see [6] and [7]) on the log oil prices ($LWTI$) which based on hypotheses:

$$\begin{cases} H_0: & \text{has unit root} \\ H_1: & \text{hasn't unit root} \end{cases}$$

with the inclusion of the test equation as intercept, trend and intercept and none. This step is essential, wich allows to identify the nature of the trend, deterministic (TS) or stochastic (DS). In effect, the analysis of unit root test, confirm that the

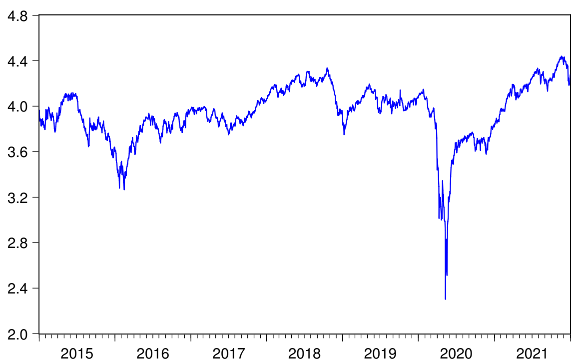

FIGURE 2. Logarithm of crude oil prices (LWTI).

WTI series has a unit root without drift ((Prob* $= 0.4210 > 0.05 : H_0$ accepted) without drift (the trend and intercept coefficient is not significant $(1.3280_{t-stat} < 3.11_{ADF})$ and the intercept coefficient is not significant $(2.0255_{t-stat} < 2.83_{ADF})$, than the serie is not stationnary and it is of the (DS) type, model one.

Note that the method of stationarization of the data series will be done by differentiation $(d(LWTI))$ presented in Figure 3.

Figure 3 represents the volatility clustering of crude oil data from January 1, 2015 to December 31, 2022, i.e., small variations tracked by small variations and large variations tracked by large variations which imply that volatility models can be formulated.

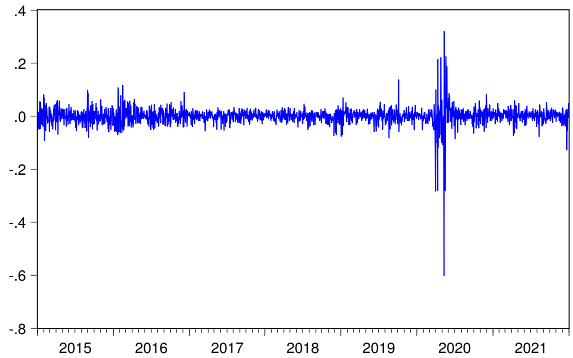

FIGURE 3. Crude oil prices returns (DLWTI).

4.3. **Statstical analysis of leverage effect and ARCH effect.** To formulate the adequate $GARCH$ model few more items need to be tested, i.e., asymmetry character, volatility clustering and presence of ARCH effects.

4.3.1. *Leverage effect (asymmetry data).* Statistical indicators of WTI series (Figure 4), in particular with the histogram and the test of Jarque-Bera, showed that there is a characteristic of asymmetry resulting from the effect of Covid-19 or leverage effect. Indeed, the Kurtosis coefficient (3.251) is lower than 3 and the skewness coefficient (-0.11) which is different from 0.

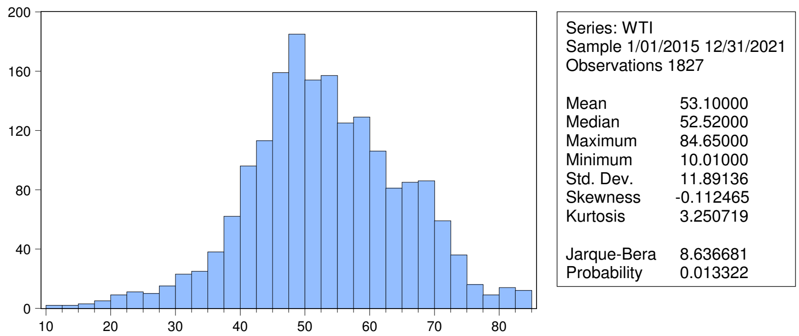

FIGURE 4. Asymmetry and peakedness character of crude oil data.

