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# A COMMON FIXED POINT THEOREM IN MENGER SPACE WITH COMPATIBLE MAPPING OF TYPE $(K)$

Ajay Kumar Chaudhary[1], Kanhaiya Jha, and K.B. Manandhar

ABSTRACT. The notion of compatible mappings of type $(K)$ in metric space has been introduced by K. Jha, V. Popa, and K.B. Manandhar in 2014. In this paper, we use Meir-Keeler contractive type condition and establish a common fixed point theorem in Menger probabilistic metric space by using compatible mappings of type $(K)$. Our results generalize and improve several similar known results in the literature.

## 1. INTRODUCTION

The idea of probability in metric space was introduced by K. Menger [13] in 1942, as an extension of the concept of metric space $(X, d)$, by replacing the notion of distance $d(x, y)$ where $(x, y \in X)$ with a distributive function $F_{x,y} : X \times X \to \mathbb{R}$, where $F_(x, y)(t)$ represents the probability that the distance between $x$ and $y$ is less than $t$. This space is called statistical metric space or Menger probabilistic metric space. B. Schweizer and A. Sklar [15] studied this concept and obtained some fundamental results on this Menger space. In 1972, V. M. Sehgal and A. T. Bharucha-Reid [16] initiated the study of contraction mappings in probabilistic

[1]*corresponding author*

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metric spaces (PM-spaces) which is an important step in the development of fixed point theorems.

In 1982, S. Sessa [19] improved the definition of commutativity in fixed point theorems by introducing the notion of weakly commuting mappings in metric space. Then in 1986, Jungck [7] introduced the concept of compatible mappings, and this notion of compatible mappings in Menger space was introduced by S.N. Mishra [14] in 1991. Further, this condition has been weakened by introducing the notion of weakly compatible mappings by G. Jungck and B.E. Rhoades [10] in 1998. Recently, B. Singh and S. Jain [17] 2005 introduced weakly compatible mappings in Menger space and established a common fixed point theorem. In 2014, the compatible mapping of type $(K)$ in metric space was introduced by K. Jha, V. Popa. and K.B. Manandhar [11].

The purpose of this paper is to introduce the notion of compatible mappings of type $(K)$ in Menger space with example and establish a common fixed point theorem for pairs of self- mappings in Menger space using this compatible mapping of type $(K)$ under Meir-Keeler contractive condition which generalizes and improves some well-known results in the literature.

## 2. Preliminaries

**Definition 2.1.** *[15] A function $F : \mathbb{R} \to \mathbb{R}^+$ is said to be* ***distribution function*** *if it*

(i) *is non-decreasing,*
(ii) *is left continuous, and*
(iii) $inf_{x\in\mathbb{R}}F(x) = 0$ *and* $sup_{x\in\mathbb{R}}F(x) = 1.$

**Definition 2.2.** *[15] Let $F : X \times X \to L$ (the set of all distribution functions) be a distribution function and $X$ be a non-empty set. Then, a pair $(X, F)$ is said to be* ***Probabilistic metric space*** *(abbreviated as PM-space) if the distribution function $F(p, q)$, where $(p, q) \in X \times X,$ also denoted by$F_(p, q)$ or $F_{p,q}$ satisfies following conditions:*

(i) $F_{p,q}(x) = 1,$ *for every* $x > 0$ *if and only if* $p = q.$
(ii) $F_{p,q}(0) = 0;$ *for every* $p, q \in X,$
(iii) $F_{p,q}(x) = F_{q,p}(x)$, *for every* $p, q \in X$, *and*

(iv) *For every* $p, q, r \in X$ *and for every*

$$x,\ y > 0,\ F_{p,r}(x) = 1,\ F_{r,q}(y) = 1\ \Rightarrow F_{p,q}(x + y) = 1.$$

*Here,* $F_(p, q)(x)$ *represents the value of* $F_(p, q)$ *at* $x \in \mathbb{R}$.

**Definition 2.3.** *[5] A function* $T : [0, 1] \times [0, 1] \to [0, 1]$ *is referred to as* ***Triangular norm*** *(shortly t-norm) if it satisfies the following conditions:*

(i) $t(0, 0) = 0$ *and* $t(a, 1) = a$ *for every* $a \in [0, 1]$,
(ii) $t(a, b) = t(b, a)$ *for every* $a, b \in [0, 1]$,
(iii) *if* $a\leq$ *c and* $b\leq d$, *then* $t(a, b) \leq t(c, d)$, *and*
(iv) $t(a, T(b, c)) = t(t(a, b), c))$ $(a, b, c \in [0, 1])$.

