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BAYESIAN ANALYSIS UNDER UNBALANCED AND BALANCED LOSS FUNCTIONS APPLYING DIFFERENT PRIOR INFORMATIONS

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ABSTRACT. In this paper, We perform a Bayesian analysis of Zeghdoudi distribution based on type II censored data. Using two type of loss functions; balanced and unbalanced loss functions, we use three different loss functions. this estimation includes three cases of prior informations; availability and lack of primary information, we obtain Bayes estimators and the corresponding posterior risks. the analytical forms of these estimators are out of reach, so, we propose Markov chain Monte-Carlo (MCMC) procedure. Moreover, given initial values for the parameters of the model,we obtain maximum likelihood estimators. Furthermore, we compare their performance with those of the Bayesian estimators using balanced and unbalanced loss functions.

1. INTRODUCTION

One of the commonly used distributions is the exponential distribution, it deals with the failures and survival times. The one parameter distribution introduce by Lindley is a mixture of exponential and gamma distributions. This distribution is commonly used in modelling lifetime data sets. In 2018 Messaadia suggested

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a one parameter distribution which is a mixtures of two gamma distributions ($\Gamma(2,\theta)$ and $\Gamma(3,\theta)$) [5], it is known as the Zeghdoudi distribution, in lifetime data sets where the Lindley distribution gave poor fit, this distribution fits well those data sets. the Bayesiann estimation of this model using diffrent loss functions using upper truncation, (Talhi and Aiachi 2021) [6], Many researchers have dealt with Bayesian analysis using balanced loss functions, A.L Firas Monther Al-Badran (2019) [2] has given the Bayes estimation under balanced Loss functions ofthe exponential model, M.J Jozani (2012) [9] studied the Bayesian robust analysis under a general class of balanced loss functions.

In This study we are interested in comparing Balanced and unbalanced loss functions while using three prior informations. the Bayesian estimations of Zeghdoudi distribution is based on type II censored data.

2. The Zeghdoudi distribution

The probability density function of the Zeghdoudi distribution (see Messaadia and Zeghdoudi (2018)) is

$$f_{ZD}(x,\theta)=\frac{\theta^3 x(1+x)e^{-\theta x}}{2+\theta},\quad x,\theta>0, \tag{2.1}$$

and its cumulative function is

$$F_{ZD}(x)=\frac{1-(x^2\theta^2+\theta(\theta+2)x+\theta+2)}{(\theta+2)e^{-\theta x}},\quad x,\theta>0. \tag{2.2}$$

3. Estimation of the unknown parameters

3.1. Maximum likelihood function. Let the sample $(x_1,x_2,\ldots,x_n)$ be generated from Zeghdoudi model, assuming that the data is type II censored, i.e., we only observe$(x_1,x_2,\ldots,x_m)$, for a given m $\in(1,2,,m)$. The likelihood function is

$$L\left(x,\theta\right)=N\times\left[1-F_{X_i}\left(x_m\right)\right]^{n-m}\times\prod_{i=1}^{m}f_{X_i}\left(x_i\right),$$

where $N=\frac{n!}{(n-m)!}$ the likelihood function is given by:

$$L\left(x,\theta\right)=\frac{N\theta^{3m}}{\left(\theta+2\right)^{n}}B^{n-m}\times\prod_{i=1}^{m}A_i e^{-\theta x_i}. \tag{3.1}$$

Here,

$$\begin{cases} A_i = x_i\,(1+x_i) \\ B = \left[\theta^2 x_m^2 + \theta\,(\theta+2)\,x_m + \theta + 2\right] \mathrm{e}^{-\theta x_m} \end{cases}.$$

The corresponding logarithm is

$$\begin{aligned} l\,(x,\theta) = \ln L\,(x,\theta) &= \ln \mathrm{N} + 3m.ln\theta + n.\ln(\theta+2) \\ &+ (n-m)\ln B + \sum_{i=1}^{m}[\ln A_i - \theta x_i]. \end{aligned} \tag{3.2}$$

The solution of the following non-linear system yields the maximum likelihood estimators θ_{MLE} of the parameter θ

$$\tfrac{\partial}{\partial\theta} l\,(x,\theta) = \tfrac{3m}{\theta} + \tfrac{n}{\theta+2} + (n-m)\,\tfrac{B_1}{B} - \sum_{i=1}^{m} x_i = 0\ , \tag{3.3}$$

where

$$B_1 = \left[\theta^2\left(x_m^3 - x_m^2\right) + \theta x_m + 1\right] \mathrm{e}^{-\theta x_m}\ .$$

The solution of the equation (3.3) seems analytically intractable. We will rely on numerical methods to obtain approximate solutions. We will use the R package BB to obtain the approximate value of the maximum likelihood estimator θ_{MLE} of the parameter θ. The R package BB is successfully used for solving non-linear system of equations; see Varadhan and Gilbert (2010).

