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THERMAL RADIATION IMPACT AND CATTANEO-CHRISTOV THEORY FOR UNSTEADY FLOW OF MAXWELL FLUID OVER STRETCHED CYLINDER WITH INCONSISTENT HEAT SOURCE/SINK

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ABSTRACT. Examination of thermal and solutal energy with sophisticated guesstimate of thermal radiation bearing. Conveyance portents in Maxwell fluid pour thru the gain of Cattaneo-Christov double diffusion theory is fulfilled in this artefact. Unsteady 2D flow of Maxwell fluid with variable thermal conductivity over the stretching cylinder with thermal emission and heat source/sink is deliberated here. We verbalize the partial differential equations (PDEs) under particular molds for the governing physical tricky of heat and mass transportation in Maxwell fluid by using double diffusion of Cattaneo-Christov model rather than classical Fourier's and Fick's law. Numerical technique 4^{th} order Runge-Kutta method is employed for the solution of ordinary differential reckonings (ODEs) which are obtained from governing PDEs under the apt resemblance transformations. In the interpretation of acquired fallouts, we beheld that for fitting upshots the tenets of unsteadiness constraint should be less than one. The higher tenets of Maxwell parameter deteriorations the flow field but increase the energy transport in the fluid flow. Both temperature and attentiveness scatterings in Maxwell liquid deterioration for sophisticated tenets of thermal and attentiveness relaxation time constraint. Alike, trifling thermal conductivity constraint also augments the temperature field. Auxiliary, the rate of heat transfer deteriorations.

2020 *Mathematics Subject Classification.* 37E35, 76B75.

Key words and phrases. Thermal radiation, Cattaneo–Christov model, Maxwell fluid, Variable thermal conductivity, Heat generation or absorption.

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1. Introduction

The heat transfer is an ample receptiveness of flora, as of the malaise contrast amid or esoteric the body as an upshot of the squabble of locomotive verve. Industrialized solicitations of heat transferal sort as of meek inert tactics to unconventional multi-loop maneuvers that prompt sundry tasks through the industrialized drill. Due to voluminous transmogrifications in the intention and headway of heat transference, an incalculably colossal figure of crafts are ill-using this proficiency, encircling air-cooled exchanger, fissile receptacle freezing, condenser, reheating and freezing vats, shrinking cistern, warmth transferal in nerve, and sundry added. The updraft conductivity act predictable by Fourier [1] pitches the rock stratum for soothsaying heat altercation presages in unrelated renown. Lone of the prominent blemishes of this act affords a parabolic warmth reckoning for malaise in nous that an early riot is fondled proximately in the unabridged run-of-the-mill. To conquer this jagged issue updraft lessening spell of warmth conveyance affixed in customary Fourier's act by Cattaneo [2], which endorsements heat transferal at curbed rapidity concluded the proliferation of updraft surfs. Updraft letup spell can be tangibly reinforced as the spell looked-for to tolerate a sturdy state-run of updraft conveyance afterwards the malaise incline has been sited. To preclude the invariant conceiving of factual, Christov [3] cohesive the upper-convicted Holroyd derived in the Cattaneo prototypical of warmness conveyance and denominated it by Cattaneo–Christov warmth unrest exemplary, quite a lot of canvassers paid out this inkling in their exploration. Ciarletta [4] explored the inimitability and operational permanency of the Cattaneo–Christov archetypal to appearance the elucidation unendingly hinge on upon fairness spell. The bearing of updraft convection on the incompressible runny stream in a conservative coat is envisaged by Straughan [5]. In the permanency of Ciarletta's exertion, Tibullo [6] mused the rareness of tenacities of this archetypal aimed at an incompressible runny. Sundry canvassers have smeared the statute of Cattaneo–Christov to mixt genera of waters, for instance viscoelastic liquefied [7–9], nanofluids [10–12], and Jeffery watery [13]. Hayat likewise painstaking the Cattaneo–Christov archetypal aimed at nonlinear overextended vastness and inertia socket pour [14–16]. Sundry facets of heat allocation are premeditated in [17–20] prevailing far detached as of frontier surroundings and geometry.

Customarily, the coarse pours inclosing non-cohesive dregs whereabouts or goblet specks are pigeonholed as non-Newtonian liquids. The grainy stream mockups for instance Bingham, Herschel–Bulkley and Cross precisely have viscoelastic conducts. In our time, the non-Newtonian elucidations are persistently bump into and enflamed the unadorned allure by motive of the widespread sort of built-up and high-tech solicitations and organism gages (e.g., thaws of polymers, biotic elucidations, dyes, as of hefty oil conveyance clinched elongated reserves, to mass handover procedures and emulsification in micro-pipes etc.). Bahiraei and Alighardashi [21] probe the non-Newtonian runny stream on the pedestals of decree of thermodynamics. Bahiraei et al. [22] heightened the hydrothermal chattels for non-Newtonian runny in the concentric-tube warmth exchanger. Khan et al. [23] expounded the facets of galvanization oomph aimed at 3D tide of non-Newtonian runny. The solicitation recycled aimed at oomph to ripen the hectic advection in rivulet on non-Newtonian runny was willful by Bahiraei et al. [24]. Bahiraei et al. [25] conferred the CFD mockup aimed at the drift of power-law nanofluid per hectic disquiet.

Raju et al. [26] wilful the stream of non-Newtonian nanofluid completed the shaft underneath the sway of compelling turf. Stream of Maxwell runny thru warmth conveyance per the sway of gluey debauchery and emission was painstaking by Hsiao [27]. Bid of Carreau nanofuid pour to enrich the warmth transference in updraft extrusion built-up dexterity was reconnoitered by Hsiao [28]. Ahmed et al. [29] statistically probed the torpor fact stream of Maxwell liquefied completed the leaky circling and elongating diskette, and their fallouts naked that outward rapidity rises thru sophisticated tenets of rapidity proportion constraint. Moshkin et al. [30] scrutinized the transitory torpor socket 2D flow of Maxwell runny.