**Definition 2.4.** *[18]* ***Menger Space****, also known as Menger Probabilistic Metric Space, is a triplet* $(X, F, T)$*, where* $(X, F)$ *is a PM space and* $t$ *is a* $T-$ *norm satisfying following condition:*

(v) $F_{p,q}(x + y) \geq t(F_{p,r}(x),\ F_{r,q}(y))$.*, for all* $p, q, r \in X$ *and* $x, y \in \mathbb{R} > 0$.

**Definition 2.5.** *[9] A mapping* $A : X \to X$ *in Menger space* $(X, F, t)$ *is said to be* ***Continuous*** *at a point* $p \in X$ *if for every* $\epsilon > 0$ *and* $\lambda > 0$*,there exists* $\epsilon_1 > 0$ *and* $\lambda_1 > 0$ *such that if* $F_{p,q}(\epsilon_1) > 1 - \lambda_1$*, then* $F_Qp, Qq(\epsilon) > 1 - \lambda$

**Definition 2.6.** *[3] Let* $(X, F, T)$ *be a* ***Menger Space*** *and* $t$ *be a continuous T-norm. Then,*

(1) *A sequence* $\{x_n\}$ *in* $X$ *is said to be* ***converge*** *to a point* $x$ *in* $X$ *(written* $x_n \to x$*) if, and only if, for every* $\epsilon > 0$ *and* $\lambda > 0$*, there exists an integer* $N = (N, \epsilon) > 0$ *such that* $F_{x_n,x}(\epsilon) > 1 - \lambda$ *for all* $n\geq N$*. In this case, we write* $\lim_{n\to\infty} x_n = x$.
(2) *A sequence* $\{x_n\}$ *in* $X$ *is said to be a* ***Cauchy sequence*** *if for every* $\epsilon > 0$ *and* $\lambda > 0$*, there exists an integer* $N = (N, \epsilon) > 0$ *such that* $F_{x_n,x_m}(\epsilon) > 1 - \lambda$ *for all* $n, m\geq N$.
(3) *A Menger space* $(X, F, T)$ *is said to be* ***Complete*** *if every Cauchy sequence in* $X$ *converges to a point in* $X$.

**Definition 2.7.** *[8] Let* $X$ *be a non-empty set and* $A, S : X \to X$ *be arbitrary mappings, then* $x \in X$ *is said to be a common fixed point of* $A$ *and* $S$ *if* $A(x) = S(x) = x$.

**Example 1.** *Let $f, g : \mathbb{R} \to \mathbb{R}$ be functions such that $f(x) = x^2$ and $g(x) = x$, then $x = 0$ is a common fixed point.*

The notion of compatible mapping in Menger Space was first introduced by S.N. Mishra [14] in 1991 as an extension of the compatible mapping in metric space introduced by G. Jungck [7] in 1986.

**Definition 2.8.** *[14] Two mappings $A, S : X \to X$ are said to be **Compatible Mappings** in Menger space $(X, F, t)$ if, and only if,*

$$lim_{n\to\infty} F_{ASx_n,SAx_n}(t) = 1 \quad \textit{for all} \quad t > 0$$

*whenever $\{x_n\}$ is a sequence in $X$ such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = z$ for some $z$ in $X$.*

**Definition 2.9.** *[3] Two mappings $R, S : X \to X$ are said to be **Compatible Mappings of type A** in Menger space $(X, F, t)$ if, and only if,*

$$lim_{n\to\infty} F_{SRx_n,RRx_n}(t) = 1 \quad and \quad lim_{n\to\infty} F_{RSx_n,SSx_n}(t) = 1 \quad \textit{for all} \quad t > 0$$

*wheneve$\{x_n\}$ is a sequence in $X$ such that $lim_{n\to\infty}Qx_n = lim_{n\to\infty}Rx_n = z$ for some $z$ in $X$.*

In 2021, A.K. Chaudhary, K. Jha, K. B. Manandhar, and P.P. Murthy [6] have introduced the following compatible mapping of type (P) in Menger space as:

**Definition 2.10.** *[6] Two mappings $A, S : X \to X$ are said to be **Compatible Mappings of type** $(P)$ in Menger space $(X, F, t)$ if, and only if,*

$$lim_{n\to\infty} F_{AAx_n,SSx_n}(t) = 1 \quad \textit{for all} \quad t > 0$$

*whenever $\{x_n\}$ is a sequence in $X$ such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = z$ for some $z$ in $X$.*

The new notion of compatible mappings of type $(K)$ in metric space was introduced by K.B. Manandhar, K. Jha, and V. Popa [11] in 2014.