3.2. Bayesian Estimation under different loss functions.

Prior and posterior distributions

Prior can be divided into two types according to the abundance of primary information as: **Informative prior and non-informative prior**, the posterior distribution for the parameters (θ) differs according to the used prior distribution; it can be evaluate by using the three mentioned prior as:

(i) Informative Prior

We assume here that the parameter θ of Zeghdoudi distribution do follow independent gamma distribution:

$$\pi_1\,(\theta) = \frac{a^b}{\Gamma\,(b)}\theta^{b-1}\exp\,(-a\theta)\,, \quad \theta > 0, a, b > 0,$$

where the constants a, b are called hyper-parameters The posterior distribution of θ reads

$$\pi_1(\theta\,|x) = K\frac{\theta^{3m+b-1}B^{n-m}}{(\theta+2)^n}\mathrm{e}^{-a\theta}\prod_{i=1}^{m}A_i e^{-\theta x_i}, \tag{3.4}$$

where K is a normalizing constant.

(ii) Non Informative Prior

$$\pi_2(\theta) = \frac{1}{\theta}, \quad \theta > 0 \tag{3.5}$$

The posterior distribution of θ reads

$$\pi_2(\theta\,|x) = K\frac{\theta^{3m-1}B^{n-m}}{(\theta+2)^n}\prod_{i=1}^{m}A_i e^{-\theta x_i}. \tag{3.6}$$

(iii) Fisher information Prior

Fisher information is defined as:

$$\pi_3(\theta) = I(\theta) = -\mathrm{E}\left[\frac{\partial^2 l(x,\theta)}{\partial\theta^2}\right], \tag{3.7}$$

$$\pi_3(\theta) = \mathrm{E}_X\left[\frac{3m}{\theta^2} + \frac{n}{(\theta+2)^2} + (n-m)\frac{B_2B - B_1^2}{B^2}\right] = E_\theta, \tag{3.8}$$

where

$$B_2 = \left[\theta^2\left(x_m^3 - x_m^4\right) + \theta\left(2x_m^3 - x_m^2\right)\right]e^{-\theta x_m}\ .$$

The posterior distribution of θ reads

$$\pi_3(\theta\,|x) = KE_\theta\frac{\theta^{3m}B^{n-m}}{(\theta+2)^n}\prod_{i=1}^{m}A_i e^{-\theta x_i}. \tag{3.9}$$

Bayesian Estimation under unbalanced different loss functions

Now we will find Bayes estimators under unbalanced loss function sequentially as follows:

(A) The generalized quadratic loss function

The generalized quadratic loss function is defined as $L\left(\hat{\theta},\theta\right) = \tau(\theta)\left(\hat{\theta}-\theta\right)^2$ and $\tau(\theta) = \theta^{\alpha-1}$.

1 - Informative Prior

In the case of the generalized quadratic loss function, the Bayes estimators are given by the formulas:

$$\hat{\theta}_{GQ1}=\frac{\int\limits_0^\infty \frac{\theta^{3m+b+\alpha-1}B^{n-m}}{(\theta+2)^n}\mathrm{e}^{-a\theta}\prod\limits_{i=1}^m A_i e^{-\theta x_i}d\theta}{\int\limits_0^\infty \frac{\theta^{3m+b+\alpha-2}B^{n-m}}{(\theta+2)^n}\mathrm{e}^{-a\theta}\prod\limits_{i=1}^m A_i e^{-\theta x_i}d\theta}.$$

The corresponding posterior risks are then

$$PR\left(\hat{\theta}_{GQ1}\right)=\mathrm{E}_{\pi_1}\left(\theta^{\alpha+1}\right)+\hat{\theta}_{GQ1}^2\mathrm{E}_{\pi_1}\left(\theta^{\alpha-1}\right)-2\hat{\theta}_{GQ1}\mathrm{E}_{\pi_1}\left(\theta^{\alpha}\right).$$

