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AN IN-DEPTH REVIEW LITERATURE OF FRACTIONAL-ORDER CHAOTIC SYSTEMS AND ITS APPLICATIONS IN SECURE TRANSMISSION SCHEMES

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ABSTRACT. This paper deals with an inclusive review of the literature on fractional-order chaotic systems and their applications in secure communications and synchronization schemes. The chaotic systems have gained more attention in the literature, which can be detected in different fields of science and engineering such as mechanical, chemical, biological, and physics. Furthermore, The fractional-order increases the degrees of complexity in chaotic systems behaviour, which leads to better degrees of security. This work can help the researchers in highlighting the most critical applications of the fractional-order chaotic systems that existed in the literature, which saves the trouble of searching and time.

1. INTRODUCTION

Among the nonlinear dynamical systems, the chaotic system has gained many authors' interest in the last thirty years. Due to its various benefits, for instance, the ability to avoid the hypotheses related to the condition of the system, the sensitivity to initial conditions, and being a deterministic system that exhibits complex and unexpected behaviour, these significant advantages stimulated the researchers to introduce and study several chaotic systems, Some of the most well-known

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systems are the unified chaotic system [1], Lorenz system [2], Chen system [3], Rössler system [4], and the Lü system [5], the chaotic attitude of these effective systems are helpful in different fields such as business [6], electrical circuits [7], medicine [8], mathematics [9], biology [10], chemical reactors [11], secure communication [12], and etc.

On the other hand, there has been a great deal of interest in studying the fractional calculus, which is the non-integer order; it has received more attention due to its precision compared to the integer one. Fractional calculus investigated that different applications can be modeled elegantly and precisely with the help of fractional derivatives such as the earthquakes, so the Researchers and the physicians depended on the strategy of using the chaos system in fractional-order. The pioneers of this significant phenomena were Grigorenko and Grigorenko in 2003 [13] then it spread out to several works like Lorenz system [13], Chen system [14], Chua's circuit [15], Duffing system [16], Lü system [17], financial system [18], and so on.

Nowadays, the secure communication branch has become a worldwide aim in encrypting images, audio, and other data, because of the necessity for many functions such as online banking, instant messaging, military and architectural plans, the recording of conferences, and business events. Among the numerous solutions, the fractional-order chaotic system is considered the most effective one. The chaotic system utilizes the synchronization between two similar chaotic systems, "emitter/receiver," with various primary conditions. This achievement made the secret information possible to recover on the receiver side after being encrypted in the emitter. Recently, the synchronization issue has become a challenging subject. therefore, many methods have been proposed by the researchers, some of them are the fractional extended Kalman filter proposed in (2009) [19], Lyapunov based methods in (2014) [20], sliding mode observer in (2013) [21], the impulsive control in (2016) [22] and the adaptive control in (2016) [23]. In the current paper, we exhibit the secure communication system's literature review using the fractional-order chaotic systems.

The rest of this paper is organized as follows. A literature review of the chaotic system and its application in secure communication is given in Section 2. Literature review of Fractional-Order chaotic system and its application in the secure transmission is introduced In section 3. Finally, brief conclusions are drawn in Section 4.

2. Literature review of chaotic system and its application in secure communication

Toshimitsu Ushio (1996) [24] suggested two control techniques for synchronising two subsystems, which are referred to as S1 and S2, respectively. S1 refers to the master system, while S2 refers to the slave system. The use of these principles results in the construction of a safe half-duplex communication system, which means both sides are able to speak with one another, but not at the same time; instead, communication can only occur in one direction at a time.

Hector Puebla and Jose Alvarez-Ramirez (2001) [25] showed a global procedure to modulate the information signals using low-dimensional chaotic oscillators. The encrypting/decrypting method [26] is set up on the inverse system masking (ISM) techniques. The secret message is recovered via a feedback approximation to the encoder inverse [27, 28]. Using hyperchaotic signals was the principle thought to inconvenience the data signal retrieving by chaotic attractor rebuilding methods (relayed on the theorem of Taken) [29]. Numerical examples of continuous systems were proposed To represent the fundamental thought.