We introduce the following definition of compatible mappings of type $(K)$ in Menger space.

**Definition 2.11.** *Two mappings $A, S : X \to X$ are said to be **Compatible Mappings of type** $(K)$ in Menger space $(X, F, t)$ if, and only if,*

$$lim_{n\to\infty} F_{AAx_n,Sz}(t) = 1 \quad and \quad lim_{n\to\infty} F_{SSx_n,Az}(t) = 1 \quad \text{for all} \quad t > 0$$

*whenever $\{x_n\}$ is a sequence in $X$ such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = z$ for some $z$ in $X$.*

**Example 2.** *Let $(X, d)$ be a metric space where $X = [0, 2]$ and let $(X, F, t)$ be Menger space with*

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+d(x,y)} & \text{for } t > 0 \\ 0 & \text{for } t = 0 \end{cases}$$

*for all $x, y \in X$. Let $A, S : X \to X$ be defined by*

$$A(x) = \begin{cases} 2 & \text{for } x \in [0, 1] - \frac{1}{2} \\ 0 & \text{for } x = \frac{1}{2} \\ \frac{2-x}{2} & \text{for } x \in (1, 2] \end{cases}$$

*and*

$$S(x) = \begin{cases} 0 & \text{for } x \in [0, 1] - \frac{1}{2} \\ 2 & \text{for } x = \frac{1}{2} \\ \frac{x}{2} & \text{for } x \in (1, 2] \end{cases}$$

*Taking sequence $\{x_n\}$ where $x_n = 1 + \frac{1}{n}, n \in N$. Then, it is neither compatible mappings of type $(A)$ nor compatible mappings of type $(P)$ but $(A, S)$ is compatible mappings of type $(K)$.*

## 3. Main Result

The following lemma helps us to prove our main theorem:

**Lemma 3.1.** *[17] Let $\{x_n\}$ be a sequence in Menger space $(X, F, t)$, where $t$ is continuous $T-$norm and $t(x, x)\geq x$ for all $x \in [0, 1]$. If there exists a constant $k \in (0, 1)$ such that $lim_{n\to\infty} F_{x_n,x_n+1}(kt)\geq F_{x_n-1,x_n}(x)$, for all $t > 0$ and $n \in N$, then $\{x_n\}$ is a Cauchy sequence in $X$.*

**Lemma 3.2.** *[17] Let $(X, F, t)$ be a Menger space. If there exists $k \in (0, 1)$ such that for all $p, q \in X$, $F_{p,q}(kt)\geq F_{p,q}(t)$then $p = q$.*

Now, we prove our main theorem by using compatible mappings of type (K) in Complete Menger Space:

**Theorem 3.1.** *Let $(X, F, t)$ be a complete Menger space with $t(x, y) = min(x, y)$ for all $x, y \in [0, 1]$ and $A, B, S, T : X \to X$ be four self mappings such that*

(3.1) $A(X) \subset T(X)$ *and* $B(X) \subset S(X)$,

(3.2) *the pairs $(A, S)$ and $(B, T)$ are compatible mappings of type $(K)$,*

(3.3) *$S$ and $T$ be continuous, and*

(3.4) *there exists a constant $k \in (0, 1)$ and for every $\epsilon \in (0, 1)$, there exists $\delta \in (0, \epsilon]$ such that $\epsilon - \delta < M_{x,y} < \epsilon$ implies $F_{Ax,By}(kt) \geq \epsilon$ and $F_{Ax,By}(kt) \geq M_{x,y}(t)$ where*

$$M_{x,y}(t) = \min\{(F_{Sx,Ty}(t), F_{Ax,Sx}(t), F_{By,Ty}(t), F_{Ax,Ty}(t)\}$$

*for all $x, y \in X$ and $t > 0$.*

*Then, $A, B, S,$ and $T$ have a unique common fixed point in $X$.*

*Proof.* Consider $x_0 \in X$. From (i), we have $A(X) \subset T(X)$and $B(X) \subset S(X)$. So if there exists a point $x_1$ in $X$ such that $Ax_0 = Tx_1$. And for $x_1 \in X$ there exist $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on.And inductively, we may construct sequence $\{y_n\}$ in $X$ such that $y_{2n-1} = Ax_{2n-2} = Tx_{2n-1}$, and $y_{2n} = Bx_{2n-1} = Sx_{2n}$, for $n = 1, 2, 3 \ldots$. Putting $x = x_{2n}$ and $y = x_{2n+1}$ in (3.4), we get