2 - Non Informative Prior

In the case of the generalized quadratic loss function, the Bayes estimators are given by the formulas:

$$\hat{\theta}_{GQ2}=\frac{\int\limits_0^\infty \frac{\theta^{3m+\alpha-1}B^{n-m}}{(\theta+2)^n}\prod\limits_{i=1}^m A_i e^{-\theta x_i}d\theta}{\int\limits_0^\infty \frac{\theta^{3m+\alpha-2}B^{n-m}}{(\theta+2)^n}\prod\limits_{i=1}^m A_i e^{-\theta x_i}d\theta}.$$

The corresponding posterior risks are then

$$PR\left(\hat{\theta}_{GQ2}\right)=\mathrm{E}_{\pi_2}\left(\theta^{\alpha+1}\right)+\hat{\theta}_{GQ2}^2\mathrm{E}_{\pi_2}\left(\theta^{\alpha-1}\right)-2\hat{\theta}_{GQ2}\mathrm{E}_{\pi_2}\left(\theta^{\alpha}\right).$$

3 - Fisher information Prior

In the case of the generalized quadratic loss function, the Bayes estimators are given by the formulas:

$$\hat{\theta}_{GQ3}=\frac{\int\limits_0^\infty E_\theta\frac{\theta^{3m+\alpha}B^{n-m}}{(\theta+2)^n}\prod\limits_{i=1}^m A_i e^{-\theta x_i}d\theta}{\int\limits_0^\infty E_\theta\frac{\theta^{3m+\alpha-1}B^{n-m}}{(\theta+2)^n}\prod\limits_{i=1}^m A_i e^{-\theta x_i}d\theta}$$

The corresponding posterior risks are then

$$PR\left(\hat{\theta}_{GQ3}\right)=\mathrm{E}_{\pi_3}\left(\theta^{\alpha+1}\right)+\hat{\theta}_{GQ3}^2\mathrm{E}_{\pi_3}\left(\theta^{\alpha-1}\right)-2\hat{\theta}_{GQ3}\mathrm{E}_{\pi_3}\left(\theta^{\alpha}\right)$$

(B) The Entropy loss function

The Entropy loss function is defined as $L\left(\hat{\theta},\theta\right)=\left(\frac{\hat{\theta}}{\theta}\right)^p-p\ln\left(\frac{\hat{\theta}}{\theta}\right)-1.$

1 - Informative Prior

Under the entropy loss function, we obtain the following estimators:

$$\hat{\theta}_{E1} = \left[K \int_{0}^{\infty} \frac{\theta^{3m+b-p-1} B^{n-m}}{(\theta+2)^{n}} \mathrm{e}^{-a\theta} \prod_{i=1}^{m} A_i e^{-\theta x_i} d\theta \right]^{-\frac{1}{p}}.$$

The corresponding posterior risks are then

$$PR\left(\hat{\theta}_{E1}\right) = p\left[\mathrm{E}_{\pi_1}\left(\ln\theta - \ln\hat{\theta}_{E1}\right)\right].$$

2 - Non Informative Prior

Under the entropy loss function, we obtain the following estimators:

$$\hat{\theta}_{E2} = \left[K \int_{0}^{\infty} \frac{\theta^{3m-p-1} B^{n-m}}{(\theta+2)^{n}} \prod_{i=1}^{m} A_i e^{-\theta x_i} d\theta \right]^{-\frac{1}{p}}.$$

The corresponding posterior risks are then

$$PR\left(\hat{\theta}_{E2}\right) = p\left[\mathrm{E}_{\pi_2}\left(\ln\theta - \ln\hat{\theta}_{E2}\right)\right].$$

3 - Fisher information Prior

Under the entropy loss function, we obtain the following estimators:

$$\hat{\theta}_{E3} = \left[K \int_{0}^{\infty} E_{\theta} \frac{\theta^{3m-p} B^{n-m}}{(\theta+2)^{n}} \prod_{i=1}^{m} A_i e^{-\theta x_i} d\theta \right]^{-\frac{1}{p}}.$$

The corresponding posterior risks are then

$$PR\left(\hat{\theta}_{E3}\right) = p\left[\mathrm{E}_{\pi_3}\left(\ln\theta - \ln\hat{\theta}_{E3}\right)\right].$$

(C) The Linex loss function

The Linex loss function is defined as $L\left(\hat{\theta},\theta\right) = \exp\left(r(\theta-\hat{\theta})\right) - r(\theta-\hat{\theta}) - 1$.