Z Li, K Li, C Wen, and YC Soh (2003) [30] proposed a digital chaotic secure communication system using the idea of the magnifying glass in order to increase the sensitivity of the cryptosystem. The synchronization of the system is based on the impulsive control strategy [31, 32] for synchronizing two similar chaotic systems established in the encryption and decryption where the extent of impulsive intervals are piecewise fixed, which makes it complicated for an outsider to detect the synchronization installment [33, 34].
The equations of Chua's circuit are given as:

$$\begin{cases} \dfrac{dx_1(t)}{dt} = ka(x_2 - x_1 - f(x_1)) \\ \dfrac{dx_2(t)}{dt} = k(x_1 - x_2 + x_3) \\ \dfrac{dx_3(t)}{dt} = k(-\beta x_2 - \gamma x_3) \end{cases} \tag{2.1}$$

f(x) is the nonlinear characteristic of Chua's diode in Chua's circuit given by:

$$f(x) = m_1 x + \frac{1}{2}(m_0 - m_1)\left\{\mid x+1 \mid - \mid x-1 \mid\right\} \tag{2.2}$$

M Chen, D Li, and A Zhang (2004) [35] presented a universal nonlinear state-observer for a class of universal chaotic systems to recognize a robust strong chaotic synchronization and the followed approach was easy alive and universal [36, 37], and it showed a satisfying result in secure communication [38, 39].

XY Wang and MJ Wang (2009) [40] presented a new scheme for secure communication stand on chaotic modulation method, this technique is depended on observer identification [41] instead of synchronization [42–44], this observer is designed to recognize the parameter and rebuild the information signal which is illustrated in the parameter of chaotic Liu system. Lyapunov exponents spectrum of chaotic system [45]has been used to analyze the parameter range to guarantee chaos. Some signals are used as examples to exhibit the efficiency and feasibility of the process.

The chaotic system that has been used is Liu's System, and the differential equations that represented the system are [46]:

$$\begin{cases} \frac{dx(t)}{dt} = -ax(t) - ey^2(t) \\ \frac{dy(t)}{dt} = by(t) - kx(t)z(t) \\ \frac{dz(t)}{dt} = -cz(t) + mx(t)y(t) \end{cases} \tag{2.3}$$

OM. Kwon, JH. Park and SM. Lee (2011) [47] presented a new synchronization scheme of Lur'e systems [48, 49] for secure communication based on delay feedback control [50]. The encryption method was achieved by the methods of N-shift cipher and public key [51]. A new delay-dependent synchronization criterion [52, 53] is built up using the Lyapunov strategy and linear matrix inequality (LMI) formulation to obtain the stability and reconstruction of the secret message.

Qu Shaocheng, Liu Di, and Wang Li (2011) [54] were the pioneers of the nonlinear controller for hyper-chaotic Lorenz system based on Lyapunov stability theory, which synchronized the emitter and the receiver. The transmitted signal is modulated in the drive system on the transmitter, leading to an improvement in communications security.

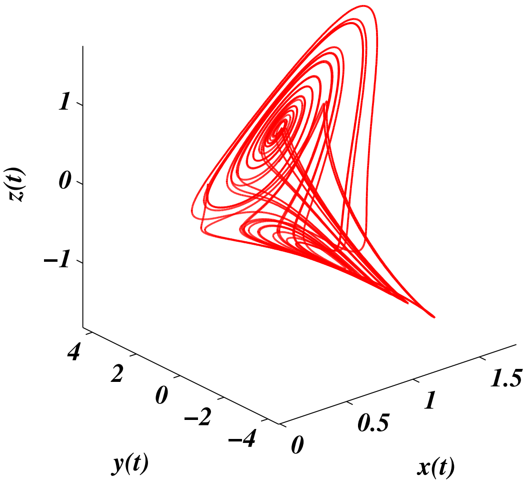

FIGURE 1. The Simulation of Liu's system (2.3).

The hyper-chaotic Lorenz system is defined as [55]:

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - y - xz + w \\ \dot{z} = xy - bz \\ \dot{w} = -kx \end{cases} \tag{2.4}$$

Xiaohui Xu (2011) [56] presented a new communication security scheme depending on generalized function projective synchronization (GFPS) [57, 58], instead of function projective synchronization (FPS) [59–62]. On the emitter side, the secret message is converted by an invertible function. Toward the chaotic system's parameter, the processed signal was modulated. The parameter and the controllers have been designed to recognize GFPS of uncertain Liu chaotic systems and identify the receiver system's unknown parameter using the Lyapunov stability theory. The secret message can be recovered successfully using the estimated parameter.

Jing Pan, Qun Ding, and Baoxiang Du (2012) [63] presented a progressed scheme for chaotic masking for communication security depending on the Lorenz system. This scheme could defeat the Lorenz system's signal by changing some features to make it more chaotic in both time and frequency domains.

Shutang Liu and Fangfang Zhang (2014) [64] designed a new scheme of communication security complex coupled chaotic systems depending on complex function projective synchronization (CFPS). The complex scaling functions were chosen due to their arbitrariness and unpredictability. The chaotic system that has been used is a coupled complex Lorenz system.