$$\begin{aligned}
&F_{y_{2n+1},y_{2n+2}}(kt) = F_{Ax_{2n},Bx_{2n+1}}(kt) \\
&\geq \min\{(F_{Sx_{2n},Tx_{2n+1}}(t), F_{Ax_{2n},Sx_{2n}}(t), F_{Bx_{2n},Tx_{2n+1}}(t), F_{Ax_{2n},Tx_{2n+1}}(t)\} \\
&\geq \min\{(F_{y_{2n},y_{2n+1}}(t), F_{y_{2n+1},y_{2n}}(t), F_{y_{2n+2},y_{2n+1}}(t), F_{y_{2n+1},y_{2n+1}}(t)\} \\
&\geq \min\{(F_{y_{2n},y_{2n+1}}(t), F_{y_{2n+1},y_{2n+2}}(t)\},
\end{aligned}$$

so, $F_{y_{2n+1},y_{2n+2}}(kt) \geq F_{y_{2n},y_{2n+1}}$. Therefore, for every $n \in N$, $F_{y_n,y_{n+1}}(kt) \geq F_{y_{n-1},y_n}$. So, by Lemma 3.1, $\{y_n\}$ is Cauchy sequence in $X$.

Since the Menger space $(X, F, t)$ is complete, so $\{y_n\}$ converges to a point $z$ in $X$ and consequently the sub sequences $\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$, also converges to $z$.

Since $S$ and $T$ are continuous and $(A, S)$ and $(B, T)$ are compatible mappings of type $(K)$. So, we have

(3.1) $AAx_{2n-2} \to Sz$ and $SSx_{2n} \to Az$ and $BBx_{2n-1} \to Tz$ and $TTx_{2n-1} \to Bz$.

Also, from (3.4), we get

$$F_{AAx_{2n-2},BBx_{2n-1}}(kt) \geq \min\{(F_{SAx_{2n-2},TBx_{2n-1}}(t), F_{AAx_{2n-2},SAx_{2n-2}}(t),$$
$$F_{BBx_{2n-1},TBx_{2n-1}}(t), F_{AAx_{2n-2},TBx_{2n-1}}(t)\}$$

As $n \to \infty$ and by using equation (3.1), we have

$$F_{Sz,Tz}(kt) \geq \min\{(F_{Sz,Tz}(t), F_{Sz,Sz}(t), F_{Tz,Tz}(t), F_{Sz,Tz}(t)\}$$

or, $F_{Sz,Tz}(kt) \geq F_{Sz,Tz}(t)$. From lemma (3.2), we get

$$Sz = Tz \tag{3.2}$$

Again, from (3.4), we have

$$F_{Az,BBx_{2n-1}}(kt) \geq \min\{(F_{Sz,TBx_{2n-1}}(t), F_{Az,Sz}(t), F_{Bz,TBx_{2n-1}}(t), F_{Az,TBx_{2n-1}}(t)\}.$$

Again, taking $n \to \infty$ and using equation (3.1) and (3.2), we get

$$F_{Az,Tz}(kt) \geq \min\{(F_{Sz,Sz}(t), F_{Az,Tz}(t), F_{Tz,Tz}(t), F_{Az,Tz}(t)\},$$

or, $F_{Az,Tz}(kt) \geq F_{Az,Tz}(t)$. From Lemma 3.2, we get

$$Az = Tz. \tag{3.3}$$

Then, from equations (3.2) and (3.3), we obtain

$$F_{Az,Bz}(kt) \geq \min\{(F_{Sz,Tz}(t), F_{Az,Sz}(t), F_{Bz,Tz}(t), F_{Az,Tz}(t)\},$$

or,

$$F_{Az,Bz}(kt) \geq \min\{(F_{Az,Az}(t), F_{Az,Az}(t), F_{Bz,Az}(t), F_{Az,Az}(t)\},$$

or, $F_{Az,Bz}(kt) \geq F_{Az,Bz}(t)$. So, we get

$$Az = Bz. \tag{3.4}$$

From equation relations (3.2), (3.3) and (3.4), we get

$$Az = Bz = Tz = Sz. \tag{3.5}$$

Now, we have to show that $Az = z$.