So the data series is leptocurtic and Skowed, which means it is not normally distributed even $Jarque - Bera$ Statistics (8.64) have also verified that the series data is leptokurtic and skowed, likewise, the associated probability lead to the same inference: $\Pr obability = 0.0133 < 0.05$ so that allows us to reject the null hypothesis of the normality test. So then, the asymmetric or leverage characteristic is proven.

4.3.2. *Heteroskedasticity and ARCH effect.*

i) Conditional Heteroskedasticity

Given the graph of the raw series (Figure 1) which reveals strong variability, we can assume the existence of conditional heteroskedasticity.

In effect, the conditional heteroskedasticity test confirm the existence of volatility, because the coefficient of the series $(DLWTI^2)$ is statistically significant (0.295). That is to say, the variation of the series of prices at time t is a function of its evolution at time $t-1$. Hence the confirmation of the existence of volatility.

ii) ARCH effect

The $ARCH$ effect can be judged from Lagrange Multiplier (LM) statistics which is represented as an observed R-squared. The observed R-square statistics is 55.3196 and it is considered significant as its probability value is < 0.05. Moreover, the F-statistics (56.9864) is also significant as its probability value is < 0.05. thus the coefficient associated with $RESID\hat{}2(-1) = 0.1741$ is statistically significant at the 5% level. This proves that there is an existence of $ARCH$ effect in the data of the crude oil from January 1, 2015 to December 31, 2021, therefore the raw series generating process is an $ARCH$ of order $1.$ which we will estimate in the next section.

4.4. **Estimation of models and search for the optimal model.**

4.4.1. *Estimation of eventual models.* After having tested the existence of $ARCH$ effects and the other essentials related to $GARCH$ modeling, three models have been formulated, $GARCH(1.1)$, $GARCH(1.2)$ and $TGARCH(1,2)$.

The specific conditions for each model are established. The models: $GARCH$ $(1,1)$ and $GARCH(1.2)$ vanish the positivity condition of coefficients of variance and stationarity equation. The $TGARCH(1,2)$ model satisfy the condition of asymmetry.

TABLE 2. ARCH effect due to the Covid-19 pandemic.

Heteroskedasticity Test: ARCH

F-statistic	56.98643	Prob. F(1,1823)	0.0000
Obs*R-squared	55.31967	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/16/22 Time: 02:22
Sample (adjusted): 1/05/2015 12/31/2021
Included observations: 1825 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.315696	0.114518	11.48902	0.0000
RESID^2(-1)	0.174100	0.023063	7.548936	0.0000

R-squared	0.030312	Mean dependent var	1.593064
Adjusted R-squared	0.029780	S.D. dependent var	4.704125
S.E. of regression	4.633551	Akaike info criterion	5.905619
Sum squared resid	39139.43	Schwarz criterion	5.911657
Log likelihood	-5386.878	Hannan-Quinn criter.	5.907846
F-statistic	56.98643	Durbin-Watson stat	2.017159
Prob(F-statistic)	0.000000		

Furthermore, the standard way to select a model is, the coefficients, $ARCH$ and $GARCH$ should be significant and there should not be the existence of heteroscedasticity or $ARCH$ effect (with the help of $ARCH$ $Lagrange$ $Multiplier$ test) and Autocorrelation (with the help of correlogram of residuals and squared residuals) after framing the model, which is well verified in any of the three models adopted for the series analyzed in this work (see Table 3, in the section "Autocorrelation residuals and heteroskedasticity").

4.4.2. *Research of the optimal model.*

A) Information criteria

The choice of the most appropriate model among the estimated models is made on the basis of minimization of the criterion $Akaike$ (AIC), $Schwarz$ (SIC), on

TABLE 3. Estimations of selected models.