In outlook of the upstairs exhaustive revisions, we offered this reminder to reconnoiter the warmth and corpus conveyance in Maxwell runny stream concluded the fraught canister in rickety ceremonial in the occurrence of updraft emission and warmth cohort or preoccupation. For the materialization of oomph and attentiveness reckonings, we exploit the Cattaneo–Christov duple dispersion notion. Arithmetical elucidations by the service of bvp4c utility are gotten aimed at prevailing carnal delinquent. The upshots aimed at haste, malaise and attentiveness arenas are accessible graphically and chatted in facet thru the assistance of corporeal defense.

2. Mathematical formulation

Ponder the tottering laminar 2D tide of an incompressible Maxwell runny thru fickle updraft conductivity swayed by elongating canister of ambit $\mathrm{R_1}$. Presume that $u\&w$ are rapidity apparatuses sideways z-axis (bloc of canister) & r-axis (standard to z-axis), singly as existing in Fig. 1. The time-dependent elongating rapidity of the canister is presumed to be $u_w(t,z) = \frac{az}{1-\gamma t}$ where $a, \gamma > 0$. Updraft and solutal oomph conveyance in the stream is steered by via Cattaneo-Christov duple dissemination notion. The Maxwell runny exemplary is look after as [31]

$$\left(1+\lambda_1\frac{D}{Dt}\right)S = \mu A_1. \tag{2.1}$$

Here λ_1 - letup spell, $\frac{D}{Dt}$ connotes the Oldroyd offshoot, S - spare trauma tensor, $A_1 = \nabla V + (\nabla V)^T$ - 1st Rivlin-Erickson tensor & μ-gluiness of runny. If $\lambda_1 = 0$ in Eq. 2.1 we mend the Newtonian molten prototypical. The CattaneoChristov prototypical for conveyance of warmth and corpus are recycled afore orthodox Fourier's & Fick's decrees. If $q\&J$ - warmth and corpus fluxes, one-to-one, then in cooperation mollify the succeeding precise lexes [32, 33]

$$q+\lambda_t\left(\frac{\partial q}{\partial t}+V\cdot\nabla q+(\nabla V)q-q\cdot\nabla V\right) = -K(T)\nabla T, \tag{2.2}$$

$$J+\lambda_c\left(\frac{\partial J}{\partial t}+V\cdot\nabla J+(\nabla V)J-J\cdot\nabla V\right) = -D_B\nabla C. \tag{2.3}$$

In the overhead reckonings λ_t - updraft time easing $\&\lambda_c$ - corpus stint letup, $K(T) = k_\infty(1+\epsilon\theta)$-variable updraft conductivity (k_∞ free torrent conductivity, ϵ trifling conductivity constraint & θ - dimensionless malaise) & $\mathrm{D_B}$ Brownian dispersion quantity. If $\lambda_t = \lambda_c = 0$ then together Eqs. 2.2 & 2.3 shrink to usual Fourier's & Ficks's decrees, singly. Aimed at an incompressible molten, Eqs. 2.2 & 2.3 come to be

$$q+\lambda_t\left(\frac{\partial q}{\partial t}+V\cdot\nabla q-q\cdot\nabla V\right) = -K(T)\nabla T, \tag{2.4}$$

$$J+\lambda_c\left(\frac{\partial J}{\partial t}+V\cdot\nabla J-J\cdot\nabla V\right) = -D_B\nabla C. \tag{2.5}$$

The plain transference reckonings for drift, warmth & bulk transportation set up by safeguarding decree

$$\rho \frac{dV}{dt} = -\nabla p + \nabla \cdot S, \tag{2.6}$$

$$(\rho c_p) \frac{dT}{dt} = -\nabla \cdot q, \tag{2.7}$$

$$\frac{dC}{dt} = -\nabla \cdot J. \tag{2.8}$$

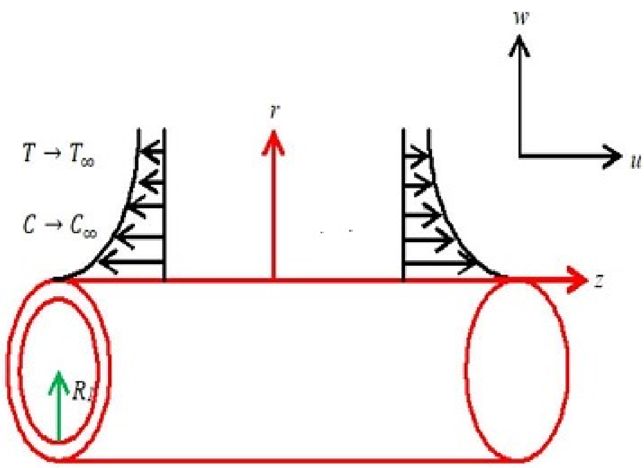

FIGURE 1. Flow configuration and coordinate system.

