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THE HIGHER FINITE DIFFERENCE METHOD FOR SOLVING THE DYNAMICAL MODEL OF COVID-19

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ABSTRACT. In the present paper, the SIR model tracks the numbers of susceptible, infected and recovered individuals during an epidemic with the help of ordinary differential equations (ODE). First, we give the model formulation of our phenomena. Secondly, a fully discrete difference scheme is derived for the SIR model.At the end of this aper, we give the simulation results of the model. A comparison of the obtained numerical results of both the models is performed in the absence of an exact solution.

1. INTRODUCTION

The novel human coronavirus disease 2019 (COVID-19) was first reported in Wuhan, China, in 2019, and subsequently spread globally to become the fifth documented pandemic since the 1918 flu pandemic. By September 2021, almost two years after COVID-19 [1] and [2] was first identified, there had been more than 200 million confirmed cases and over 4.6 million lives lost to the disease. Here, we take an in-depth look at the history of COVID-19 from the first recorded case to the current efforts to curb the spread of the disease with worldwide vaccination programs.

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The first official cases of COVID-19 were recorded on the 31st of December, 2019, when the World Health Organization (WHO) was informed of cases of pneumonia in Wuhan, China, with no known cause. On the 7th of January, the Chinese authorities identified a novel coronavirus, temporally named 2019-nCoV, as the cause of these cases. Weeks later, the WHO declared the rapidly spreading COVID-19 outbreak as a Public Health Emergency of International Concern on the 30th of January 2020. It wasn't until the following month, however, on the 11th of February that the novel coronavirus got its official name - COVID-19. Nine days later, the US Centers for Disease Control and Prevention (CDC) confirmed the first person to die of COVID-19 in the country. The individual was a man in his fifties who lived in Washington state.

A finite difference method [6]- [12] proceeds by replacing the derivatives in the differential equations by finite difference approximations. This gives a large algebraic system of equations to be solved in place of the differential equation [14]- [18], something that is easily solved on a computer.

Mathematical modeling can be thought of as an iterative process made up of the following components. (Note that the word tep is intentionally avoided to highlight the lack of a prescribed ordering of these components, as some may occur simultaneously and some may be repeated.)

The remainder of this paper is structured as follows. Section 2 discusses the formulation of the model. In the section 3 we present the forward second order accurate approximation to the first derivative. In section 4 we propose a new numerical scheme for a spatially discrete model of total variation of indice i. Finally, in the last section, We give some numerical results including both simulation and an empirical example to study the proposed testing procedure in different times.

2. Model formulation

The COVID-19 pandemic, among other pandemics from the past, has attracted great attention not only from mathematicians but researchers from numerous fields. It is assumed that the sum of the four categories S,I,R is equal to the total population (M) at time t=0 (system parameters relate to the time t in days). Besides, nowadays the researchers are devoting their research work to the fractional-order COVID-19 mathematical models. A huge number of good research papers related to fractional-order COVID-19 mathematical models can be found in the literature, some of them are the following [1]- [2].

For nonlinear systems, we consider the effects of three unknown functions on each other. A three by three system of nonlinear ordinary differential equations has the form:

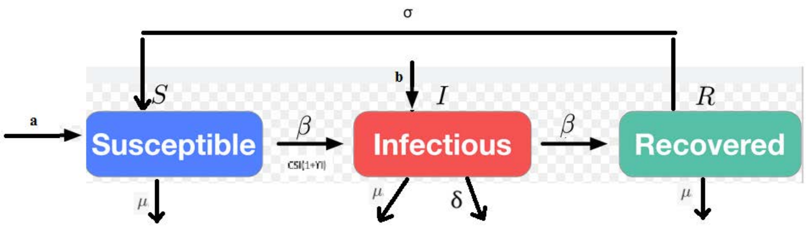

FIGURE 1. The Model of SIR

This is because of two exposures over a small time period: a single contact produces infection at the rate CIS, while the new infective individuals arise from double exposures with CI^2S. It produces further chance that the recovered individual againmay catch infection.

Here we remark that the function $\Phi(S, I) = CI(t)S(t)(1 + \gamma I(t))$, where both C, γ are positive constants. This is an interesting example for nonlinear incidence rate already used by some authors $[17, 31, 32]$.

