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MATHEMATICAL MODEL TO EVALUATE NON-DETERMINISTIC CUMULATIVE DAMAGE IN AN ORGANIZATION

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ABSTRACT. Unsystematic events like shocks can harm an organization and lead to its demise. The damage is not harmful by itself because of the one-failure time. The organization could eventually collapse due to the alternating damage. The organization fails once the total harm reaches a particular point. To ascertain when the organization's response plan is necessary, the inter-arrival time of harm is estimated. This study will employ the Exponentiated Exponential Binomial Distribution.

1. INTRODUCTION

Itemized shock models, according to Nakagawa [1], are frequently utilized in a range of industries, including dependability, infrastructure engineering, insurance, credit risk, and more. Nakagawa also took into account the cumulative damage model and the independent damage model, which both assume that total damage is additive. This study focuses on the cumulative damage model, in which the exponentiated exponential binomial distribution (EEBD) represents the entire damage as additive. Exponentiated Gammam Distribution (EGD) was utilized by Vijaya and Jaikar [2] to compute a cumulative damage shock model in

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an organization throughout the renewal process. The inference showed that the predicted time dropped as the inter-arrival increased when the parameters were set for different time periods.

An employee's exit from an organization was studied using a shock model technique by Ahmed Al Kuwaiti et al. [3], who came to the conclusion that recruitment should take place on a regular basis after the person leaves the company. They came to the conclusion that the employee's anticipated time of departure dropped as the inter-arrival time rose. In order to assess the demand for recruitment and the time to harm in an organization by determining the expected time, Kannadasan et al. [4] employed the Three Parameter Generalized Exponential Distribution (GED). In our present simulation work, we obtained a similar outcome using EEBD.

In an EEBD scenario, failure results from the occurrence of a random variable N initial flaws of the same type, according to Bakouch et al [5]. Other distributions, including the exponential distribution, exponentiated exponential distribution, exponential binomial distribution, and beta generalized exponential distribution, appear when the EEBD's parameters are altered. Additionally, it was determined that EEBD offers the best match out of all the distributions examined. Assumptions are widely used in the development of mathematical models, therefore these models are best and good based on the assumptions established. By employing a random variable, our model will be beneficial for calculating the likelihood of EEBD outcomes in an organization. Forecasting and identifying unanticipated or hidden elements that have an impact on the organization.

2. Description of Exponentiated Exponential Binomial Distribution

A continuous distribution with decreasing, increasing, and upside-down bathtub hazard rate shapes that offers a more adaptable distribution for modeling lifetime data, notably in terms of reliability. X_i : is a continuous random variable that represents the amount of Failure to the organization on the i^{th} contact.

The Distribution Function of EEBD

$$F(x) = \frac{1 - (1 - \theta(1 - e^{-\lambda x})^{\alpha})^{n}}{1 - \theta^{-n}}. \tag{2.1}$$

The corresponding Survival Function is

$$H(\overline{x}) = 1 - F(x) = 1 - \frac{\theta}{1-\theta^{-1}} \sum_{r=0}^{n} (-1)^r \binom{\alpha}{r} e^{-\lambda r x}. \tag{2.2}$$

It is possible that inspecting an individual item to determine the failure threshold Y is impractical. Y: a continuous random variable denoting the threshold level with EEBD. In general, EEBD with parameter θ is followed by the threshold Y. The threshold in this case must be a random variable. g(.) The probability density functions of X_i; g_k (.): The k- fold convolution of g (.) i.e., p.d.f. of $\sum_{j=1}^{k} X_i$; $g_k^*(.)$ Laplace transform of $g_k(.)$; $g^*(.)$ Laplace transform of g(.). The shock survival probability are given by

$$P(X_i < Y) = \int_0^\infty g_k(x)\overline{H}(x) = [g^*(1)]^k - \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{\alpha} (-1)^r \binom{\alpha}{r} [g^*(\lambda)]^k. \tag{2.3}$$

The probability that the cumulative damage of threshold will fail only after time t is indicated by the survival function. The interval between two successive shocks, the harm a shock causes, and the breakdown of an organization serve as the criteria for shock models. When the overall cumulative damage surpasses a predetermined threshold, it is likely that any factor that is subject to shocks that harm the organization will fail. This threshold is known as the failure rate. Even if the shocks are independent, it's feasible that they will gradually grow more damaging even if they aren't. So,

$$\begin{aligned} S(t) = P(T > t) &= \text{Probability that the total damage survives beyond t} \\ &= \sum_{k=0}^{\infty} \{\text{there are exactly } k \text{decisions in} (0,t] \\ &\quad * P(\text{the total cumulative threshold } (0,t]\} . \end{aligned}$$