By smearing the decree of upkeep of corpus, impetus and oomph to the unabridged coordination of drift delinquent & jettisoning S in Eqs. 2.1 & 2.6, q in Eqs. 2.2 & 2.7 and J in Eqs. 2.3 & 2.8, singly, we attained at ensuing fixed of prevailing partial differential reckonings (Figure 1):

$$\frac{\partial(ru)}{\partial z} + \frac{\partial(rw)}{\partial r} = 0, \tag{2.9}$$

$$\begin{aligned} &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + w\frac{\partial u}{\partial r} = v\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] \\ &- \lambda_1\left[\frac{\partial^2 u}{\partial t^2} + 2u\frac{\partial^2 u}{\partial t \partial z} + 2w\frac{\partial^2 u}{\partial r \partial t} + 2uw\frac{\partial^2 u}{\partial r \partial z} + w^2\frac{\partial^2 u}{\partial r^2} + u^2\frac{\partial^2 u}{\partial z^2}\right]. \end{aligned} \tag{2.10}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial z}+w\frac{\partial T}{\partial r}+\lambda_t\left[\begin{array}{l}\frac{\partial^2 T}{\partial t^2}+\frac{\partial u}{\partial t}\frac{\partial T}{\partial z}+2u\frac{\partial^2 T}{\partial t\partial z}+\frac{\partial w}{\partial t}\frac{\partial T}{\partial r}\\+2w\frac{\partial^2 T}{\partial t\partial r}+2uw\frac{\partial^2 T}{\partial r\partial z}+w^2\frac{\partial^2 T}{\partial r^2}\\+u^2\frac{\partial^2 T}{\partial z^2}+u\frac{\partial u}{\partial z}\frac{\partial T}{\partial z}+w\frac{\partial u}{\partial r}\frac{\partial T}{\partial z}+\\u\frac{\partial w}{\partial z}\frac{\partial T}{\partial r}+w\frac{\partial w}{\partial r}\frac{\partial T}{\partial r}\end{array}\right]$$

(2.11)

$$=\frac{1}{(\rho c_p)_f}\frac{1}{r}\frac{\partial}{\partial r}\left[K(T)\left(r\frac{\partial T}{\partial r}\right)\right]+\frac{16\sigma^*T_\infty^3}{3k*(\rho c_p)_f}\frac{\partial^2 T}{\partial r^2}$$
$$+\frac{ku_w}{zv_f}\left[A^*\left(T_w-T_\infty\right)\exp\left(\sqrt{\frac{a}{v(1-\gamma t)}}\left(\frac{r^2-R_1^2}{2R_1}\right)\right)+B^*\left(T-T_\infty\right)\right],$$

(2.12)

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial z}+w\frac{\partial C}{\partial r}+\lambda_c\left[\begin{array}{l}\frac{\partial^2 C}{\partial t^2}+\frac{\partial u}{\partial t}\frac{\partial C}{\partial z}+2u\frac{\partial^2 C}{\partial t\partial z}+\frac{\partial w}{\partial t}\frac{\partial C}{\partial r}\\+2w\frac{\partial^2 C}{\partial t\partial r}+2uw\frac{\partial^2 C}{\partial r\partial z}+w^2\frac{\partial^2 C}{\partial r^2}\\+u^2\frac{\partial^2 C}{\partial z^2}+u\frac{\partial u}{\partial z}\frac{\partial C}{\partial z}+w\frac{\partial u}{\partial r}\frac{\partial C}{\partial z}\\+u\frac{\partial w}{\partial z}\frac{\partial C}{\partial r}+w\frac{\partial w}{\partial r}\frac{\partial C}{\partial r}\end{array}\right]$$
$$=D_B\frac{1}{r}\frac{\partial}{\partial r}\left[\left(r\frac{\partial C}{\partial r}\right)\right]$$

The agreeing periphery disorders aimed at agreed snags are

(2.13) $$u(\mathrm{t,z,r})=\mathrm{u}_w(t,z)=\frac{az}{1-\gamma t},w(t,z,r)=0,T=T_w,C=C_w\text{ at }r=R_1,$$

(2.14) $$u\to 0,T\to T_\infty,C\to C_\infty\text{ as }r\to\infty.$$

Here v - kinematic gluiness, $\mathrm{T_w}$&$\mathrm{C_w}$ - malaise & attentiveness at the hedge, singly, T_∞&C_∞ the free torrent malaise & attentiveness, singly, ρ - watery solidity & c_p - explicit heat bulk at relentless heaviness.

With the relief of succeeding adaptation constraints

(2.15)

$$u=-\frac{R_1}{r}\sqrt{\frac{av}{1-\gamma t}}f(\eta),w=\frac{az}{1-\gamma t}f'(\eta),\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$
$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\eta=\sqrt{\frac{a}{v(1-\gamma t)}}\left(\frac{r^2-R_1^2}{2R_1}\right).$$

Reckoning 2.9 is mollified inevitably and Eqs. 2.10 – 2.14 harvest

(2.16)

$$(1+2\alpha\eta)f'''+2\alpha ff''-\frac{S}{2}\eta f''-Sf'-f'^2+ff''-\frac{7}{4}\beta_1S^2\eta f''-\frac{\beta_1}{4}\eta^2S^2f'''$$
$$-2\beta_1S^2f'-2S\beta_1f'^2-\beta_1\eta Sf'f''+S\beta_1\eta ff'''+3S\beta_1ff''+2\beta_1ff'f''$$
$$-\frac{\alpha\beta_1}{1+2\alpha\eta}f^2f''-\beta_1f^2f'''=0,$$

3. Numerical procedure

$$
\begin{aligned}
&(1+2\alpha\eta)\theta''\left(1+\frac{4}{3}Rd\right)+\Pr\left(f\theta'-\frac{S}{2}\eta\theta'\right)+(1+2\alpha\eta)\left(\theta\theta''+\theta'^2\right)\varepsilon \\
&+2\alpha\theta'+2\alpha\varepsilon\theta\theta'-\Pr\beta_t\begin{pmatrix}\frac{3}{4}S^2\eta\theta'-\frac{3S}{2}\theta' f-\frac{S}{2}\eta\theta' f' \\ \frac{1}{4}S^2\eta^2\theta''-S\eta f\theta''+\theta'' f^2+\theta' f f'\end{pmatrix} \\
&+\Pr\left(A^*\exp(-\eta)+B^*\theta\right)=0,
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
&(1+2\alpha\eta)\phi''+Le\Pr\left(f\phi'-\frac{S}{2}\eta\phi'\right)+2\alpha\phi' \\
&-\mathrm{PrLe}\,\beta_c\begin{pmatrix}\frac{3}{4}S^2\eta\phi'-\frac{3S}{2}\phi' f-\frac{S}{2}\eta\phi' f' \\ \frac{1}{4}S^2\eta^2\phi''-S\eta f\phi''+\phi'' f^2+\phi' f f'\end{pmatrix}=0,
\end{aligned}
\tag{3.2}
$$