	GARCH(1,1)	GARCH(1,2)	TGARCH(1,2)
Main Equation			
C (Prob)	0.292819 (0.0186)	0.252381 (0.0416)	0.062799 (0.5999)
WTI(-1) (Prob)	0.995287 (0.0000)	0. 995940 (0.0000)	0.999146 (0.0000)
Variance Equation			
w (Prob)	0.041242 (0.0000)	0.065049 (0.0000)	0.065460 (0.0000)
α_1 (Prob)	0.082836 (0.0000)	0.125001 (0.0000)	0.044011 (0.0023)
B_1 (Prob)	0.894473 (0.0000)	0.191560 (0.0062)	0.199395 (0.0073)
B_2 (Prob)	*	0.646135 (0.0000)	0.652199 (0.0000)
γ (Prob)	*	*	0.121670 (0.0000)
Autocorrelation Residuals and Heteroskedasticity			
Autoc. Residuals	Non	Non	Non
Autoc. Squared Residuals	Non	Non	Non
Heteroskedasticity (ARCH LM-test)	No Signifacant	No Signifacant	No Signifacant

the one hand, and maximization of $Log - likelihood$, on the other hand. All these specificities which helped in the preliminary selection of the appropriate models are represented in Table 4 bellow.

TABLE 4. Informations criteria to select the suitable model.

	GARCH(1,1)	GARCH(1,2)	TGARCH(1,2)
AIC	3.167363	3.163288	3.153744
SIC	3.182450	3.181393	3.174866
LL	-2886.802	-2882.082	-2872.368
Lj-Bx squar (Prob)	34.248 (0.552)	33.695 (0.579)	33.661 (0.580)

According to Table 4 the $TGARCH(1,2)$ model has the lowest values of the minimization criteria (AIC and SIC) and has the highest value of the maximization criterion ($Log - likelihood$) compared to the other two models.

The comparison will end after analyzing the criteria of good prediction in the following point.

B) Forecasting criteria

We use the good forecast criteria, RMSE (Root Mean Square Error), MAE (Mean Absolut Error), MAPE (Mean Absolute Percentage Error) and Theil inequality coefficient (close to zero), shown in Talbe 5 below, to select the optimal model among the three selected models, According to the criteria of Table 5, the model who having more minimal values is the $GARCH(1,2)$, so it is the better then $GARCH(1,1)$ and $TGARCH(1,2)$. Moreover, as can be deduced from the Table 6, this adopted model satisfies the necessary conditions,

TABLE 5. Forecast standard criteria to select the suitable model.

	GARCH(1,1)	GARCH(1,2)	TGARCH(1,2)
R.M.S.E	14.08002	13.95559	14.98503
MAE	11.29239	11.18234	11.80188
MAPE	27.08691	26.82942	28.70260
THEIL	0.121919	0.120988	0.127465

$$\alpha_1 \geq 0,\ \beta_j \geq 0,\ \forall j = 1, 2,$$

and the suffisient stationnarity condition (volatility persistence)

$$\alpha_1 + \sum_{j=1}^{2} \beta_j = 0.9627006 < 1. \tag{4.1}$$

Hence, the model retained for predicting crude oil price volatility is the $GARCH(1,2)$. The its estimation results are presented in the Table 6 bellow

Table 6 shows the estimation results of the $GARCH(1,2)$ model. The results contain two parts. The upper part shows the main equation and the lower part represents the variance equation. In the main equation, both the constant (C) and the co-efficient of first lag $[WTI(-1)]$ are significant as the probability value is < 0.05. In case of variance equation, $C(3)$ is the constant, $C(4)$ is the $ARCH$ co-efficient, $C(5)$ and $C(6)$ are the $GARCH$ coefficients of first and second lag respectively. All the coefficients in the variance equation are significant as their probability values are < 0.05.