The survival function S(t) which is the probability that an individual survives for a time t. Next, $V_k(t)$ is the probability that there are exactly k contacts; and $V_k(t) : F_k(t) - F_(k+1)(t); F_k(.)$ is the k-fold convolution functions of F (.). It is also known from renewal process that

$$
\begin{aligned}
P(T>t) &= \sum_{k=0}^{\infty} V_k(t) P(X_i<y) \\
&= \sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)]\left[(g^*(1))^k - \frac{\theta}{1-\theta^{-1}}\sum_{r=1}^{n}(-1)^r\binom{\alpha}{r}[g^*(\lambda r)]^k\right] \\
&= 1-(1-g^*(1))\sum_{k=1}^{\infty}F_k(t)(g^*(1))^{k-1} \\
&\quad -\sum_{r=1}^{n}(-1)^r\binom{\alpha}{r}\left[1-(1-g^*(\lambda r))\sum_{k=1}^{\infty}F_k(t)(g^*(\lambda r))^{k-1}\right] \\
&= 1-(1-g^*(1))\sum_{k=1}^{\infty}F_k(t)(g^*(1))^{k-1}-\frac{\theta}{1-\theta} \\
&\quad +\sum_{k=1}^{\infty}\sum_{r=1}^{n}(-1)^{r+1}\binom{\alpha}{r}\frac{\theta}{1-\theta^{-1}}(1-g^*(\lambda r))F_k(t)(g^*(\lambda r))^{k-1}.
\end{aligned} \tag{2.4}
$$

3. Assessing failure rate of life time L(t)

Lifetimes, failure times, and survival data are all terms used to describe data that measures "the length of time" until an event occurs. Now, the life time is given by

$$P(T<t)=L(t)=1-S(t) = \text{the distribution function of life time } (t).$$

Using convolution theorem for Laplace transforms, $F_0(t)=1$ and on simplification, as seen in equation (2.4) is equal to

$$
\begin{aligned}
&(1-g^*(1))\sum_{k=1}^{\infty}F_k(t)(g^*(1))^{k-1}+\frac{\theta}{1-\theta^{-1}} \\
&+\frac{\theta}{1-\theta^{-1}}\sum_{k=1}^{\infty}\sum_{r=1}^{n}(-1)^{r+1}\binom{\alpha}{r}(1-g^*(\lambda r))^{k-1}F_k(t)(g^*(\lambda r))^{k-1}.
\end{aligned} \tag{3.1}
$$

By taking Laplace-Stieltjes transform, it can be show that

$$l^*(s)=\frac{[(1-g^*(1))f^*(s)]}{[(1-g^*(1))f^*(s)]}+\frac{\theta}{1-\theta^{-1}}\sum_{r=1}^{n}(-1)^{r+1}\binom{\alpha}{r}\frac{[(1-g^*(\lambda r))c]}{[(c+s-g^*(\lambda r))c]}. \tag{3.2}$$

Let a random variable denoting the inter-arrival times between contact with c.d.f. $F_i(.), i = 1, 2, 3, \ldots, k$ which follows exponential with parameter. Now $f^*(S) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (3.2) we get,

$$= \frac{[(1 - g^*(1))c]}{[c + s - g^*(1)c]} + \frac{\theta}{1 - \theta^{-1}} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{[(1 - g^*(\lambda r))c]}{[(c + s - g^*(\lambda r))c]}$$

The **EBBD** parameter is embedded in the proposed model. Some mathematical properties, as well as an estimate, are used to calculate the expected time:

$$\begin{aligned} E(T) &= \frac{-d}{ds} l^*(s)/s \\ &= \left[\frac{1}{c[1 - g^*(1)]} + \frac{\theta}{1 - \theta^{-1}} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{1}{c[1 - g^*(\lambda r)]} \right] = 0, \end{aligned} \tag{3.3}$$

$g^*(1) \approx \frac{1}{\mu}$, $g^*(\lambda r) \approx \frac{\mu}{\mu + 2r}$, and

$$E(T) = \frac{\mu}{c[\mu - 1]} + \frac{\theta}{1 - \theta^{-1}} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{\mu + \lambda r}{c[\lambda r]}. \tag{3.4}$$

The simulation part is carried out using the Mathcad 7 professional software, based on the expected time as shown in (3.4).

4. Conclusion

The cumulative harm to the anticipated recruitment time diminishes in an organization when the parameter distribution is fixed over various time frames, as illustrated in all four figures. Less failure was seen in the initial stage as the inter-arrival time c rose when the parameter values were fixed. It is revealed that the projected time to recruitment in the organization is longer as the occurrence of damage in the organization is more as the parameter value grows in all instances. When all of the parameters were altered, a comparable outcome was attained. One of the different distributions used to estimate the anticipated time in an organization using a shock model is the EEBD distribution. The Exponentiated Exponantial Distribution (EED), created by [6], can also be used to determine the anticipated time decreases in an organization. As can be observed, a decrease in an organization's cumulative harm was implied by all three distributions (EED, EGD, and GED). The conclusion drawn in [5] that the EEBD offers the greatest fit among the other distributions investigated is substantially supported by the findings of this study.