The dynamics of the population are described by the following differential equations:

$$
\begin{aligned}
\frac{dS(t)}{dt} &= a - CI(t)\big(1 + \gamma I(t)\big) - \mu S(t) + \alpha R(t), \\
\frac{dI(t)}{dt} &= CI(t)S(t)\big(1 + \gamma I(t)\big) - (\beta + \mu + \delta - b)I(t), \qquad (2.1)\\
\frac{dR(t)}{dt} &= \beta I(t) - (\alpha + \mu)R(t).
\end{aligned}
$$

The parameters involved in model (1) are described as in Table 1.

3. FORWARD SECOND ORDER ACCURATE APPROXIMATION TO THE FIRST DERIVATIVE

Develop a forward difference formula for $f_i^{(1)}$ which is $E = O(h)^2$ accurate. First derivative with $O(h)$ accuracy then the minimum number of nodes is 2. Then, the first derivative with $O(h)$ accuracy then need 3 nodes

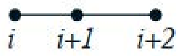

FIGURE 2. 3 NODES

The first forward derivative can therefore be approximated to $O(h)$ as:

$$\frac{df}{dx}\bigg|_{x=x_i} - E = \frac{\alpha_1 + \alpha_2 f_{i+1} + \alpha_3 f_{i+2}}{h}.$$

The T.S. expansions about x_i are:

$$\begin{aligned} f_i &= f_i, \\ f_{i+1} &= f_i + hf_i^{(1)} + \frac{h^2}{2} f_i^{(2)} + \frac{h^3}{6} f^{(3)} + O(h)^4, \\ f_{i+2} &= f_i + 2hf_i^{(1)} + 2h^2 f_i^{(2)} + \frac{4}{3} h^3 f_i^{(3)} + O(h)^4. \end{aligned}$$

We substituting into our assumed form of and re-arranging

$$\begin{aligned} \frac{\alpha_1 + \alpha_2 f_{i+1} + \alpha_3 f_{i+2}}{h} = \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{h} f_i + (\alpha_2 + 2\alpha_3) f_i^{(1)} + \Big(\frac{\alpha_2}{2} + 2\alpha_3\Big) h f_i^{(2)} \\ + \Big(\frac{1}{6}\alpha_2 + \frac{4}{3}\alpha_3\Big) h^2 f_i^{(3)} + O(h)^3. \end{aligned}$$

Desire $f_i^{(1)}$ and 2^{nd} order accuracy then coefficient of $f_i^{(1)}$ must equal unity and coefficients of f_i and $f_i^{(2)}$ must vanish

$$\begin{aligned} \frac{\alpha_1 + \alpha_2 + \alpha_3}{h} &= 0, \\ (\alpha_2 + 2\alpha_3) &= 1, \\ \Big(\frac{\alpha_2}{2} + 2\alpha_3\Big) h &= 0. \end{aligned}$$

We solve these simultaneous equations

$$\alpha_1 = -\frac{3}{2}, \quad \alpha_2 = 2, \quad \alpha_3 = -\frac{1}{2}.$$

Thus the equation now becomes

$$\frac{-\frac{3}{2} f_i + 2 f_{i+1} - \frac{1}{2} f_{i+2}}{h} = (0) f_i + (2-1) f_i^{(1)} + (0) f_i^{(2)} + \Big(\frac{1}{6}.2 - \frac{4}{3}.\frac{1}{2}\Big) h^2 f_i^{(3)} + O(h)^3,$$

then we get

$$f_i^{(1)} = \frac{-3 f_i + 4 f_{i+1} - f_{i+2}}{2h} + \frac{1}{3} h^2 f(3) + O(h)^3.$$

The forward difference approximation of 2nd order accuracy

$$f_i^{(1)} = \frac{-3f_i + 4f_{i+1} - f_{i+2}}{2h} + E \text{ where } E = \frac{1}{3}h^2 f_i^{(3)}. \tag{3.1}$$