$$
f(0)=0, f'(0)=1, \theta(0)=1, \phi(0)=1, \tag{3.3}
$$

$$
f'(\infty)=0, \theta(\infty)=0, \phi(\infty)=0. \tag{3.4}
$$

In the beyond reckonings $S\left(\frac{\gamma}{a}\right)$ - wobbliness constraint, $\alpha\left(\frac{1}{R_1}\sqrt{\frac{v(1-\gamma t)}{a}}\right)$ - arch bound, $\beta_1\left(\frac{\lambda_1 a}{1-\gamma t}\right)$

- Maxwell constraint, $\Pr\left(\frac{v}{\alpha_1}\right)$ - Prandtl number, $Rd\begin{pmatrix}4\sigma & T_\infty^3 \\ k & k\end{pmatrix}$ – emission constraint, $\beta_t\left(\frac{\lambda_t a}{1-\gamma t}\right)$ - updraft letup while constraint, $\beta_c\left(\frac{\lambda_c a}{1-\gamma t}\right)$ - mass letup stint constraint & $\left(\frac{\alpha_1}{D_B}\right)$ Lewis number.

The condensed Eqs. 2.16– 3.4 are vastly non-linear and tied in flora, so their padlocked custom elucidations are not conceivable. They can be deciphered arithmetically via Runge-Kutta-4 (RK4) thru shelling routine aimed at unlike tenets of constraints. The paraphernalia of the unindustrialized constraints on the dimensionless haste, malaise, attentiveness, heat and mass transferal tariffs are willful realistically. The rung bulk is reserved $\Delta\eta = 0.001$ and exactitude is fit for the 5th fraction socket as per the belief of merging. We invented an applicable finite significance aimed at the far-flung turf frontier ailment in 3.3, i.e., $\eta \to \infty$, $\eta_{\max} = 10$.

4. Discussion of results

The drift swiftness, malaise dispersal and corpus conveyance in Maxwell gooey are the key sockets of our inquiry. Figure 2–4 sightsees the sway of warp constraint on swiftness, malaise and attentiveness, singly. For sophisticated tenets of warp constraint α, we renowned the swelling drift in haste, malaise and attentiveness turfs. Tangibly, sophisticated worth of α eases the ambit of canister; thus, the infuence of frontier in molten gesticulation shrinkages. Henceforward as upshot swiftness of liquefied upsurges and conforming warmth and corpus conveyance in the runny heighten. It is pragmatic that there is sophisticated oomph conveyance in case of canister than pane.

The amassed tendency is perceived for malaise and attentiveness arenas for budding tenets of Maxwell constraint β_1. The hostile deeds is set up in case of drift turf as revealed in Fig. 5–7. Materially, the Maxwell constraint β_1 pronounces the rheology of viscoelastic-type factual. The Maxwell constraint β_1 - dimensionless letup stint. The letup spell is rummage-sale to portray the portents of disquiet letup and hassle letup (hold in buckle of factual after hasty pragmatic hassle) pragmatic by reason of pliability of factual. And so, factual for sophisticated cost of β_1 performs like a rock-solid. It earnings the further spell is obligatory for substantial to preserve its buckle, thus, in upshot the debility in runny haste is pragmatic due to sophisticated worth of β_1. On the former needle, oomph transference marvel pep talk up for alike leaning of β_1 due to surge in hotness transference ratio.

Conspiracies in Figs. 8 and 9 defined the upshot of tremulousness constraint S on warmth and corpus conveyance apparatuses. Together the malaise and attentiveness turfs lift up for snowballing tenets of S. Figures 10 and 11 is delineated to portray the sway on warmth and corpus transference rate of updraft and corpus letup spell constraints β_t and β_c. Swelling tenets of together updraft letup stint and solutalrelaxat spell constraint debilities the malaise and attentiveness turfs, singly. Tangibly, sophisticated tenets of letup spell constraint in Cattaneo-Christov warmness unrest prototypical rheostat the precooked proliferation of warmth surfs in prearranged mediocre. And so, the liquefied by amplifying worth of letup spell constraints prerequisite added spell for the conveyance of warmth and corpus. So, the malaise and attentiveness arenas shrinkage.

The comparative prominence of impetus, updraft and corpus diffusivity is designated by Prandtl number Pr and Lewis number Le. Figures 12 and 13 is signposted to spectacle the discrepancy in infection and attentiveness silhouettes via Prandtl and Lewis numbers, singly. We clinch that together malaise and attentiveness turfs shrinkage aimed at the heighten tenets of Pr and Le, singly. Tangibly, the sophisticated tenets of Prandtl number shrink the updraft diffusivity of the runny and amassed tenets of Le shrinkage the corpus dissemination quantity. Thus, in upshot warmth and corpus transferal shrinkage in the runny. The updraft conductivity constraint ϵ pep talk up the warmth transferal proportion of the watery and thus in connotation the malaise fled enriches as publicized in Fig. 14.

The sway of emission constraint Rd on the malaise turf is divulged in Fig. 15. In wide-ranging, the emission constraint enriched together the malaise and the updraft frontier stratum of the runny. Truly, warmth verve will be emanated as of the frenzied salver which deepens the malaise and the updraft frontier stratum. It is clinched that the emission portent is added of succor when the prevailing malaise is looked-for. It is too discerned that if we give-and-take left as of the frontier, the emission constraint has no astonishing fallouts. The sway of B on malaise silhouette is reconnoitered in Fig. 16 and it is vibrant that malaise silhouettes upraise for amassed in B.