TABLE 1. Expected values of all the paramter change

	α			λ			μ			θ		
c	50	75	100	2	3	4	2	3	4	0.2	0.3	0.4
10	1.5	2.25	3	1.875	1.75	1.688	22.5	28	34.5	1.2	2.2	2.8
20	1	1.5	2	1.25	1.167	1.125	8.75	11.375	14.25	1.1	1.6	1.9
30	0.722	1.083	1.44	0.903	0.843	0.813	5.278	7	8.833	0.844	1.178	1.378
40	0.563	0.844	1.125	0.703	0.656	0.633	3.75	5.031	6.375	0.675	0.925	1.075
50	0.46	0.69	0.92	0.575	0.537	0.517	2.9	3.92	4.98	0.56	0.76	0.88
60	0.389	0.583	0.778	0.486	0.454	0.437	2.361	3.208	4.083	0.478	0.644	0.744
70	0.337	0.505	0.673	0.421	0.393	0.379	1.99	2.714	3.459	0.416	0.559	0.645
80	0.297	0.445	0.594	0.371	0.346	0.334	1.719	2.352	3	0.369	0.494	0.569
90	0.265	0.398	0.531	0.332	0.31	0.299	1.512	2.074	2.648	0.331	0.442	0.509
100	0.24	0.36	0.48	0.3	0.28	0.27	1.35	1.855	2.37	0.3	0.4	0.46