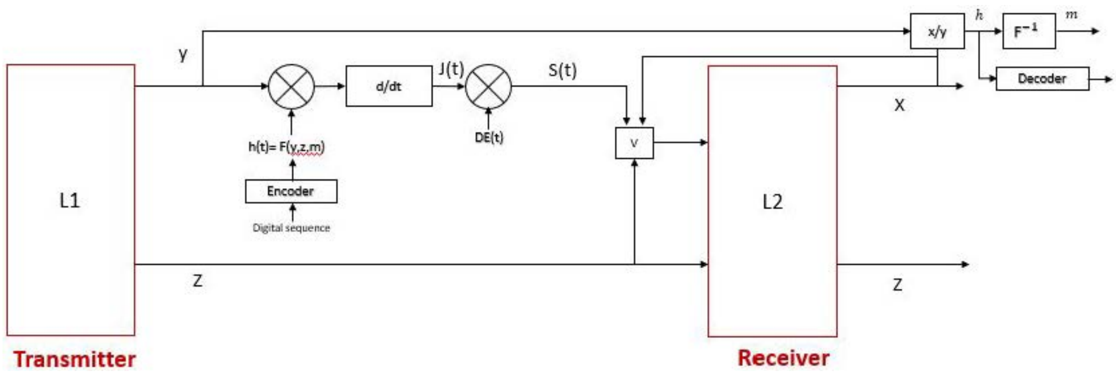

FIGURE 2. The block diagram of the proposed communication scheme [64].

Nie Chun Yan, Tian Hua, and Sun Hai Xin (2014) [65] proposed a new system for secure communication that combines the synchronization theory and the fourth-order hyper-chaotic system. In addition to producing the unexpected pseudo-random sequences with this hyper-chaotic system.

Ammami Sonia, Djemai Mohamed, et al. (2015) [66] investigated the synchronization of a unified chaotic system to encrypt several forms of information signals. Via the Lyapunov theory, they confirmed the asymptotic convergence between the emitter and the receiver.

Naderi Bashir and Kheiri Hossein (2016) [67] presented the application of the exponential synchronization of the chaotic system without linear term in communication security, the convergence time was shorter because of the exponential stability mechanism while the security and synchronization were accomplished through convenient robust controllers.

Israr Ahmad Muhammad Shafiq and M. Mossa Al-Sawalha (2018) [68] proposed a novel strategy and scheme of the global exponential multi switching combination synchronization (GEMSCS) within three unlike chaotic systems, two of them for the emitter system and one for the receiver system. The Lyapunov theorem was applied to achieve the global exponential stability of the synchronization error, which complicated the level of the digital message and made the decryption process faster and more precise.

Adnan Javeed, Tariq Shah, and Attaullah. (2020) [69] They presented a novel technique to achieve secure communication using a chaotic oscillator generated by a second-order differential equation OED. They formulated an algorithm to build a substitution box utilizing this OED. In addition to proposing a lightweight process to encrypt images using the duffing oscillation and the S-box to create confusion and diffusion in the cryptosystem in an advanced way.

YJ Chen, HG Chou, WJ Wang, SH Tsai (2020) [70] presented a new synchronization of multi-scroll Chen chaotic systems based on polynomial fuzzy control [71] for disturbance refusal, in addition to keeping the performance into account and Inputs restraints. This control [72] is performed according to sum-of-squares (SOS) [73, 74] that can also be managed by the polynomial optimization Matlab toolbox SOSOPT. The Chen's system is defined as:

$$\begin{cases} \dfrac{dx(t)}{dt} = a(y(t) - x(t)) \\ \dfrac{dy(t)}{dt} = (c-a)x(t) - x(t)z(t) + cy(t) \\ \dfrac{dz(t)}{dt} = x(t)y(t) - bz(t) \end{cases} \tag{2.5}$$

Van Nam Giap, et al. (2021) [75] proposed a new form of Lorenz chaotic system by changing the original form into a Takagi-Sugeno (T-S) fuzzy model with two sub-linear systems in order to decrease the cost of experimental equipment without affecting the generalizability of master and slave system features. A new adaptive disturbance observer (ADOB) with a fast convergence rate was presented for the synchronisation system; this method was created to remove the perturbation values on both the master and slave sides of the system. Furthermore, the transmitter and receiver systems of a secure communication system are synchronized with the help of adaptive sliding-mode control (ASMC).

Joseph Chang Lun Chan, et al. (2022) [76] proposed a secure communication scheme based on chaotic systems on the transmitter side and a sliding-mode observer (SMO) on the receiver side for purposes of estimating system states and synchronisation. The encryption process is divided into two stages: first, using an N-cipher and encryption key signal, and subsequently injecting the encrypted message into the chaotic dynamics of the transmitter system. Instead of injecting the message directly into the chaotic system on the transmitter side to encrypt the message. For further safety, the encryption key is created separately from the transmitter's status information. The proposed scheme is developed to isolate the effects of the disturbances from the recovered messages. The secret information was reconstructed using the decryption key signal and the recovered encrypted messages (N-cipher).