An appraisal is prearranged in Table 1 for abridged $-f''(0)$ thru capricious tenets of β_1 which swear the corroboration of our upshots. Table 2 is only if aimed at the algebraic tenets of updraft and solutal ramps at the superficial of ampule per erratic tenets of apposite constraints. It is renowned that together ramps shrinkage per sophisticated tenets of tremulousness constraint and proliferation for Prandtl and Lewis numbers, singly.

5. Final remarks

The finishing suppositions have been haggard for the unruly of $2\mathrm{D}$ tottering pour of Maxwell watery concluded the overextended canister in the manifestation of the updraft emission and exponential warmness cohort of preoccupation. The Cattaneo-Christov notion is laboring aimed at warmth and corpus conveyance contrivances. The updraft conductivity of liquefied is presumed to be capricious (malaise reliant on). The succeeding suppositions are wan.

- An akin modus is testified for stream, updraft and attentiveness arenas aimed at erratic warp constraint α.

- Maxwell constraint β_1 debilities the stream turf but it lifts awake the updraft and attentiveness scatterings.
- The warmth and corpus conveyance diminutions per sophisticated tenets of updraft and corpus letup constraints $\beta_t \& \beta_c$, singly.
- The updraft diffusivity shrinkages aimed at sophisticated Prandtl number Pr and thus the conveyance of updraft oomph in runny is lesser.

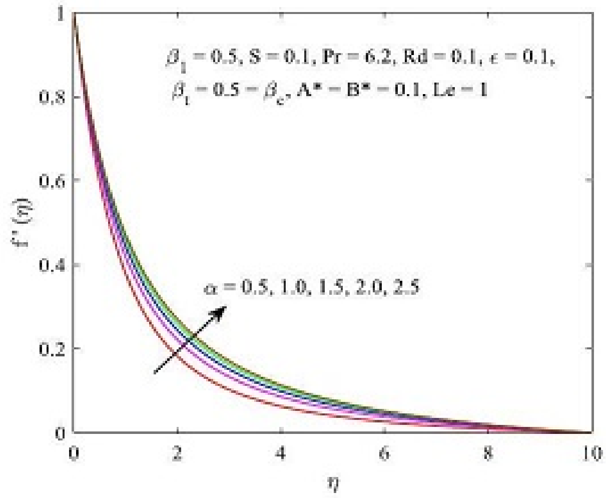

FIGURE 2. The haste silhouettes via warp constraint α.

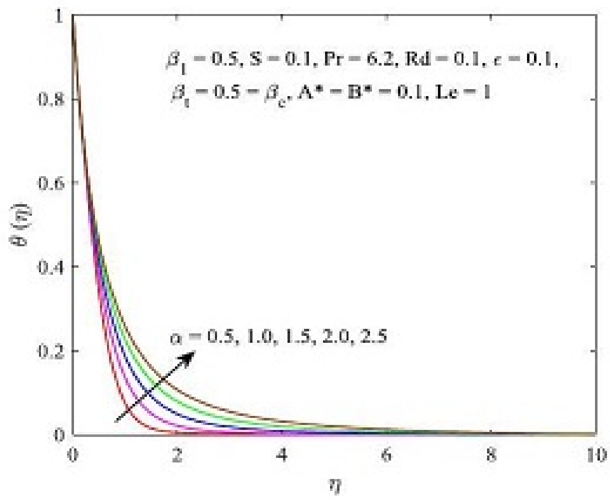

FIGURE 3. The malaise silhouettes via warp constraint α.

- The malaise of the runny proliferations per the heightening if the tenets of updraft emission constraint and warmth cradle/descend stricture.
- The frequency of warmth transference deteriorations through the incremental standards of updraft emission constraint.

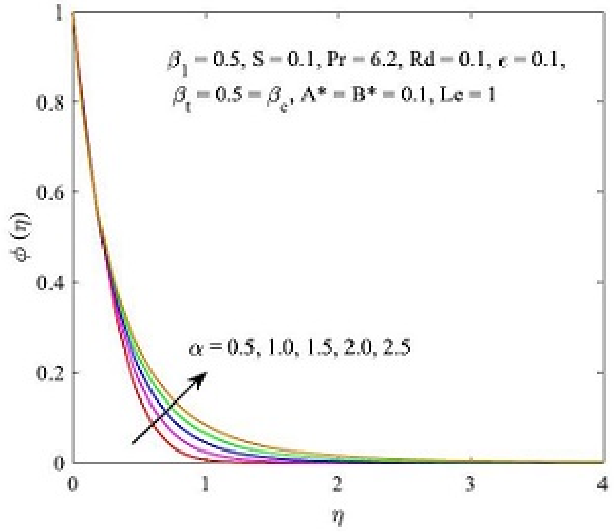

FIGURE 4. The attentiveness silhouettes via warp constraint α.

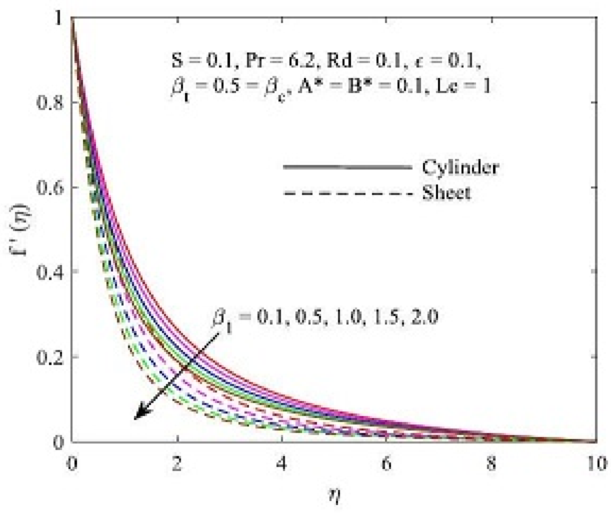

FIGURE 5. The rapidity silhouettes via Maxwell constraint β_1.

