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SOME DISTINGUISHING CHARACTERISTICS OF THE LINEAR IMMUTABILITY OF CONTINUOUS TIME SERIES VIA BIVARIATE VECTOR VALUED STOCHASTIC PROCESSES

A.I. El-Deosokey[1], M.A. Ghazal, and A.M. Ben Aros

ABSTRACT. During the process of analyzing the continuous expanded finite Fourier transforms of strictly stable $(i + j)$ vector-valued time series, it is presumed that some of the observations have been misplaced. This is done on the basis of an assumption. This is due to the fact that the method entails looking at continuous extended finite Fourier transforms. This is done so that the findings can be interpreted in a manner that is as precise as is practical given the information that is available. The reason for this is so that the findings can be used to make better decisions. Consequently of this additional data, the continuous Fourier transform will become the focal point of the researchers' achievements. At the present time, the concept of asymptotic moments is garnering a large amount of interest from researchers all around the world. In this investigation we will apply our theoretical study in case study on the subject of Electricity Energy,

1. INTRODUCTION

Assuming a linear relationship between $X(t)$ and $Y(t)$, we investigate the statistical features of the extended finite Fourier transform similar to the ones proposed by D.R. Brillinger, M.Rosenblatt, and others in 1967, D.R. Brillinger in 2001,

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Ghazal and Farag in 2005, Ghazal in 1999, Teamah in 2004, Ghazal, et al., in 2005, and Elhassain in (2014). A quick overview of the paper's structure: Section (1) provides introduction, Section 2 investigates the Approximated Characteristics of the Observed Procedure, Section 3 explores the Approximated Characteristics of the Unobserved Procedure, and Section 4 puts our theoretical concepts into practical, in the period between January 2006 and December 2015, we used this technique to analyze the General Electric Company's average monthly Imported and exported Energy.

Consider a fixed series with vector valued $(i+j)$,

$$\mathfrak{R}(t) = \begin{bmatrix} X(t) & Y(t) \end{bmatrix}^T \tag{2.1}$$

$t = 0, \pm 1, \pm 2, \ldots$ with $X(t)$, i- vector-valued and $Y(t)$ j-vector-valued. The series (2.1) is assumed to be a firmly fixed $(i+j)$ vector-valued series with components $\begin{bmatrix} X_r(t) & Ys(t) \end{bmatrix}^T, \quad r = 1, 2, \ldots, j, s = 1, 2, .., i$, all of whose moments exist, and then we define the mean function as:

$$EX(t) = 0, \quad EY(t) = 0$$

and covariance

$$E\left\{[X(t+g) - \tau_x][X(t) - \tau_x]^T\right\} = \tau_{\text{xx}}(g)$$
$$E\left\{[X(t+g) - \tau_x][Y(t) - \tau_y]^T\right\} = \tau_{\text{xy}}(g)$$
$$E\left\{[Y(t+g) - \tau_y][Y(t) - \tau_y]^T\right\} = \tau_{\text{yy}}(g),$$

with spectral densities

$$f_{\text{xx}}(h) = \int_{-\infty}^{\infty} \frac{1}{(2\pi)} \sum_{g=-\infty}^{\infty} \tau_{\text{xx}}(g) Exp(-ihg)$$
$$f_{\text{xy}}(h) = \int_{-\infty}^{\infty} \frac{1}{(2\pi)} \sum_{g=-\infty}^{\infty} \tau_{\text{xy}}(g) Exp(-ihg)$$
$$f_{\text{yy}}(h) = \int_{-\infty}^{\infty} \frac{1}{(2\pi)} g \sum_{u=-\infty}^{\infty} \tau_{\text{yy}}(g) Exp(-ihg), \quad -\infty < h < \infty.$$

For any t, there exists $\beta_a(t), a = 1, 2, \ldots, i, (t \in R)$ which is not dependent on

$$\mathfrak{R}(t)P[\beta_a(t) = 1] = p_a,$$
$$P\left[\beta_a(t) = 0\right] = q_a.$$

Noting that

$$E\left\{\beta_a(t)\right\} = P.$$

The success of an independent data does not depend on the success of another. The modified series definition could be as follows:

$$\delta(t) = \beta(t)\mathfrak{R}(t),$$

where

$$\delta_a(t) = \beta_a(t)\mathfrak{R}_a(t), \tag{2.2}$$

and

$$\beta_a(t) = \begin{cases} 1, & \text{if } y\ X_a(t), Y_a(t) \text{ are recorded}, \\ 0, & \text{otherwise} \end{cases} . \tag{2.3}$$