Chih-Hsueh Lin, et al. (2022) [77]. Secure network-based video and audio streaming is proposed using a novel cryptosystem-based, four-dimensional hyper-chaotic Lorenz system. In order to get more and better random numbers, the 4-D hyper-chaotic Lorenz system is used instead of the 3-D system. The SHA3 (Secure hash algorithm 3) method is used to make randomness much better and to make synchronised dynamic key generators possible. With the synchronised dynamic key generators, the AES CFB (Advanced Encryption Standard Cipher Feedback) algorithm is used to encrypt the secret message. The secret information is sent to the slave system through the public channel, where it will be completely rebuilt using the dynamic random keys that are made at the slave system at the same time. The 4D-LS system that has been used (2.6) is described as follows:

$$
\begin{aligned}
\dot{x}_1(t) &= -ax_1(t) + ax_2(t) + \lambda x_3(t)\\
\dot{x}_2(t) &= dx_1(t) + \gamma x_2(t) - x_1(t)x_4(t),\\
\dot{x}_3(t) &= -cx_1(t) - x_3(t)\\
\dot{x}_4(t) &= x_1(t)x_2(t) - bx_4(t)
\end{aligned}
\tag{2.6}
$$

3. Literature review of Fractional-Order chaotic system and its application in secure communication

A Kiani-B, K Fallahi,et al (2009) [19] investigated for the first time secure communication using chaotic system with Fractional-order in order to increase nonlinearity and complexity of the power spectrum. For data encryption, fractional-order Lorenz system has been used. An extended fractional Kalman filter (EFKF) was used for the synchronization and recovering the original message [78]. The results of fractional Lorenz system were compared by numerical example to the integer one [79, 80].

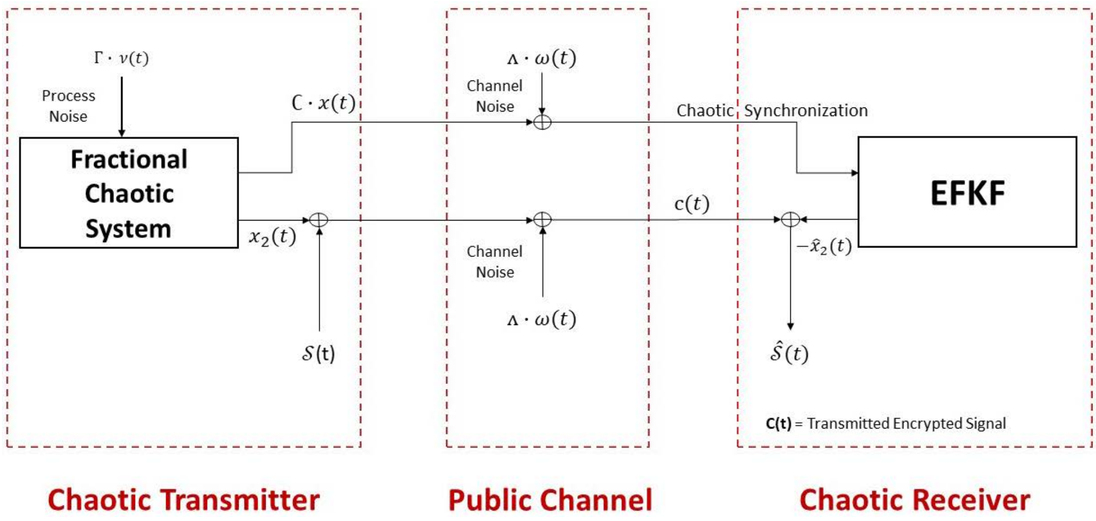

FIGURE 3. The proposed scheme [19].

A Kiani-B, K Fallahi, et al. (2009) [19] investigated for the first time secure communication using the chaotic system with Fractional-order in order to increase nonlinearity and complexity of the power spectrum. For data encryption, a fractional-order Lorenz system has been used. An extended fractional Kalman filter (EFKF) was used to synchronize and recover the original message [78]. The fractional Lorenz system results were compared by numerical example to the integer one [79, 80].

Y Xu, H Wang, et al. (2014) [81] Showed the synchronized of tow fractional-order Lorenz-like chaotic system is established by Pecora and Carroll (PC) control method [82] based on Laplace transformation theory [83]. A new cryptosystem technique for the image was proffered depending on chaos synchronization [84, 85]. As a result, the original image is well encrypted, and decrypted.