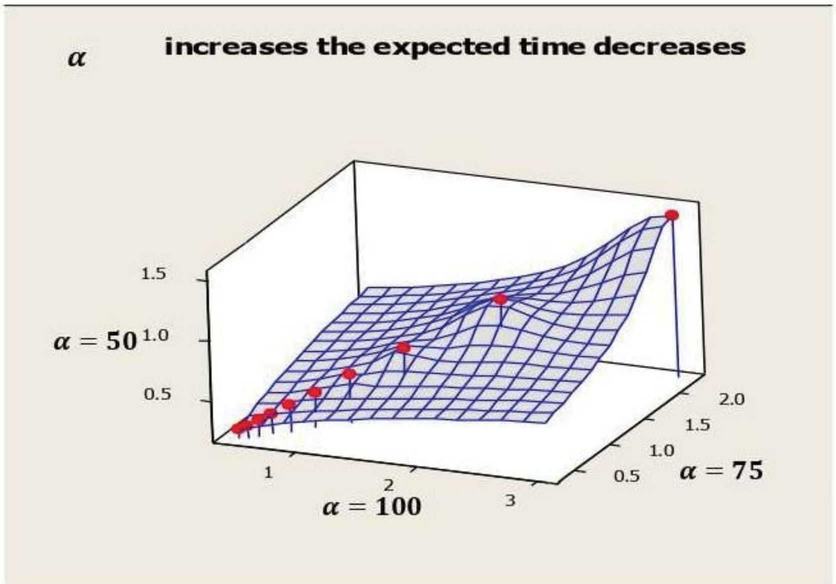

Figure 1 Parameter α increases the expected time decreases

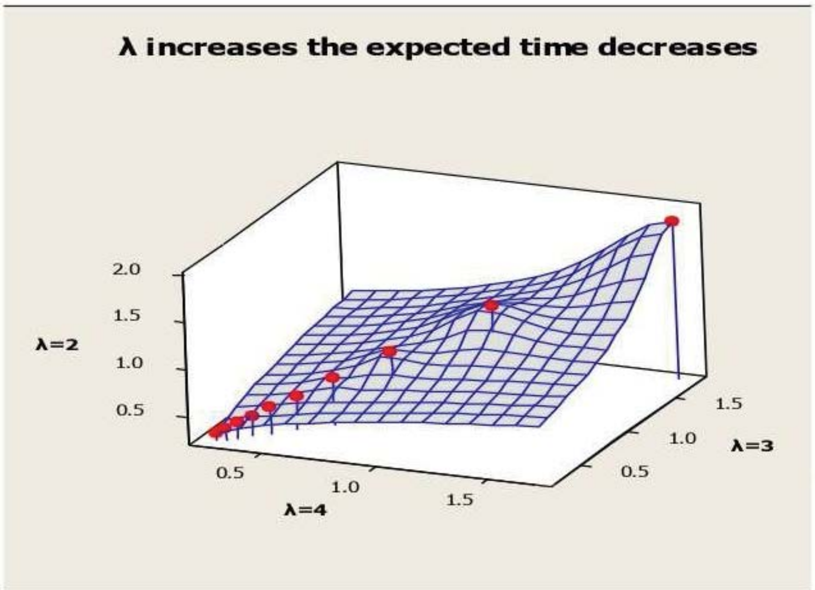

Figure 2 Parameter λ increases the expected time decreases

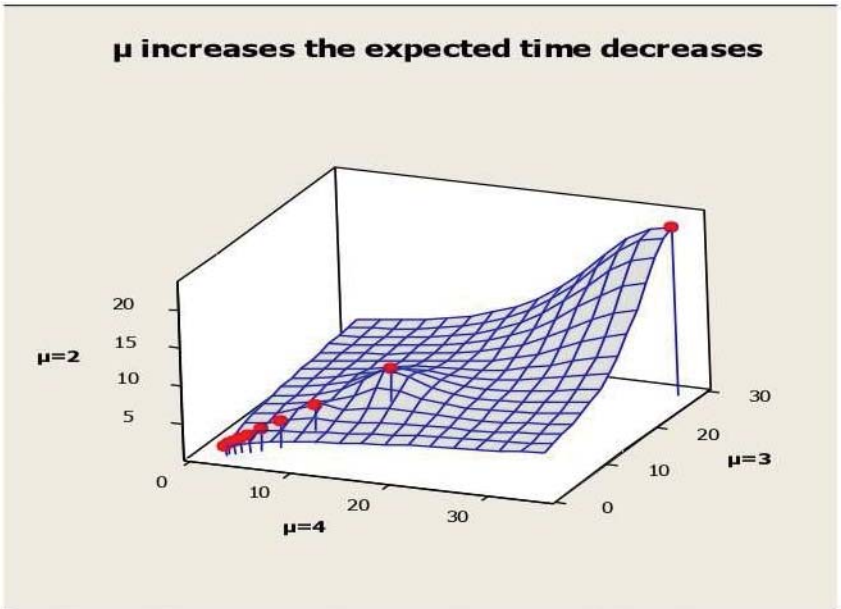

Figure 3 Parameter μ increases the expected time decreases

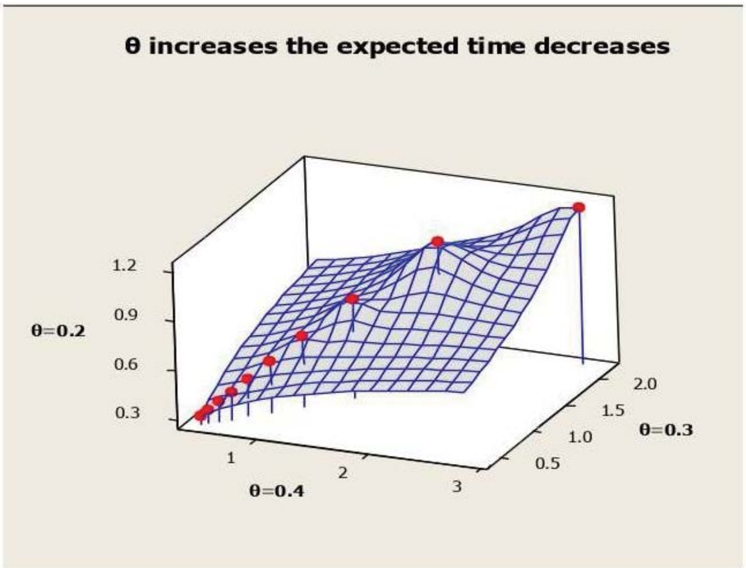

Figure 4 Parameter θ increases the expected time decreases

References

[1] T. Nakagaw: *Shocks and Damage Models in Reliability Theory*, Springer, London, 2007.

[2] S. Vijaya, R. Jaikar: *Observing the renewal period in an organization through mathematical model*, Journal of Reliability and Statistical Studies, **10** (2017), 137-142.

[3] A.K. Ahmed, R. Vinoth, A.V. Subbarayalu, R.M. Palanivel, P. Sivasankar: *Predicting The Exit Time of Employees in an Organization Using Statistical Model*, International Journal of Scientific and Technology Research, **5** (2016), 213-217.

[4] K. Kannadasan, P. Pandiyan, R. Vinoth, R. Saminathan:*Time to Recruitment in an Organization through Three Parameter Generalized Exponential Model*, Journal of Reliability and Statistical Studies, **6** (2013), 21-28.

[5] H.S. Bakouch, M.M. Ristic, A. Asgharzadeh, L. Esmaily: *An Exponentiated exponential binomial distribution with application*, Statistics and Probability Letters, **82** (2012), 1067-1081.

[6] S. Parthasarathy, M.K. Ravichandran, R. Vinoth: *An Application of Stochastic Models – Grading System in Manpower Planning*, International Business Research, bf3 (2010), 79-86.

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