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NEUTRINO MASSES IN A LEFT-RIGHT MODEL WITH A LIGHT NEUTRINO MIRROR

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ABSTRACT. In the framework of the extension of the Standard Model with gauge symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$ and additional exotic fermions known as mirror fermions, neutrino masses and mixing are studied. Starting from the most general Majorana neutrino mass matrix, we explore the possibility that one neutrino mirror plays the role of a sterile neutrino with mass on the scale of a few $eV's$, that is, $\hat{m}_1 \approx O(1\, eV)$.

1. INTRODUCTION

Neutrino oscillation experiments on atmospheric, solar, reactor and accelerators neutrinos reveal that neutrinos have nonzero masses. The tiny neutrino masses may be the first clear evidence of physics beyond the standard model. Cosmology and Short Baseline Oscillation experiments lead to the possible existence of light sterile neutrinos [1].

Here we study neutrino masses within the context of the "Left Right Mirror Model" (LRMM) [4]. Applying a double seesaw approximation to the most general Majorana-type neutrino mass matrix, we perform an approximate analytical

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diagonalization, allowing that one of the mirror neutrinos may get a mass of a few eV´s.

2. Model and Spontaneous symmetry breaking

The LRMM formulation is based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$. with the fermion content:

$$\begin{aligned} l^0_{iL} &= \begin{pmatrix} \nu^0_i \\ e^0_i \end{pmatrix}_L, e^0_{iR}, \nu^0_{iR}; \qquad \widehat{l}^0_{iR} = \begin{pmatrix} \widehat{\nu}^0_i \\ \widehat{e}^0_i \end{pmatrix}_R, \widehat{e}^0_{iL}, \widehat{\nu}^0_{iL}, \\ Q^0_{iL} &= \begin{pmatrix} u^0_i \\ d^0_i \end{pmatrix}_L, u^0_{iR}, d^0_{iR}; \qquad \widehat{Q}^0_{iR} = \begin{pmatrix} \widehat{u}^0_i \\ \widehat{d}^0_i \end{pmatrix}_R, \widehat{u}^0_{iL}, \widehat{d}^0_{iL}. \end{aligned}$$

The quantum numbers of fermions under the gauge group G defined above are given by

$$\begin{aligned} l^0_{iL} \sim (1,2,1,-1)_{iL}, \nu^0_{iR} \sim (1,1,1,0)_{iR}, e^0_{iR} \sim (1,1,1,-2)_{iR} \\ \widehat{\nu}^0_{iL} \sim (1,1,1,0)_{iL}, \widehat{e}^0_{iL} \sim (1,1,1,-2)_{iL}, \widehat{l}^0_{iR} \sim (1,1,2,-1)_{iR} \\ u^0_{iR} \sim (3,1,1,\frac{4}{3})_{iR}, d^0_{iR} \sim (3,1,1,\frac{2}{3})_{iR} \\ \widehat{u}^0_{iL} \sim (3,1,1,\frac{4}{3})_{iL}, \widehat{d}^0_{iL} \sim (3,1,1,\frac{2}{3})_{iL} \\ Q^0_{iL} \sim (3,2,1,\frac{1}{3})_{iL}, \widehat{Q}^0_{iR} \sim (3,1,2,\frac{1}{3})_{iR}., \end{aligned}$$

where the last entry corresponds to the hypercharge (Y') with the electric charge $Q = \mathrm{T}_{3L} + \mathrm{T}_{3R} + \frac{Y'}{2}$.

We assume the "Spontaneous Symmetry Breaking" (SSB) stages

$$\mathrm{G} \xrightarrow{\langle\hat{\Phi}\rangle} \mathrm{G}_{SM} \xrightarrow{\langle\Phi\rangle} \mathrm{SU(3)}_C \otimes \mathrm{U(1)}_Q, \tag{2.1}$$

where $G_{SM} = SU(2)_L \otimes SU(3)_C \otimes U(1)_Y$ is the SM gauge group and $\frac{Y}{2} = T_{3R} + \frac{Y'}{2}$. The Higgs sector to induce this SSB, Eq.(2.1), involves two doublets of scalar fields:

$$\Phi = (1,2,1,1), \qquad \hat{\Phi} = (1,1,2,1), \tag{2.2}$$

with the "Vacuum Expectation Values" (VEV's)

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad \langle\hat{\Phi}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \hat{v}\end{pmatrix}. \tag{2.3}$$

The most general potential that we can write with the above scalars, Eq.(2.2), which develops the pattern of VEV's, Eq.(2.3), and then the SSB, Eq.(2.1), is

$$V = -(\mu\,\Phi^\dagger\,\Phi + \hat{\mu}\,\hat{\Phi}^\dagger\,\hat{\Phi}) + \frac{\lambda_1}{2}\,[(\Phi^\dagger\,\Phi)^2 + (\hat{\Phi}^\dagger\,\hat{\Phi})^2] + \lambda_2\,(\Phi^\dagger\,\Phi)\,(\hat{\Phi}^\dagger\,\hat{\Phi})\,, \tag{2.4}$$

and the scalar Lagrangian for the model is written as

$$\mathcal{L}_{sc} = (D_\mu\,\Phi)^\dagger(D^\mu\,\Phi) + (\hat{D}_\mu\,\hat{\Phi})^\dagger(\hat{D}^\mu\,\hat{\Phi}), \tag{2.5}$$

where D_μ and $\hat{D}_\mu$ are the covariant derivatives for the SM and the mirror part, respectively.