The equation of fractional-order Lorenz's system is defined as [86]:

$$\begin{cases} {}_0D_t^{q1}x(t) = \sigma(y(t) - x(t)) \\ {}_0D_t^{q2}y(t) = x(t)(\rho - z(t)) - y(t) \\ {}_0D_t^{q3}z(t) = x(t)y(t) - \beta z(t) \end{cases} \tag{3.1}$$

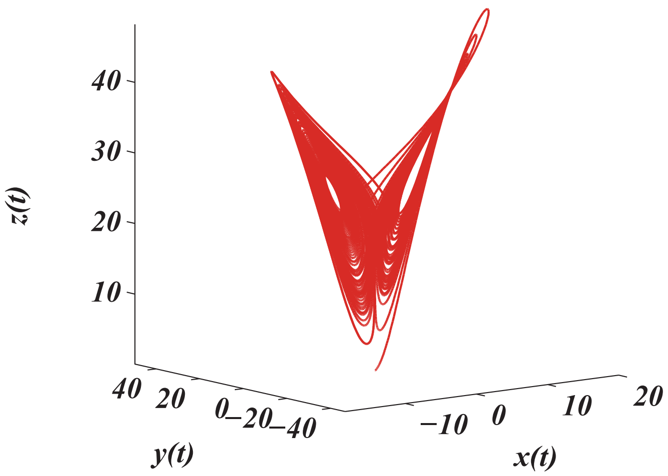

FIGURE 4. The simulation outcome (3.1)

Xia Huang, et al. (2014) [87] investigated a novel encryption algorithm of color image based on a fractional order hyper chaotic system. The secret keys include the parameters and the initial conditions, while the image is encrypted using the XOR and shuffling operations. As a result, this new study can resist the statistical attacks and it achieved a higher security.

Luo chao (2015) [88] investigated A new method instead of synchronization, which is the unsynchronised communication architecture, in order to guarantee better robustness and make the transmission errors able to be recognized in real-time and self-corrected. He also presented a procedure with a high probability against the lack of a data-dependent error management mechanism. The author used the fractional Order [89, 90] shifting Chaotic system [19, 91] as a transmission signal generator to achieve a higher complication and non-linearity in both fields, time and frequency.

The fractional-order Chen's system is defined as [92]:

$$
\begin{cases}
{}_0D_t^{q1}x(t) = a(y(t) - x(t)) \\
{}_0D_t^{q2}y(t) = (c - a)x(t) - x(t)z(t) + cy(t) \\
{}_0D_t^{q3}z(t) = x(t)y(t) - bz(t)
\end{cases}
\tag{3.2}
$$

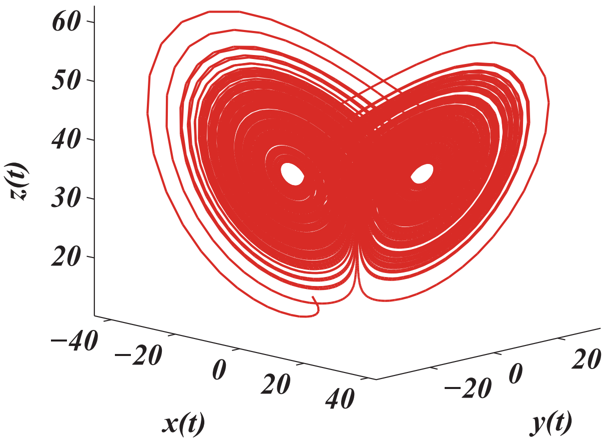

FIGURE 5. (3.2)

Delavari Hadi and Mohadeszadeh Milad (2016) [93] presented a novel adaptive sliding mode control (ASMC) method for finite-time synchronization between two Fractional-order chaotic and hyper-chaotic systems [94], in addition to the presence of foreign disturbances and system uncertainties. Stable synchronization [95] is achieved by adequate conditions of the Lyapunov stability theorem. Relevant adaptive laws and Particle Swarm Optimization (PSO) are used in order to estimate the unknown controller parameters and optimize the parameters of the (ASMC), respectively.

Ali Durdua and Yılmaz Uyaroglu (2017) [96] investigated optimal fractional-order on a new chaotic system. By Pecaro Carroll [82, 97] synchronization algorithm, the encryption was implemented with optimal fractional order, giving precision rapidness in the synchronization time. The proposed fractional-order chaotic system is defined as:

$$\begin{cases} \dfrac{d^q x}{dt^q} = a(x - yz) \\ \dfrac{d^q y}{dt^q} = by + xz \\ \dfrac{d^q z}{dt^q} = z(c - x) + xy \end{cases} \tag{3.3}$$

Mohammadzadeh Ardashir and Ghaemi Sehraneh (2017) [98] proposed a new scheme of robust controller relayed on a novel self-evolving non-singleton type-2 fuzzy neural network (SE-NT2FNN) in order to achieve the synchronization of the undetermined fractional-order hyper-chaotic systems and estimating the unsure functions in the dynamic of the system. [99, 100].