4. The Discrete Model

A finite difference method proceeds by replacing the derivatives in the differential equations by finite difference approximations. This gives a discrete model as fellows:

$$\begin{aligned} \frac{-3S_i + 4S_{i+1} - S_{i+2}}{2h} &= a - CI_i\big(1+\gamma I_i\big) - \mu S_i + \alpha R_i, \\ \frac{-3I_i + 4I_{i+1} - I_{i+2}}{2h} &= CI_i S_i\big(1+\gamma I_i\big) - (\beta+\mu+\delta-b)I_i, \\ \frac{-3R_i + 4R_{i+1} - R_{i+2}}{2h} &= \beta I_i - (\alpha+\mu)R_i. \end{aligned} \tag{4.1}$$

After arrangement of the previous equations, we obtain:

$$\begin{aligned} S_{i+2} &= -3S_i + 4S_{i+1} - 2h(a - CI_i\big(1+\gamma I_i\big) - \mu S_i + \alpha R_i, \\ I_{i+2} &= -2h(CI_i S_i\big(1+\gamma I_i\big) - (\beta+\mu+\delta-b)I_i) - 3I_i + 4I_{i+1}, \\ R_{i+2} &= -2h(\beta I_i - (\alpha+\mu)R_i) - 3R_i + 4R_{i+1}. \end{aligned} \tag{4.2}$$

The initial conditions (ICs) for the above model are given as follows: $S(0) \geq 0$, $I(0) \geq 0$ and $R(0) \geq 0$.

5. Numerical Results

In this section, we present some numerical results obtained by applying the new methods. These results indicate the efficiency of the methods. Consider model (4.2) with the parameters given in Figure 3.

Using the differential equations of the SIR model and converting them to numerical discrete forms, one can set up the recursive equations and calculate the S, I, and R populations with any given initial conditions but accumulate errors over a long calculation time from the reference point. Sometimes a convergence test is needed to estimate the errors. Given a set of initial conditions and the disease-spreading data, one can also fit the data with the SIR model and pull out the three reproduction numbers when the errors are usually negligible due to the short time

Parameters	The physical interpretation
$S(t)$	Susceptible compartment
$I(t)$	Infected compartment
$R(t)$	Recovered compartment
a	The recruitment rate
μ	Natural death
δ	Death due to corona
b	The immigration rate of infected individuals
β	Corona infection recovery rate
C	The infection rate
γ	Rate at which recovered individuals lose immunity
α	Rate of recovery from infection

FIGURE 3. The Parameters in our Model

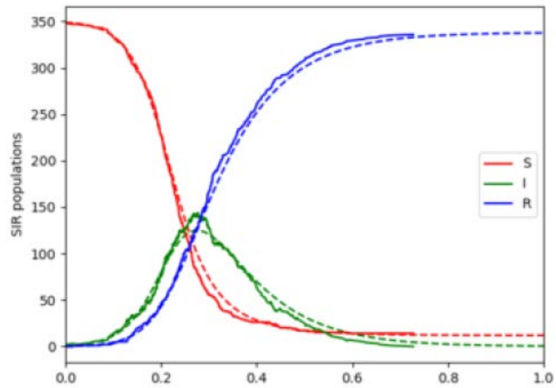

FIGURE 4. Time arbitrary units

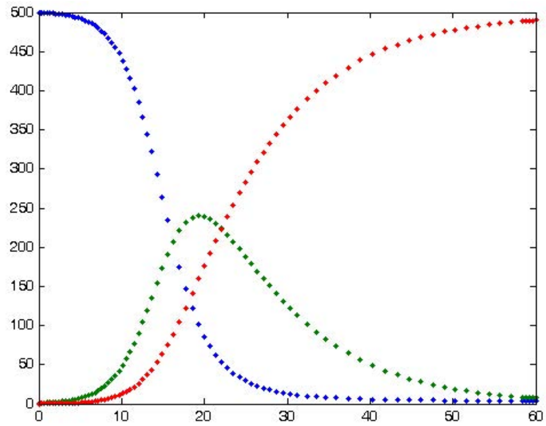

FIGURE 5. Time days between 0 to 60

step from the reference point. Let us now implement the model in **MATLAB, using the ode45** command to numerically solve differential equations. The script **SIR.m** provided on the web page will also help you to plot the results as in Fig. 4 and Fig. 5 with runing the model with the preset parameters.

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