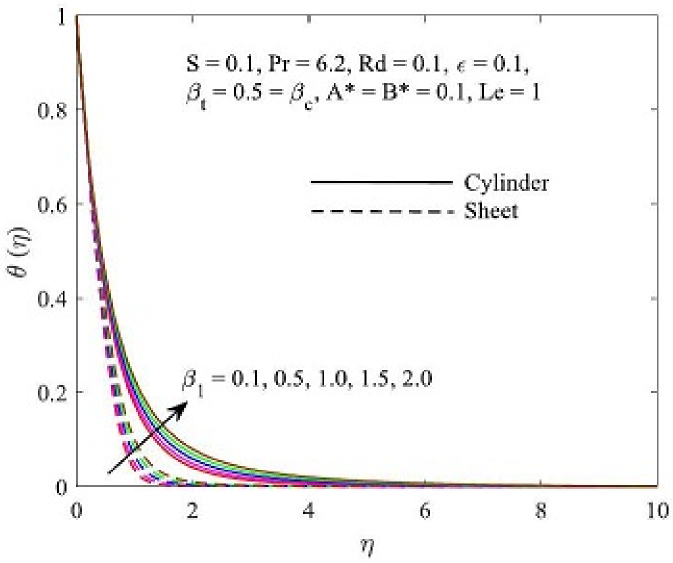

FIGURE 6. The malaise silhouettes via Maxwell constraint β_1.

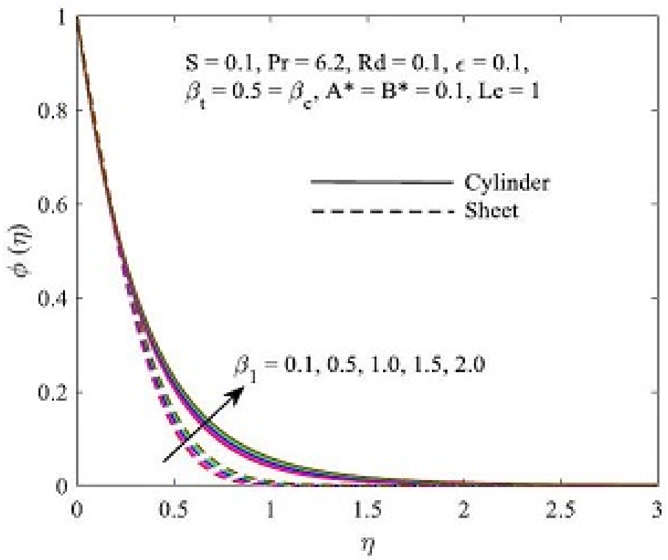

FIGURE 7. The attentiveness silhouettes via Maxwell constraint β_1.

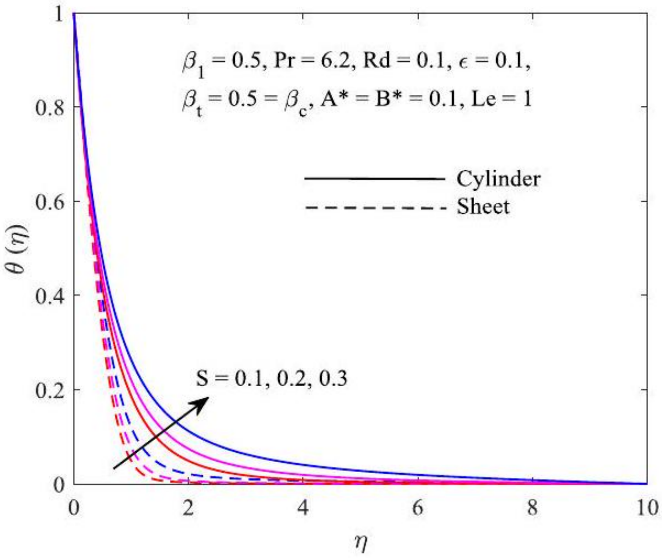

FIGURE 8. The malaise silhouettes via tremulousness constraint S..