1- Informative Prior

Under the Linex loss function, we obtain the following estimators:

$$\hat{\theta}_{L1} = -\frac{1}{r}\ln\left[K \int_{0}^{\infty} \frac{\theta^{3m+b-1} B^{n-m}}{(\theta+2)^{n}} \mathrm{e}^{-\theta(a+r)} \prod_{i=1}^{m} A_i e^{-\theta x_i} d\theta \right].$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{L1}\right) = r\left[\hat{\theta}_{GQ1} - \hat{\theta}_{L1}\right].$$

2- Non Informative Prior

Under the Linex loss function, we obtain the following estimators:

$$\hat{\theta}_{L2} = -\frac{1}{r}\ln\left[K\int_{0}^{\infty}\frac{\theta^{3m-1}B^{n-m}}{(\theta+2)^{n}}\mathrm{e}^{-\theta r}\prod_{i=1}^{m}A_{i}e^{-\theta x_{i}}d\theta\right],$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{L2}\right) = r\left[\hat{\theta}_{GQ2} - \hat{\theta}_{L2}\right].$$

3 - Fisher information Prior

Under the Linex loss function, we obtain the following estimators:

$$\hat{\theta}_{L3} = -\frac{1}{r}\ln\left[K\int_{0}^{\infty}E_{\theta}\frac{\theta^{3m}B^{n-m}}{(\theta+2)^{n}}\mathrm{e}^{-\theta r}\prod_{i=1}^{m}A_{i}e^{-\theta x_{i}}d\theta\right],$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{L3}\right) = r\left[\hat{\theta}_{GQ3} - \hat{\theta}_{L3}\right].$$

Bayesian Estimation under Balanced different loss functions

Balanced loss function

We use comprehensive criterion put down by (Zellner, 1994), which is the Balanced criterion or equilibrium criterion. The objective of achieving equilibrium in the loss function is to increase accuracy and conformity in the estimation process. The loss functions that are discussed above are consider an unbalanced loss functions.

Apart from the symmetry criterion, the loss function can be a balanced according to Zellner's formula as follows:

$$L_{L,\omega,\hat{\theta}_{GQ}}\left(\hat{\theta}_{GQB},\theta\right) = \omega L\left(\hat{\theta}_{GQ},\hat{\theta}_{GQB}\right) + (1-\omega)\,L\left(\hat{\theta}_{GQB},\theta\right),$$

where:

- $L_{L,\omega,\overset{\wedge}{\theta}_{GQ}}\left(\overset{\wedge}{\theta}_{GQB},\theta\right)$ Balanced loss function;
- ω weighted coefficient, $w\in(0,1)$;
- θ_0 Primary estimator for the parameter θ depends on the observations;
- $L\left(\hat{\theta}_{GQ},\hat{\theta}_{GQB}\right)$ Unbalanced loss function;
- $L\left(\hat{\theta}_{GQ},\hat{\theta}_{GQB}\right)$Unbalanced loss function for the likelihood function.

Clearly that the balanced loss function heavily depends on the weighted coefficient (θ), and the initial estimator θ_0.

Now we will find bayes estimators under balanced loss function sequentially as follows:

(A) The generalized quadratic losss function

The general formula of the balanced generalized quadratic loss function:

$$L_{L,\omega,\hat{\theta}_{GQ}}\left(\hat{\theta}_{GQB}-\theta\right)=\omega L\left(\hat{\theta}_{GQ},\hat{\theta}_{GQB}\right)+(1-\omega)\,L\left(\hat{\theta}_{GQB}-\theta\right).$$

1 - Informative Prior

The Bayes estimator under the balanced generalized quadratic loss function are given by the formula:

$$\hat{\theta}_{GQB1}=\frac{\omega\left[\hat{\theta}_{GQ1}\right]^{\alpha}+(1-\omega)\,\mathrm{E}_{\pi_1}\left(\theta^{\alpha}\right)}{\omega\left[\hat{\theta}_{GQ1}\right]^{\alpha-1}+(1-\omega)\,\mathrm{E}_{\pi_1}\left(\theta^{\alpha-1}\right)},$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{GQB1}\right)=\mathrm{E}^{*}_{\pi_1}\left(\tau(\theta)\left(\theta-\hat{\theta}GQB1\right)\right)$$