Jia Hongyan, Guo Zhiqiang, et al. (2018) [101] discussed the chaotic behaviors in a four-wing fractional-order system [102] based on frequency-domain [15] and time-domain approach [103]. The chaos synchronization of the fractional-order system is implemented by a novel analog circuit that has been planed. The synchronization is realized depending on the control proposal of the observer in his previous work [104].

Shukla MK and Sharma BB (2018) [105] investigate a backstepping-based feedback control strategy for synchronization of two similar fractional-order Rossler's system in master-slave configuration [105, 106], and also this strategy has been used widely for stabilization [107]. Two signals have been transmitted, one for the message and the other for the synchronization purpose and recover the original message.

The fractional-order Rossler's system is defined as [14]:

$$\begin{cases} {}_0D_t^{q1} x(t) = -(y(t) + z(t)) \\ {}_0D_t^{q2} y(t) = x(t) + ay(t) \\ {}_0D_t^{q3} z(t) = b + z(t)(x(t) - c) \end{cases} \tag{3.4}$$

Bettayeb Maamar, Al-Saggaf Ubaid Muhsen, and Djennoune Said (2018) [108] designed a new step-by-step sliding mode observer [109] to obtain robust synchronization of two fractional-order modified Chua's systems and to recover the original message infinite time. The encrypted and the synchronization signals are transmitted through one channel.

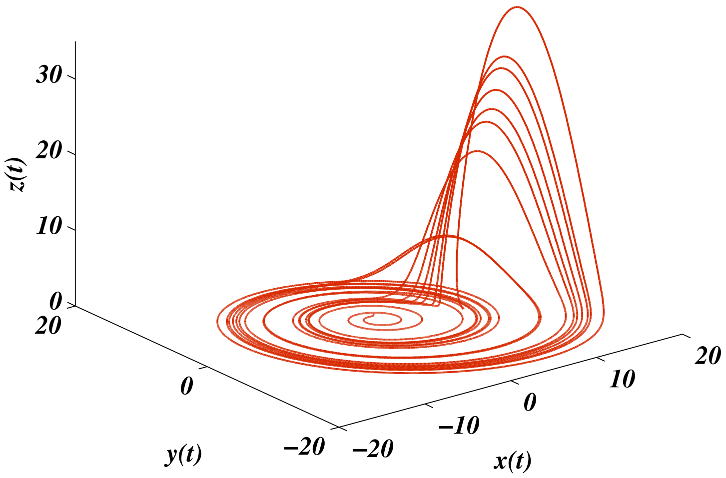

FIGURE 6. The simulation outcome (3.4)

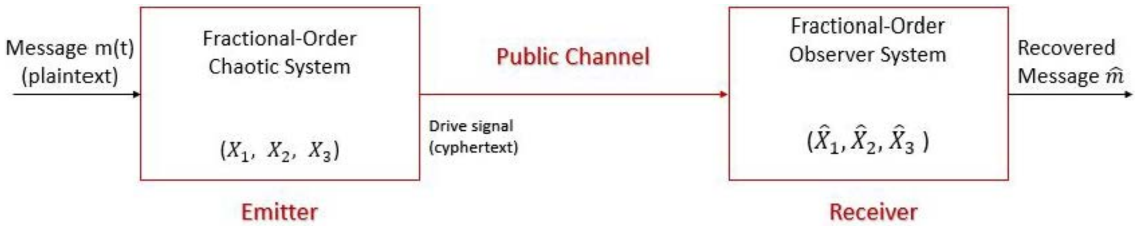

FIGURE 7. The proposed secure communication scheme [108].

D-P Sergio M, M-G Rafael, et al. (2018) [110] proposed a new methodology for a class of non-linear system depending on the fractional algebraic observability and defined the degree of concealment to demonstrate the capacity of hiding a signal into the dynamic of a system. In addition to indicating where the message is settled so that it can be estimated [106, 111], they introduced a closed-loop fractional-order differentiator developed by the sliding mode process in order to obtain secure communication [112], the proposed methodology helps to get the security in the presence of uncertainty.

Zouad Fadia, Kemih Karim, and Hamiche Hamid (2019) [113] presented a new scheme of secure communication using the Chen fractional-order delayed chaotic system under the receiver perturbations, this approach is developed, and the electronics circuit is simulated with Multisim [114]. Using an H-infinity controller. achieved The synchronization and recovering the transmitted signal. The secret message was injected in the Chen fractional order's dynamics that delayed the emitter side's chaotic system.