From condition (3.4), we have

$$F_{Az,Bx_{2n-1}}(kt) \geq \min\{(F_{Sz,Tx_{2n-1}}(t), F_{Az,Sz}(t), F_{Bx_{2n-1},Tx_{2n-1}}(t), F_{Az,Tx_{2n-1}}(t)\}.$$

Taking $n \to \infty$ and using equations (3.2) and (3.3), we get

$$\begin{aligned} F_{Az,z}(kt) &\geq \min\{(F_{Sz,z}(t), F_{Az,Sz}(t), F_{z,z}(t), F_{Az,z}(t)\} \\ &\geq \min\{(F_{Az,z}(t), F_{Az,Az}(t), F_{z,z}(t), F_{Az,z}(t)\} \end{aligned}$$

or, $F_{Az,z}(kt) \geq F_{Az,z}(t)$, or, by Lemma 3.2 $Az = z$.

Hence, from equation (3.5), we get $z = Az = Bz = Tz = Sz$ and $z$ is a common fixed point of $A, B, S$, and $T$.

**Uniqueness:** Suppose $w \neq z$ is another common fixed point of $A, B, S$, and $T$. Then $Aw = Bw = Sw = Tw = w$. Therefore, from condition (3.4), we have

$$F_{z,w}(kt) = F_{Az,Bw}(kt) \geq \min\{(F_{Sz,Tw}(t), F_{Az,Sz}(t), F_{Bw,Tw}(t), F_{Az,Tw}(t)\},$$

or, $F_{z,w}(kt) \geq \min\{(F_{z,w}(t), F_{z,z}(t), F_{w,w}(t), F_{z,w}(t)\}$, or, $F_{z,w}(kt) \geq F_{z,w}(t)$.

By Lemma 3.2, we get $z = w$. Hence, $z = Az = Bz = Tz = Sz$ and $z$ is unique in $X$. This completes the proof. □

## 4. Verification and Application

We verify our main result through following example:

**Example 3.** *Let $(X, F, t)$ be a complete Menger Space with $t(x, y) = min(x, y)$ for all$x, y \in [0, 1]$ where $X = [1, 10]$ with metric $d$ defined by $d(x, y) = |x - y|$ and $F$ is defined by*

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+d(x,y)} & \textit{for } t > 0 \\ 0 & \textit{for } t = 0 \end{cases}$$

*for all $x, y \in X$. We define $A, B, S, T : X \to X$ as:*

$$A(x) = \begin{cases} 1 & \textit{for } x \leq 4 \\ 2 & \textit{for } x > 4, \end{cases},$$

$$B(x) = \begin{cases} 1 & \textit{for } x \leq 5 \\ 2 & \textit{for } x > 5 \end{cases},$$

*and $S(x) = T(x) = x$ for all $x \in X$. Taking sequence $\{x_n\}$ where $x_n = 1 + \frac{1}{n}, n \in N$. Then, the mappings $A, B, S$ and $T$ satisfy all the conditions of the above Theorem 3.1 and have a unique common fixed point $x = 1$.*

Our main result help to prove following corollary. In the theorem 3.1, if we take $A = B, T = S$, then we have following result:

**Corollary 4.1.** *Let $A$ and $S$ be self-mappings in complete Menger space (X,F,t) with continuous $t(x, y) = min(x, y)$ for all $x, y \in [0, 1]$ satisfying the following conditions:*

(3.1) $A(X) \subset S(X)$,

(3.2) *the pairs $(A, S)$ be compatible mappings of type $(K)$,*

(3.3) *$S$ be continuous, and*

(3.4) *there exists a constant $k \in (0, 1)$ and for every $\epsilon \in (0, 1)$, there exists $\delta \in (0, \epsilon]$ such that $\epsilon - \delta < M_{x,y} < \epsilon$ implies $F_{Ax,Ay}(kt) \geq \epsilon$ and $F_{Ax,Ay}(kt) \geq M_{x,y}(t)$ where $M_{x,y}(t) = \min\{(F_{Sx,Sy}(t), F_{Ax,Sx}(t), F_{Ay,Sy}(t), F_{Ax,Sy}(t)\}$,*

*for all $x, y \in X$ and $t > 0$.Then, $A$ and $S$ have a unique common fixed point in $X$.*

**Remark 4.1.** *Our results generalize the results of Jungck et.al [7], Meir Keeler [12] and Jha et al [11]. Also, this result improves other similar results in the literature.*

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## REFERENCES

[1] S. BANACH: *Sur les Operations dans les ensembles abstraits et leur applications aux equations integral*, Fund. Math., **3** (1922), 87–92.