2 - Non Informative Prior

The Bayes estimator under the balanced generalized quadratic loss function are given by the formula:

$$\hat{\theta}_{GQB2}=\frac{\omega\left[\hat{\theta}_{GQ2}\right]^{\alpha}+(1-\omega)\,\mathrm{E}_{\pi_2}\left(\theta^{\alpha}\right)}{\omega\left[\hat{\theta}_{GQ2}\right]^{\alpha-1}+(1-\omega)\,\mathrm{E}_{\pi_2}\left(\theta^{\alpha-1}\right)},$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{GQB2}\right) = \mathrm{E}^{*}_{\pi_2}\left(\tau\left(\theta\right)\left(\theta - \hat{\theta}GQB2\right)\right).$$

3 - Fisher information Prior

The Bayes estimator under the balanced generalized quadratic loss function are given by the formula:

$$\hat{\theta}_{GQB3} = \frac{\omega\left[\hat{\theta}_{GQ3}\right]^{\alpha} + (1-\omega)\,\mathrm{E}_{\pi_3}\left(\theta^{\alpha}\right)}{\omega\left[\hat{\theta}_{GQ3}\right]^{\alpha-1} + (1-\omega)\,\mathrm{E}_{\pi_3}\left(\theta^{\alpha-1}\right)},$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{GQB3}\right) = \mathrm{E}^{*}_{\pi_3}\left(\tau\left(\theta\right)\left(\theta - \hat{\theta}GQB3\right)\right).$$

(B) The Entropy loss function

The general formula of the balanced Entropy loss function:

$$L_{L,\omega,\hat{\theta}_E}\left(\hat{\theta}_{EB},\theta\right) = \omega L\left(\hat{\theta}_{E},\hat{\theta}_{EB}\right) + (1-\omega)\,L\left(\hat{\theta}_{EB},\theta\right).$$

1 - Informative Prior

The Bayes estimator under the balanced Entropy loss function are given by the formula:

$$\hat{\theta}_{EB1} = \left[\frac{\omega}{\left(\hat{\theta}_{E1}\right)}^{p} + (1-\omega)\,\mathrm{E}_{\pi_1}\left(\frac{1}{\theta^{p}}\right)\right]^{-\frac{1}{p}},$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{EB1}\right) = p\left[\mathrm{E}^{*}_{\pi_1}\left(\ln\theta - \ln\hat{\theta}_{EB1}\right)\right].$$

2 - Non Informative Prior

The Bayes estimator under the balanced Entropy loss function are given by the formula:

$$\hat{\theta}_{EB2} = \left[\frac{\omega}{\left(\hat{\theta}_{E2}\right)}^{p} + (1-\omega)\,\mathrm{E}_{\pi_2}\left(\frac{1}{\theta^{p}}\right)\right]^{-\frac{1}{p}},$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{EB2}\right) = p\left[\mathrm{E}^{*}_{\pi_2}\left(\ln\theta - \ln\hat{\theta}_{EB2}\right)\right].$$

3 - Fisher information Prior

The Bayes estimator under the balanced Entropy loss function are given by the formula:

$$\hat{\theta}_{EB3} = \left[\frac{\omega}{\left(\hat{\theta}_{E3}\right)^{p}} + (1-\omega)\,\mathrm{E}_{\pi_3}\left(\frac{1}{\theta^{p}}\right)\right]^{-\frac{1}{p}},$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{EB3}\right) = p\left[\mathrm{E}^{*}_{\pi_3}\left(\ln\theta - \ln\hat{\theta}_{EB3}\right)\right].$$

(C) The Linex loss function

The general formula of the balanced Entropy loss function:

$$L_{L,\omega,\hat{\theta}_L}\left(\hat{\theta}_{LB},\theta\right) = \omega L\left(\hat{\theta}_{L},\hat{\theta}_{LB}\right) + (1-\omega)\,L\left(\hat{\theta}_{LB},\theta\right).$$

1 - Informative Prior

The Bayes estimator under the balanced Linex loss function are given by the formula:

$$\hat{\theta}_{LB1} = -\frac{1}{r}\ln\left[\omega\exp\left(-r\hat{\theta}_{L1}\right) + (1-\omega)\,\mathrm{E}_{\pi_1}\left(\exp\left(\,-r\theta\right)\right)\right],$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{LB1}\right) = r\left[\hat{\theta}_{GQB1} - \hat{\theta}_{LB1}\right].$$

2 - Non Informative Prior

The Bayes estimator under the balanced Linex loss function are given by the formula:

$$\hat{\theta}_{LB2} = -\frac{1}{r}\ln\left[\omega\exp\left(-r\hat{\theta}_{L2}\right) + (1-\omega)\,\mathrm{E}_{\pi_2}\left(\exp\left(\,-r\theta\right)\right)\right],$$

and the corresponding posterior risks are

$$PR\left(\hat{\theta}_{LB2}\right) = r\left[\hat{\theta}_{GQB2} - \hat{\theta}_{LB2}\right].$$