Assumption. In the data window, $l_a^{(T)}(t)$ is constrained to have finite range, finite variation, and disappear at time$0 < t < T - 1$. Let

$$\gamma^{(T)}_{a_1,\ldots,a_k}(h) = \int_0^T \left[\prod_{r=1}^{N} l^{(T)}_{\text{ar}}(t)\right] \exp\left\{-iht\right\} \text{dt}$$

2. Approximated Characteristics of the Unobserved Procedure

Theorem 2.1. *Given that $X_a(t), Y_a(t), a = 1, 2, \ldots, \min(i, j)$ represents a fixed stochastic procedure and that $\delta_a(t) = \beta_a(t)\mathfrak{R}_a(t), a = 1, 2, \ldots. \min(i, j)$ represents data points that were missing and that $\beta_a(t)$ is a Bernoulli sequence of random variables satisfying (2.2) and (2.3), we get the following.*

$$E\left\{\delta_a(t)\right\} = 0,$$

$$\mathit{Cov}\left\{\delta_{a_1}(t_1), \delta_{a_2}(t_2)\right\} = P_{a_1a_2}\begin{bmatrix} \tau_{xx}(g) & \tau_{xx}(g)K(h)^T \\ K(h)\tau_{xx}(g) & K(h)\tau_{xx}(g)K(h)^T \end{bmatrix}.$$

Lemma 2.1. *If we fix* $\omega_a^{(T)}(h), a = 1, \ldots, \min(j, i)$ *equal to*

$$\omega_a^{(T)}(h) = \left[2\pi \int_0^T \left(l_a^{(T)}(t)\right)^2\right]^{-\frac{1}{2}} \int_{-\infty}^{\infty} l_a^{(T)}(t)\delta_a(t) \exp\{-iht\}\, dt, \tag{3.1}$$

for $h \in R$*, then we find that* $\omega_a^{(T)}(h)$ *has the following dispersion:*

$$D\omega_a{}^{(T)}(h) \tag{3.2}$$

$$= P_{aa} \times \begin{bmatrix} \int_{-\infty}^{\infty} f_{aa}(h-\psi)\times\zeta_{aa}(\psi)d\psi & \int_{-\infty}^{\infty} f_{aa}(h-\psi)K(h)^T\times\zeta_{aa}(\psi)d\psi \\ \int_{-\infty}^{\infty} K(h)f_{aa}(h-\psi)\times\zeta_{aa}(\psi)d\psi & \int_{-\infty}^{\infty} K(h)f_{aa}(h-\psi)K(h)^T\times\zeta_{aa}(\psi)d\psi \end{bmatrix},$$

where

$$\zeta_{aa}{}^{(T)}(x) = \left[\int_0^T (2\pi)(l_a{}^{(T)}(t)dt\right]^{-1} \left|\partial_a{}^{(T)}(x)\right|,$$

$$\partial_a{}^{(T)}(x) = \int_0^T l_a{}^{(T)}(t) \exp(-ixt)dt,$$

$x \in R$.

Proof. Using equation (3.1) we have

$$D\omega_a{}^{(T)}(h) = p_{a_1a_2} \begin{bmatrix} p_1 & p_2 \\ p_3 & p_4 \end{bmatrix},$$

where

$$p_1 = \int_{-\infty}^{\infty} f_{a_1a_2}(u)\times\zeta_{a_1a_2}(h_1-u, h_2-u)du,$$
$$p_2 = \int_{-\infty}^{\infty} f_{a_1a_2}(u)K(h)^T\times\zeta_{a_1a_2}(h_1-u, h_2-u)du$$
$$p_3 = \int_{-\infty}^{\infty} K(h)f_{a_1a_2}(u)\times\zeta_{a_1a_2}(h_1-u, h_2-u)du,$$
$$p_4 = \int_{-\infty}^{\infty} K(h)f_{a_1a_2}(u)K(h)^T\times\zeta_{a_1a_2}(h_1-u, h_2-u)du.$$

When $a_1 = a_2 = a, a = 1, 2, \ldots, \min(i, j)$, and $h_1 = h_2 = h, h \in R$, by substituting $h - u = \psi$, then equation (3.2) is obtained. □

Theorem 2.2. *Given the spectral density function* $f_{aa}(x), a = 1, \ldots, \min(i, j)$*,* $x \in R$ *is bounded and continuous at a point* $x = h, h \in R$*, and if the function* $\zeta_{aa}{}^{(T)}(x)$*,*

$a = 1, \ldots, \min(i,j), x \in R$ *meets these conditions, then*

$$\underset{T\to\infty}{Lim}\, D\omega_a{}^{(T)}(h) = \begin{bmatrix} f_{aa}(h)) & f_{aa}(h)K(\upsilon)^T \\ K(\upsilon)f_{aa}(h)) & K(\upsilon)f_{aa}(h)K(\upsilon)^T \end{bmatrix}, \tag{3.3}$$

$a = 1, \ldots, \min(i,j)$.