[2] R.K. BISHT, V. RAKOCEVIC: *On a probabilistic version of Meir-Keeler type fixed point theorem for a family of discontinuous operators*, Appl. Gen. Topology, **22**(2) (2021), 435–446.

[3] Y.J. CHO, P.P. MURTHY, M. STOJAKOVIC : *Compatible Mappings of type (A) and Common fixed point in Menger space*, Comm. Korean Math. Soc., **7**(2) (1992), 325–339.

[4] M. FRECHET: *Sur quelques points du calcul fonctionnel*, Rendic. Circ. Mat. Palermo, (1906), 1–74.

[5] O. HADZIC, E. PAP : *Fixed-Point Theory in Probabilistic Metric Space*, Kluwer Academic Publisher, London, 536, 2010.

[6] A.K. CHAUDHARY, K.B. MANANDHAR, K. JHA, P.P. MURTHY: *A common fixed point theorem in Menger space with compatible mapping of type (P)*, International Journal of Math. Sci.and Engg. Appls., **15**(2) (2021), 59–70.

[7] G. JUNGCK: *Compatible Mapping and common fixed points*, Internat. J. Math. Sci. **9**(4) (1986), 771–779.

[8] G. JUNGCK: *Common fixed points for non-continuous non self-maps on non-metric spaces*, Far East. J. Math. Sci. **4**(2) (1996), 199–212.

[9] G. JUNGCK, P.P. MURTHY, Y.J. CHO: *Compatible mappings of type (A) and common fixed points*, Math. Japonica **38** (1993), 381–390.

[10] G. JUNGCK, B.E. RHOADES: *fixed point for set-valued functions without continuity*,Indian J. Pure Appl. Math. **29**(3) (1998), 227–238.

[11] K. JHA, V. POPA, K.B. MANANDHAR: *A common fixed point theorem for compatible mappings of type (K) in metric space*,Internat J. Math. Sci. and Engg. Appls **8**(1) (2014), 383–391.

[12] A. MEIR, E. KEELER: *A theorem on Contraction Mappings*,J. Math. Anal. Appl. **28** (1969), 326–329.

[13] K. MENGER: *Statistical Matrices*, Proceedings of National Academy of Sciences of USA, **28** (1942), 535–537.

[14] S.N. MISHRA: *Common fixed points of Compatible Mappings in probabilistic Metric Space*, Math.Japonica. **36** (1991), 283–289.

[15] B.SCHWEIZER, A. SKLAR : *Probabilistic Metric space*, Dover Publications, INC, Mineola, New York, 2005.

[16] V.M. SEHGAL, A.T. BHARUCHA-REID : *Fixed Point contraction mapping in Probabilistic Metric Space*, Math. System Theory, **6** (1972), 97–102.

[17] B. SINGH, S. JAIN : *Common Fixed Point theorem in Menger Space through Weak Compatibility*, J. Math. Anal. Appl. **301** (2005), 439–448.

[18] B. SCHWEIZER, A. SKLAR : *Statistical metric space*, Pacific J. of Math. **10** (1960), 314–334.

[19] S. SESSA: *On a weak commutativity condition of mappings in fixed point considerations*, Publ. Inst. Math. (Beograd) (N.S.) **32**(46) (1982), 149–153.

[20] S. SESSA, B.E. RHOADES, M.S. KHAN: *On common fixed points of compatible mappings in metric and Banach Spaces*, Internat. J. Math. Math. Sci. **11**(2) (1988), 375–392.

DEPARTMENT OF MATHEMATICS, TRICHANDRA MULTIPLE CAMPUS, TRIBHUVAN UNIVERSITY GHANTAGHAR, KATHMANDU, NEPAL AND DEPARTMENT OF MATHEMATICS, SCHOOL OF SCIENCE, KATHMANDU UNIVERSITY, KAVRE, NEPAL.
*Email address*: akcsaurya81@gmail.com

DEPARTMENT OF MATHEMATICS, SCHOOL OF SCIENCE, KATHMANDU UNIVERSITY, KAVRE, NEPAL.
*Email address*: jhakn@ku.edu.np

DEPARTMENT OF MATHEMATICS, SCHOOL OF SCIENCE,KATHMANDU UNIVERSITY, KAVRE, NEPAL.
*Email address*: kb.manandhar@ku.edu.np