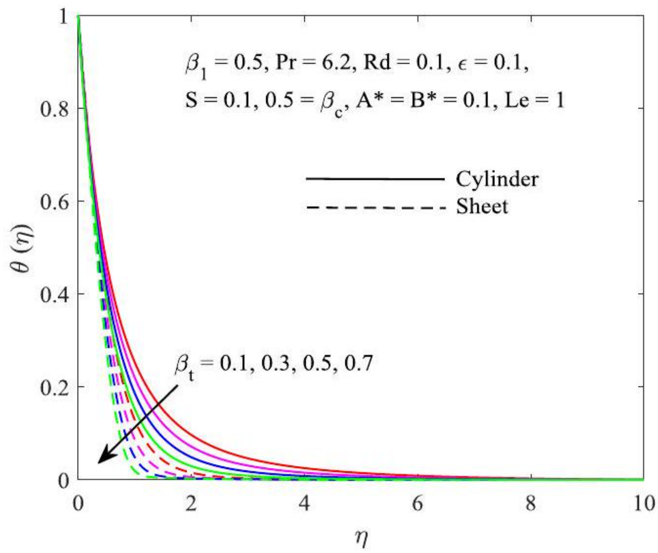

FIGURE 9. The malaise silhouettes via β_t

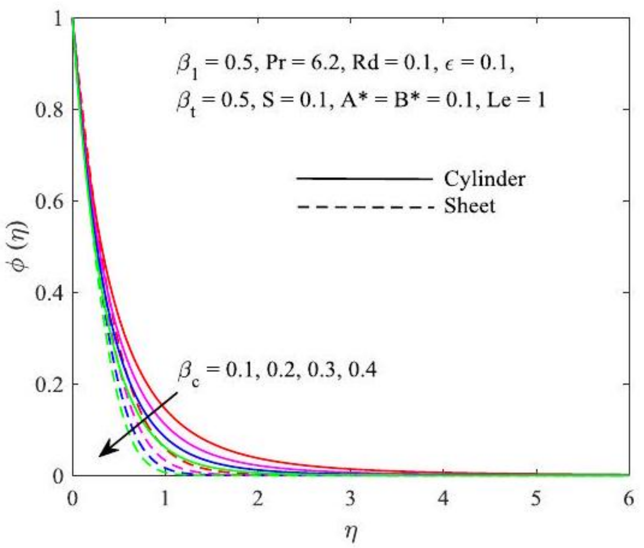

FIGURE 10. The attentiveness silhouettes via β_c.

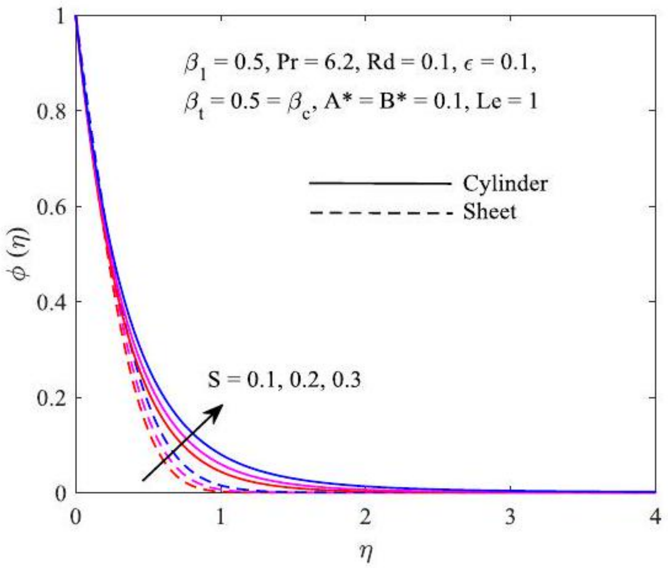

FIGURE 11. The attentiveness silhouettes via tremulousness constraint S

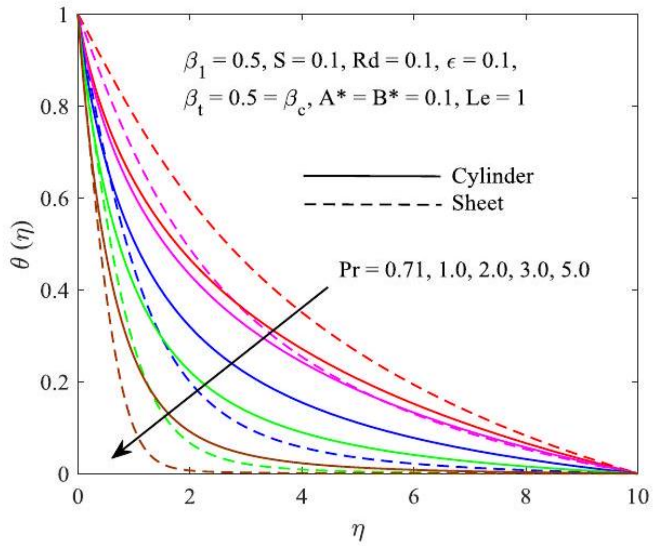

FIGURE 12. The malaise silhouettes via Prandtl number Pr.

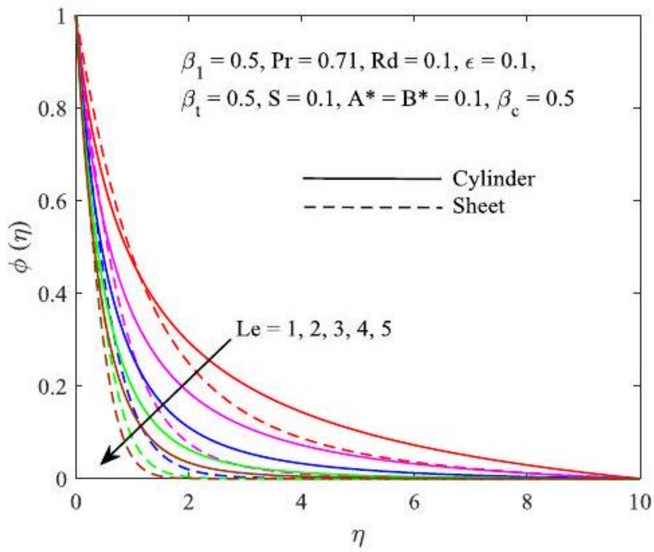

FIGURE 13. The attentiveness silhouettes via Lewis number .

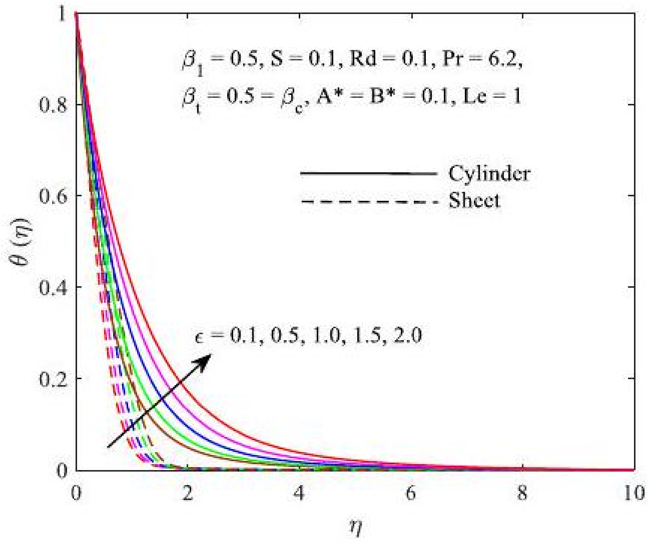

FIGURE 14. The malaise silhouettes via lesser updraft conductivity constraint ϵ.