Proof. In order to establish formula (3.3), we must demonstrate that

$$\underset{T\to\infty}{\text{Lim}} \left| D\omega_a{}^{(T)}(h) - p_{\text{aa}} \begin{bmatrix} f_{\text{aa}}(h)) & f_{\text{aa}}(h)K(\upsilon)^T \\ K(\upsilon)f_{\text{aa}}(h)) & K(\upsilon)f_{\text{aa}}(h)K(\upsilon)^T \end{bmatrix} \right| = 0,$$

Now from Lemma (3.1) we have

$$\left| D\omega_a{}^{(T)}(h) - p_{\text{aa}} \begin{bmatrix} f_{\text{aa}}(h)) & f_{\text{aa}}(h)K(\upsilon)^T \\ K(\upsilon)f_{\text{aa}}(h)) & K(\upsilon)f_{\text{aa}}(h)K(\upsilon)^T \end{bmatrix} \right|$$

$$\leq p_{aa} \begin{bmatrix} \int_{-\infty}^{\infty} \left| f_{aa}\ (h-\psi) \right| & \int_{-\infty}^{\infty} \left| f_{aa}\ (h-\psi)K(\upsilon)^{(T)} \right| \\ \int_{-\infty}^{\infty} \left| K(\upsilon) f_{aa}\ (h-\psi) \right| & \int_{-\infty}^{\infty} \left| K(\upsilon) f_{aa}\ (h-\psi)K(\upsilon)^{(T)} \right| \end{bmatrix}$$

$$- \begin{bmatrix} \int_{-\infty}^{\infty} \left| f_{aa}\ (h) \right| & \int_{-\infty}^{\infty} \left| f_{aa}\ (h)K(\upsilon)^{(T)} \right| \\ \int_{-\infty}^{\infty} \left| K(\upsilon) f_{aa}\ (h) \right| & \int_{-\infty}^{\infty} \left| K(\upsilon) f_{aa}\ (h-\psi)K(\upsilon)^{(T)} \right| \end{bmatrix} \eta_{aa}{}^{(T)}(\psi)d\psi$$

$$\leq p_{aa} \begin{bmatrix} \int_{-\infty}^{\psi} \left| f_{aa}\ (h-\psi) \right| & \int_{-\infty}^{\psi} \left| f_{aa}\ (h-\psi)K(\upsilon)^{(T)} \right| \\ \int_{-\infty}^{\psi} \left| K(\upsilon) f_{aa}\ (h-\psi) \right| & \int_{-\infty}^{\psi} \left| K(\upsilon) f_{aa}\ (h-\psi)K(\upsilon)^{(T)} \right| \end{bmatrix}$$

$$- p_{aa} \begin{bmatrix} \int_{-\infty}^{\psi} \left| f_{aa}\ (h) \right| & \int_{-\infty}^{\psi} \left| f_{aa}\ (h)K(\upsilon)^{(T)} \right| \\ \int_{-\infty}^{\psi} \left| K(\upsilon) f_{aa}\ (h) \right| & \int_{-\infty}^{\psi} \left| K(\upsilon) f_{aa}\ (h-\psi)K(\upsilon)^{(T)} \right| \end{bmatrix} \eta_{aa}{}^{(T)}(\psi)d\psi$$

$$+p_{aa}\begin{bmatrix} \int_{-\psi}^{\psi}\left| f_{aa}(h-\psi)\right| & \int_{-\psi}^{\psi}\left| f_{aa}(h-\psi)k(\upsilon)^{T}\right| \\ \int_{-\psi}^{\psi}\left| k(\upsilon)f_{aa}(h-\psi)\right| & \int_{-\psi}^{\psi}\left| k(\upsilon)f_{aa}(h-\psi)k(\upsilon)^{T}\right| \end{bmatrix}$$

$$-\ p_{aa}\begin{bmatrix} \int_{-\psi}^{\psi}\left| f_{aa}(h)\right| & \int_{-\psi}^{\psi}\left| f_{aa}(h)K(\upsilon)^{(T)}\right| \\ \int_{-\psi}^{\psi}\left| K(\upsilon)f_{aa}(h)\right| & \int_{-\psi}^{\psi}\left| K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}\right| \end{bmatrix}\eta_{aa}{}^{(T)}(\psi)d\psi$$