Liu Jiaxun, Wang Zuoxun, et al. (2019) [115] wanted to make Secure Communication of Fractional Complex Chaotic Systems (FCCS) possible. Hence, they used Fractional Difference Function Synchronization (FDFS) [116] by following a specific method, which is Extending the (DFS) to (FDFS) and investigating the general controller. In order to validate the usefulness and benefits of (FCCS), a new protected communication system (FDFS) was implemented. Four kinds of signals are transmitted, which include analog signal [117], voice signal, and digital, in addition to proposing a new cryptosystem with (FDFS) for image signal [118].

The fractional-order Genesio-Tesi's system is defined as [119]:

$$\begin{cases} {}_0D_t^{q1}x(t) = y(t) \\ {}_0D_t^{q2}y(t) = z(t) \\ {}_0D_t^{q3}z(t) = -\beta_1 x(t) - \beta_2 y(t) - \beta_3 z(t) + \beta_4 x^2(t) \end{cases} \tag{3.5}$$

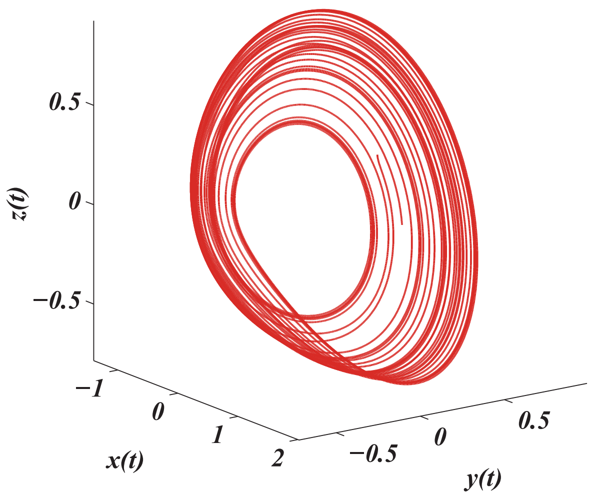

FIGURE 8. The simulation outcome (3.5).

Li Ye, Wang Haoping, and Tian Yang (2019) [120] investigated a fractional-order adaptive non-singular terminal sliding mode control (FONTSMC). The synchronization of two nonlinear fractional-order chaotic systems under the external disturbance was achieved by (FONTSMC) method [121, 122]. In addition to get better performance of the controller, they proposed terminal sliding mode (TSM) [123]. The Lyapunov theorem has been used for analyzing the stability of the system. This method was applied to a dual-channel secure communication system, and the simulation results guarantee the shortness in the time of synchronization, and the secret message can be built successfully [124].

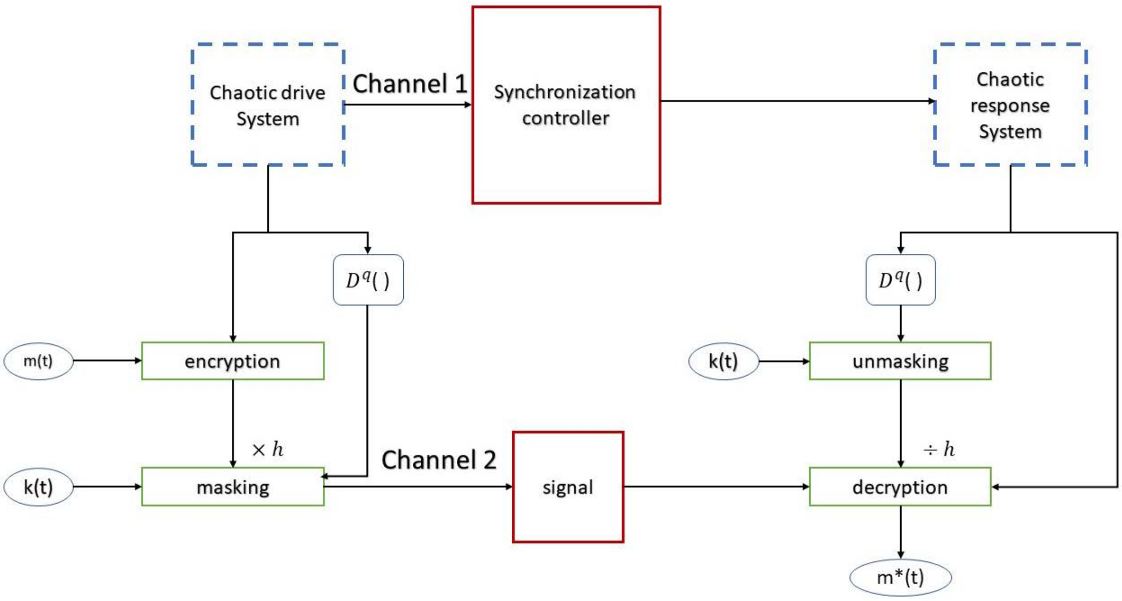

FIGURE 9. the proposed secure communication scheme [120].