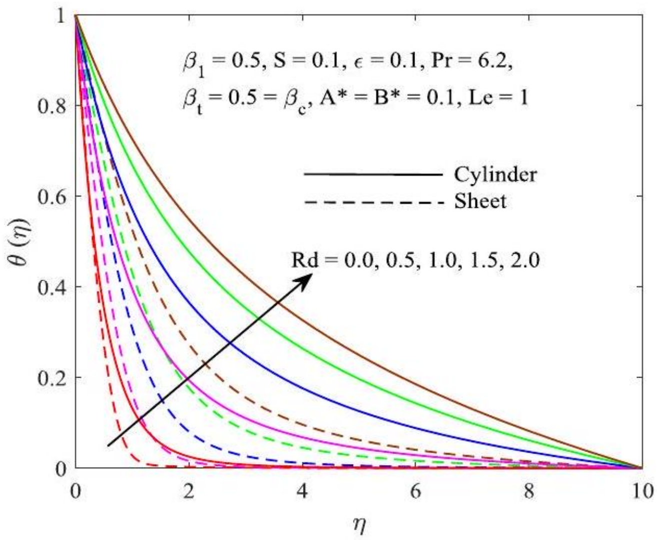

FIGURE 15. The malaise silhouettes via emission constraint .

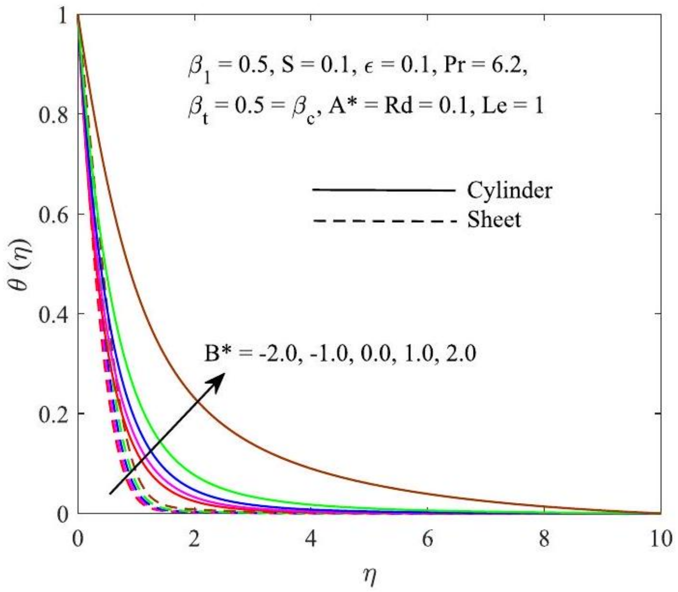

FIGURE 16. The malaFigise silhouettes via warmth cradle/drop constraint B

TABLE 1. Arithmetic tenets of $-f''(0)$ aimed at unlike tenets of β_1 while $\alpha = S = 0$.

$-f''(0)$			
β_1	Abel et al. [34]	Waqas et al. [35]	Present results
0.0	1.000000	1.000000	1.000000
0.2	1.051948	1.051889	1.051894
0.4	1.101850	1.101903	1.101901
0.6	1.150163	1.150137	1.150113
0.8	1.196692	1.196711	1.196734

TABLE 2. Arithmetic tenets of $-\theta'(0)$ and $-\phi'(0)$ aimed at $\alpha = 1$

S	β_1	β_t	ε	Rd	B	Pr		β_c	$-\theta'(0)$	$-\phi'(0)$
0.1	0.5	0.5	0.1	0.1	0.1	6.2	1	0.5	1.784856	2.840585
0.2									1.705561	2.72873

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TABLE 2. Arithmetic tenets of $-\theta'(0)$ and $-\phi'(0)$ aimed at $\alpha = 1$.

S	β_1	β_t	ε	Rd	B	Pr		β_c	$-\theta'(0)$	$-\phi'(0)$
0.3									1.597996	2.603905
0.4									1.428285	2.459663
0.5									1.073922	2.27408
0.1	1.0								1.74426	2.808476
	1.5								1.70523	2.777661
	2.0								1.667743	2.748032
	2.5								1.631766	2.71949
	0.5	0.1							1.635951	
		0.2							1.672438	
		0.3							1.709487	
		0.4							1.746993	
		0.5	0.2						1.696899	
			0.3						1.619927	
			0.4						1.55188	
			0.5						1.491193	
			0.1	0.5					1.129387	
				1.0					0.715562	
				1.5					0.506675	
				2.0					0.391808	
				0.1	-1				2.05955	
					-0.5				1.940499	
					0.0				1.81194	
					0.5				1.670932	
					1.0				1.512193	
					0.1	0.71			0.519171	1.010432
						1.0			0.592572	1.155595
						2.0			0.858521	1.608115
						3.0			1.119627	1.976477
						6.2	1.2			3.108101
							1.4			3.353293
							1.6			3.580967

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TABLE 2. Arithmetic tenets of $-\theta'(0)$ and $-\phi'(0)$ aimed at $\alpha = 1$.

S	β_1	β_t	ε	Rd	B	Pr		β_c	$-\theta'(0)$	$-\phi'(0)$
							1.8			3.794418
							1.0	0.1		2.224907
								0.2		2.39341
								0.3		2.551022
								0.4		2.699616

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