3 - Fisher information Prior

The Bayes estimator under the balanced Linex loss function are given by the formula:

$$\hat{\theta}_{LB3} = -\frac{1}{r} \ln \left[\omega \exp \left(-r\hat{\theta}_{L3} \right) + (1-\omega) \, \mathrm{E}_{\pi_3} \left(\exp \left(\, -r\theta \right) \right) \right],$$

and the corresponding posterior risks are

$$PR \left(\hat{\theta}_{LB3} \right) = r \left[\hat{\theta}_{GQB3} - \hat{\theta}_{LB3} \right].$$

4. Simulation and results

In this simulation study, we have chosen samples sizes $(n = 10, 25, 50, 100)$, several parameter values $(\theta = 0.2, 0.5, 1, 5)$, also $(a = b = 2)$ and $(\alpha = p = r = 2)$, also we select $(w = 0.5)$ in order to override the aligned in the estimation process, i.e., this weighted value will give the same loss to the initial estimator and Bayes estimator in their customize formulas. The number of replications used was $(K = 1000)$. The simulation program written by using (R3.5.1) program. After the parameter estimated.

Table (1): Estimated parameters of Zeghdoudi distribution

θ,	n	$\hat{\theta}_{BS1}$	$\hat{\theta}_{BBS1}$	$\hat{\theta}_{BE1}$	$\hat{\theta}_{BBE1}$	$\hat{\theta}_{BS2}$	$\hat{\theta}_{BBS2}$	$\hat{\theta}_{BE2}$	$\hat{\theta}_{BBE2}$
0.2	10	0.2217216	0.21563552	0.236936799	0.22324312	0.299624533	0.204586986	0.208681309	0.209115373
	25	0.224128161	0.221645598	0.229525038	0.224344036	0.214039847	0.216601441	0.218425995	0.218794515
	50	0.20735369	0.206280153	0.209590225	0.20739842	0.203083284	0.20414495	0.205104525	0.20515557
	100	0.207274759	0.206738385	0.2083694	0.207285706	0.205100011	0.205651012	0.206140606	0.206171309
0.5	10	0.634151472	0.602443898	0.713420406	0.642078365	0.517280271	0.544008298	0.56430575	0.567521037
	25	0.601444212	0.589415328	0.62759396	0.602490202	0.553135596	0.56526102	0.574410042	0.575898243
	50	0.590509727	0.58460463	0.602812013	0.590755773	0.566057243	0.572378	0.577156	0.577928
	100	0.51623197	0.51365081	0.521499643	0.516284647	0.505951	0.50851	0.51096	0.511015
1	10	1.508920699	1.433474664	1.697535786	1.527782207	1.215023857	1.286526243	1.325480572	1.3417546
	25	1.149113	1.12613036	1.19907403	1.151111069	1.05847047	1.080809289	1.099180873	1.10116449
	50	1.059284969	1.048692119	1.081353406	1.059726338	1.017403144	1.027751207	1.037352	1.037726
	100	1.075467413	1.070090076	1.08644157	1.075577154	1.05363994	1.059176339	1.064072019	1.064392379
5	10	5.682602878	5.398472734	6.392928238	5.753635414	5.095285492	5.104814041	5.103947809	5.1091452
	25	5.314219249	5.207935	5.54527226	5.323461	5.02004674	5.06084861	5.097740846	5.099695662
	50	5.556690528	5.501123623	5.672454914	5.559005815	5.332266074	5.388911396	5.436820311	5.441188514
	100	5.380825631	5.353921503	5.435732015	5.381374694	5.271585661	5.299301518	5.323779579	5.325398476

The results of the simulation study are summarized and tabulated in table (1).

The estimated values of the parameters are very close to the real values as the sample size increase.

When θ is increasing, the estimated parameter values will pull away from the real values.

In the case of the non-informative prior, the estimated values of the parameter under the balanced loss functions are closer to the real values than these estimators that are estimated by the unbalanced loss functions, but the opposite is true in the case of the informative prior.

5. Conclusions

We conclude through the experimental part results that the balanced loss functions provide an efficient Bayesian estimators in the absence (not defined) of information about the studied phenomenon, as in the case of non-informative prior. While in the case of the informative prior, balanced loss functions may not be as efficient; which means the unbalanced loss functions have a better performance.

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