Ghiasi Amir Rikhtegar, Gharamaleki Mona Saber, et al. (2019) [125] presented a new type of optimized time-delayed feedback control method which the Optimizing method was based on particle swarm optimization method (PSO) [126, 127]. The primary purpose of Using this controller is to stabilize [128] the chaotic behavior of fractional order electrical oscillator [129] by adding a control signal to the chaotic system. The proposed method was compared to the general (TDFC) and succeeded.

$$\begin{cases} {}_0D_t^{q1}x(t) = y(t) \\ {}_0D_t^{q2}y(t) = z(t) \\ {}_0D_t^{q3}z(t) = -\beta_1 x(t) - \beta_2 y(t) - \beta_3 z(t) + \beta_4 x^2(t) \end{cases} \tag{3.6}$$

Abd El-Maksoud Ahmed J, Hassan, Bahy G, et al. (2019) [130] presented the FPGA implementation of fractional order multi-scrolls [131] Tang, Yalcin [132] and Ozoğuz [133] chaotic systems . After comparing all the fractional-order (Yalcin, Ozoğuz, Tang) chaotic systems on FPGA [134] they found that Yalcin is the highest throughput. Therefore, is used as the chaotic and the chaotic generator in a speech encryption algorithm, this algorithm was introduced using stream ciphering blocks and its robustness was shown by analysis technique.

Lahdir Mourad, Hamiche Hamid, et al. (2019) [135] investigated a new cryptosystem for image which is compression and encryption using SPIHT coding-decoding [136, 137] and fractional-order chaotic respectively. considering that it is a fast technique for image compression and more complexity to the cryptosystem. The synchronization was achieved by the basic Parallel processing BPP method. The results show that the encryption and decryption of the image were successfully achieved.

Karthikeyan Rajagopal, Ali Durdu, et al. (2019) [138] the Adams-Bashforth-Moulton algorithm was used to derive a new scheme of synchronization for the fractional-order multi scroll system. So they proposed a multi scroll attractor and were able to control the number of scrolls by proper choice of the nonlinear function. The Lyapunov exponents of the proposed scheme are obtained to ensure the chaotic behavior of the system. This system has many real-time applications like image and voice encryption, random number generators, and so on.

Li Rui-Guo and Wu Huai-Ning (2019) [139] Investigated a synchronization based on fractional-order chaotic systems in parameter uncertainty and disturbance. This process went by designing a feedback controller using the adaptive control method and the fractional calculus theory. In addition to providing a policy of optimization to the controller, then they improved the quantum particle swarm optimization (QPSO) algorithm by introducing an interval estimation strategy to it then applying it to optimize the controller parameters. As a final step, they offered a feedback mechanism with the encrypted signal in secret communication.

Ismail Samar M, Said Lobna A, et al. (2020) [140] investigated a novel image encryption algorithm without any data loss based on chaotic maps and edge maps to add extra degrees of freedom to the system and offer fewer correlation coefficients, respectively [141, 142]. Using fractional-order edge detection filters [143] in order to get better noise performance over the conventional integer filter. The system's flexibility can be used with different edge detectors and suitable for medical imaging security. This algorithm has been compared with other existing cryptosystems.

Ayub Khan, Lone Seth Jahanzaib, and Pushali Trikha (2020) [144] introduced a novel 4-D fractional-order chaotic system that has been synchronized with its parallel systems in dislocated phase and anti-phase synchronization by building non-linear control functions [145]. Consider the novel fractional-order chaotic system as (3.7):

$$\begin{cases} D^q z_1 = a z_2 - a z_1 + z_3 z_4 \\ D^q z_2 = z_1(a - z_3) - z_2 + z_1 \\ D^q z_3 = z_1 z_2 - b z_3 + |z_3| \\ D^q z_4 = z_2 z_3 - c z_4 \end{cases} \tag{3.7}$$

MA Atoussi, et al. (2021) [146] investigated a new communication strategy to enhance the security of a message between two chaos systems; in the transmitter system fractional-order chaotic Chua's system was used in order to encrypt the secret message. On the other hand an optimal step by step sliding mode observer with chua's system and fractional order was used in the receiver system to achieve the synchronization between the transmitter and receiver and also to recover the secret message. In addition to the significant meta-heuristic grey wolf optimizer algorithm (GWO) that had been used in the receiver part to estimate the parameter of the step-by-step sliding mode observer.

4. CONCLUSION

In this paper, the literature review on the secure communication system using the chaotic system and the fractional-order chaotic system has been illustrated due to the numerous researchers that have been trying to develop the application of the chaotic system in this field, "communication security." In addition to the fractional calculus that received more attention, especially with the chaotic system, due to its precision and sensitivity, this review explains that most of the studies achieved recently in secure communication are depending on the fractional-order chaotic system.

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