Hence the variance equation can be shown as given below

$$h_t = 0.065049 + 0.125001\varepsilon_{t-1}^2 + 0.191560h_{t-1} + 0.646135h_{t-2}. \tag{4.2}$$

TABLE 6. Estimation results of GARCH(1,2) model.

Dependent Variable: WTI
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 05/16/22 Time: 19:53
Sample (adjusted): 1/02/2015 12/31/2021
Included observations: 1826 after adjustments
Convergence achieved after 38 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1) + C(6)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.252381	0.123890	2.037134	0.0416
WTI(-1)	0.995940	0.002209	450.8203	0.0000
Variance Equation				
C	0.065049	0.013273	4.900944	0.0000
RESID(-1)^2	0.125001	0.010435	11.97901	0.0000
GARCH(-1)	0.191560	0.070014	2.736033	0.0062
GARCH(-2)	0.646135	0.068545	9.426414	0.0000

R-squared	0.988707	Mean dependent var	53.10022
Adjusted R-squared	0.988701	S.D. dependent var	11.89461
S.E. of regression	1.264370	Akaike info criterion	3.163288
Sum squared resid	2915.903	Schwarz criterion	3.181393
Log likelihood	-2882.082	Hannan-Quinn criter.	3.169966
Durbin-Watson stat	2.083557		

4.5. **Forecasting using the formulated model.** By using the above-formulated model, the crude oil prices and volatility of crude oil prices have been forecasted in the forecasting section.

The Figure 5 explains how the forecasting graph made by the adopted model (WTIF) coincides with that made by the actual WTI data (Figure 1), also for the graphs of the predictive volatility and the graph of the volatility of the actual data, which proves the validity of the adopted model. The Table 7, can be confirmed these results by comparing the values of the real data (actual) and the values provided by the equation of the $GARCH(1,2)$ model adopted (forecast).

Figure 6 shows the forecasted prices of crude oil and the forecasted variance for crude oil for the modeling period from october 01, 2021 to December 31, 2021. The first graph of forecasted prices of crude oil depicts that its general trend has seen two essential phases, the first is characterized by growth, crude oil prices

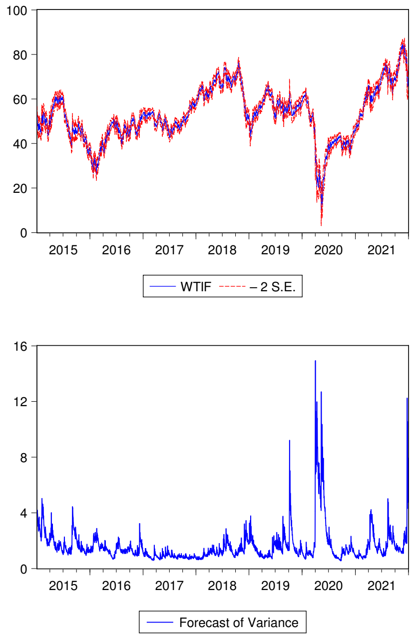

FIGURE 5. WTIF and Volatility Forecast by using the formulated model.

TABLE 7. Actual and forecasted oil peices of last 3 months of 2021.