3. Majorana neutrino mass matrix

After symmetry breaking, we may write the most general neutrino Majorana mass matrix

$$\left(\overline{\Psi}_{\nu L}, \overline{\Psi^c}_{\nu L}\right)\begin{pmatrix} M_L & M_D\\ M_D^T & M_R\end{pmatrix}\begin{pmatrix}(\Psi^c_\nu)_R\\ (\Psi_\nu)_R\end{pmatrix} \tag{3.1}$$

with

$$(\Psi_\nu)_{L,R} = \begin{pmatrix}\nu_i\\ \hat{\nu}_i\end{pmatrix}_{L,R}, \qquad (\Psi^c_\nu)_{L,R} = \begin{pmatrix}(\nu^c_i)\\ (\hat{\nu}^c_i)\end{pmatrix}_{L,R}, \tag{3.2}$$

$$M_L = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}}\,\sigma\\ \frac{v}{\sqrt{2}}\,\sigma^T & \hat{M}\end{pmatrix}, \qquad M_R = \begin{pmatrix} \chi & \frac{\hat{v}}{\sqrt{2}}\,\pi\\ \frac{\hat{v}}{\sqrt{2}}\,\pi^T & 0\end{pmatrix}, \tag{3.3}$$

$$M_D = \begin{pmatrix} \frac{v}{\sqrt{2}}\,\lambda & 0\\ h & \frac{\hat{v}}{\sqrt{2}}\,\eta\end{pmatrix}, \tag{3.4}$$

where σ, $\hat{M}$, χ, π, λ, h and η in Eqs.(3.3 , 3.4) are unknown matrices of 3×3 dimension.

3.1. **Double Seesaw approximation.**

By assuming the natural hierarchy $|(M_L)_{ij}| \ll |(M_D)_{ij}| \ll |(M_R)_{ij}|$ for the mass terms, the mass matrix $\begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$, Eq.(3.1), can be diagonalized in the Seesaw approximation [8], yielding

$$\left(\overline{\Psi'}_{\nu L}, \overline{\Psi'^c}_{\nu L}\right) \begin{pmatrix} M_\nu & 0 \\ 0 & M_R \end{pmatrix} \begin{pmatrix} (\Psi'^c_\nu)_R \\ (\Psi'_\nu)_R \end{pmatrix}, \tag{3.5}$$

where, neglecting $\mathcal{O}\left(M_D\, M_R^{-1}\right)$ terms, we may write with good approximation $\Psi'_{\nu L,R} \approx \Psi_{\nu L,R}$, and $\Psi'^c_{\nu L,R} \approx \Psi^c_{\nu L,R}$.

The Majorana mass matrix for the left handed neutrinos may be written in this seesaw approximation as

$$M_\nu \approx M_L - M_D\, M_R^{-1}\, M_D^T\,. \tag{3.6}$$

We assume a scenario where the dominant contribution for the active known neutrinos comes from the M_L matrix, Eq.(3.6), having the same structure of a Type I seesaw. We can explicitly write

$$M_\nu \approx M_L = \begin{pmatrix} m & \mu \\ \mu^T & \hat{m} \end{pmatrix}, \tag{3.7}$$

where

$$m = \begin{pmatrix} 0 & 0 & 0 & \sigma'_{11} \\ 0 & 0 & 0 & \sigma'_{21} \\ 0 & 0 & 0 & \sigma'_{31} \\ \sigma'_{11} & \sigma'_{21} & \sigma'_{31} & \hat{M}_{11} \end{pmatrix}, \; \mu = \begin{pmatrix} \sigma'_{12} & \sigma'_{13} \\ \sigma'_{22} & \sigma'_{23} \\ \sigma'_{32} & \sigma'_{33} \\ \hat{M}_{12} & \hat{M}_{13} \end{pmatrix}, \; \hat{m} = \begin{pmatrix} \hat{M}_{22} & \hat{M}_{23} \\ \hat{M}_{23} & \hat{M}_{33} \end{pmatrix},$$

and $\sigma'_{ij} = \frac{v}{\sqrt{2}}\, \sigma_{ij}$. In this way we explore the possibility that one of the mirror neutrinos may acquire a mass of the order of a few $eV's$. Therefore, applying the seesaw approximation again to M_L, Eq.(3.7), we obtain

$$(M^{light})_{4\times 4} = m - \mu\, \hat{m}^{-1}\, \mu^T. \tag{3.8}$$

The matrix M_L in Eq.(3.7), may be diagonalized by using a unitary transformation

$$U^\dagger\, M_L\, U = Diag\,(m_1, m_2, m_3, \hat{m}_1, \hat{m}_2, \hat{m}_3)\ , \tag{3.9}$$

where the mixing matrix U compatible with our framework may be written as

$$U_{6\times6} \approx \begin{pmatrix} U_{4\times4} & \mu\,\hat{m}^{-1} \\ -(\mu\,\hat{m}^{-1})^T & I_{2\times2} \end{pmatrix}, \tag{3.10}$$

where

$$U_{4\times4} \approx \begin{pmatrix} & & & U_{14} \\ & (U_{TB})_{3\times3} & & U_{24} \\ & & & U_{34} \\ U_{41} & U_{42} & U_{43} & O(\lesssim 1) \end{pmatrix}, \quad |U_{i4}| \approx |U_{4i}| \lesssim 0.1, \tag{3.11}$$

$U_{4\times4}$, Eq.(3.11), being the neutrino mixing involving the three SM active neutrinos and one light neutrino mirror.

4. Conclusions

Applying a double seesaw approximation to the most general Majorana-type neutrino mass matrix, an approximate analytical diagonalization was performed for light neutrinos, Eqs.(3.8 , 3.11), allowing the possibility that one neutrino mirror plays the role of a sterile neutrino with mass on the scale of a few $eV's$, that is, $\hat{m}_1 \approx O(1\,eV)$.

A numerical fit of parameters is in progress in order to accommodate simultaneously the active neutrino masses and mixing consistent with the current neutrino oscillation data.

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