Day	October Actual	October Forecast	Day	November Actual	November Forecast	Day	December Actual	December Forecast
01/10	75.88	74.98	01/11	84.05	83.48	01/12	65.57	66.16
04/10	77.62	75.82	02/11	83.91	83.96	02/12	66.5	65.56
05/10	78.93	77.56	03/11	80.86	83.82	03/12	66.26	66.48
06/10	77.43	78.86	04/11	78.81	80.78	06/12	69.49	66.24
07/10	78.3	77.37	05/11	81.27	78.74	07/12	72.05	69.46
08/10	79.35	78.23	08/11	81.93	81.19	08/12	72.36	70.23
11/10	80.52	79.28	09/11	84.15	81.85	09/12	70.94	68.89
12/10	80.64	80.45	10/11	81.34	84.06	10/12	71.67	70.02
13/10	80.44	80.57	11/11	81.59	81.26	13/12	71.29	69.88
14/10	81.31	80.37	12/11	80.79	81.51	14/12	70.73	69.12
15/10	82.28	81.23	15/11	80.88	80.71	15/12	70.87	71.11
18/10	82.44	82.20	16/11	80.76	80.80	16/12	72.38	70.95
19/10	82.96	82.36	17/11	78.36	80.68	17/12	70.86	68.75
20/10	83.87	82.88	18/11	79.01	78.29	20/12	68.23	69.65
21/10	82.5	83.788	19/11	76.1	78.94	21/12	71.12	70.44
22/10	83.76	82.42	22/11	76.75	76.04	22/12	72.76	73.01
25/10	83.76	83.672	23/11	78.5	76.69	23/12	73.79	72.78
26/10	84.65	83.672	24/11	78.39	78.43	27/12	75.57	74.02
27/10	82.66	84.56	25/11	77.41	78.32	28/12	75.98	76.21
28/10	82.81	82.58	26/11	68.15	77.35	29/12	76.56	77.05
29/10	83.57	82.73	29/11	69.95	68.13	30/12	76.99	75.36
			30/11	66.18	69.92	31/12	75.21	76.08

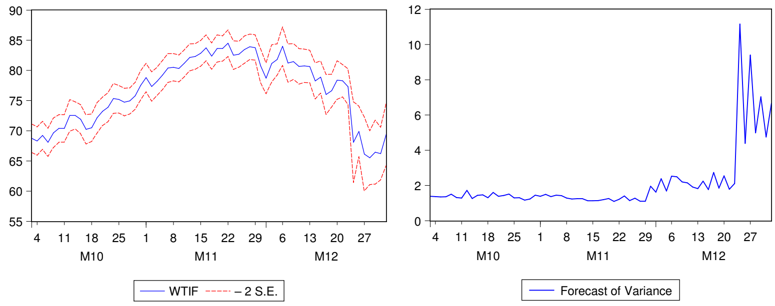

FIGURE 6. Forecasting crude oil volatility based on modified 3 months data from 1st October 2021 to 31 th December 2021 by using the formulated model.

have increased since the beginning of October 2021 until reaching a high peak on November 22, while the second phase is characterized by a decrease, in a single month the prices fell in a continuous and remarkable way with fluctuations, from November 22 until the last of December of the year, when they started to rise again another time.

While the predictive variance graph in turn experienced the same two phases, the first witnessing a quiet period of variance from October 1 to November 22, which corresponds exactly to the upward phase of crude oil prices (good news: The expansion of the Corona virus epidemic recovery and the removal of the global containment measure),the second witness to a wildly fluctuating period of variance (volatility) from November 22 until the end of December, which corresponds to the bearish phase of crude oil prices (bad news: Increasing global demand for energy production, especially oil and gas, due to the conflict in Eastern Europe).

5. CONCLUSION

From this study we can conclude several important results, it is that the leverage effect or the asymmetry of volatility is observed due to the spread of the pandemic that has an impact on the volatility of crude oil prices, which is evidenced by the number of asymmetric models discovered in the data analysis, other than those presented in this article, such as $TGARCH(1,1)$, $TGARCH\ (2,1)$, $TGARCH(3,1)$, $EGARCH(1,2)$, $EGARCH(2,1)$ and $EGARCH(3,1)$, which belong to asymmetric models, such that the asymmetric coefficient λ is negative. But they are excluded from the choice for the absence of minimization or maximization criteria, despite this there are some among whom can be adapted to the data studied.

The forecasting graphs of thrée months above of crude oil prices indicate, notably in the last of this priode (Décembre 2021), that It is difficult to predict the expected volatility of prices as the volatility graph is extremely fluctuating. The investors in the commodities market focusing on investing in the crude oil can use the formulated models to take investment decisions.

Soon we are trying to develop a study on the impact, with more techniques, of the current conflict in Western Europe on oil prices and natural gas volatilites, the demand for which has been steadily increasing since the beginning of the crisis.

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