$$-p_{aa}\begin{bmatrix} \int_{\psi}^{\infty}\left| f_{aa}(h-\psi)\right| & \int_{\psi}^{\infty}\left| f_{aa}(h-\psi)k(\upsilon)^{T}\right| \\ \int_{\psi}^{\infty}\left| k(\upsilon)f_{aa}(h-\psi)\right| & \int_{\psi}^{\infty}\left| k(\upsilon)f_{aa}(h-\psi)k(\upsilon)^{T}\right| \end{bmatrix}$$

$$-\ p_{aa}\begin{bmatrix} \int_{\psi}^{\infty}\left| f_{aa}(h)\right| & \int_{\psi}^{\infty}\left| f_{aa}(h)K(\upsilon)^{(T)}\right| \\ \int_{\psi}^{\infty}\left| K(\upsilon)f_{aa}(h)\right| & \int_{\psi}^{\infty}\left| K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}\right| \end{bmatrix}\eta_{aa}{}^{(T)}(\psi)d\psi$$

$$= B_1 + B_2 + B_3.$$

We shall explain each of them. Since $f_{\text{ab}}(\psi)$ is continuous at point $\Psi =, a, b = 1, \ldots, \min(i, j)$, then we get

$$B_2 = p_{aa}\begin{bmatrix} \int_{-\psi}^{\psi}\left| f_{aa}(h-\psi)\right| & \int_{-\psi}^{\psi}\left| f_{aa}(h-\psi)k(\upsilon)^{T}\right| \\ \int_{-\psi}^{\psi}\left| k(\upsilon)f_{aa}(h-\psi)\right| & \int_{-\psi}^{\psi}\left| k(\upsilon)f_{aa}(h-\psi)k(\upsilon)^{T}\right| \end{bmatrix}$$

$$- p_{aa} \begin{bmatrix} \int_{-\psi}^{\psi} \left| f_{aa}(h) \right| & \int_{-\psi}^{\psi} \left| f_{aa}(h) K(\upsilon)^{(T)} \right| \\ \int_{-\psi}^{\psi} \left| K(\upsilon) f_{aa}(h) \right| & \int_{-\psi}^{\psi} \left| K(\upsilon) f_{aa}(h-\psi) K(\upsilon)^{(T)} \right| \end{bmatrix} \eta_{aa}{}^{(T)}(\psi) d\psi$$

$$= p_{aa} \begin{bmatrix} \int_{-\psi}^{\psi} \left| f_{aa}(h-\psi) - f_{aa}(h) \right| & \int_{-\psi}^{\psi} \left| f_{aa}(h-\psi) k(\upsilon)^{T} - f_{aa}(h) k(\upsilon)^{T} \right| \\ \int_{-\psi}^{\psi} \left| k(\upsilon) f_{aa}(h-\psi) - f_{aa}(h) k(\upsilon) \right| & \int_{-\psi}^{\psi} \left| k(\upsilon) f_{aa}(h-\psi) k(\upsilon)^{T} - k(\upsilon) f_{aa}(h) k(\upsilon)^{T} \right| \end{bmatrix}$$

$$B_2 \leq \int_{-\psi}^{\psi} \eta_{\mathrm{aa}}{}^{(T)}(\psi) d\psi \leq \Omega \int_{-\infty}^{\infty} \eta_{\mathrm{aa}}(\psi) d\psi \eta_{\mathrm{aa}}{}^{(T)}(\psi) d\psi.$$

Given that $f_{\mathrm{ab}}(\psi)$ is continuous at $\psi = h, a, b = 1, \ldots, \min(i, j)$, we have $B_2 \leq \Omega$. Now, B_2 is extremely low according any Ω is very small thus, $\mathrm{B}_2 = 0$. If $f_{\mathrm{aa}}(h)$, $a = 1, \ldots, \min(i, j)$, $h \in R$ is constrained to a finite value by a constant G, then,

$$B_1 \leq 2G \int_{-\infty}^{-\psi} \eta_{\mathrm{aa}}{}^{(T)}(\psi) d\psi \underset{T \to \infty}{\to} 0,$$

Similarly, $B_3 \underset{T \to \infty}{\to} 0$. Therefore

$$\left| D\omega_a{}^{(T)}(h) - p_{\mathrm{aa}} \begin{bmatrix} f_{\mathrm{aa}}(h)) & f_{\mathrm{aa}}(h) K(\upsilon)^T \\ K(\upsilon) f_{\mathrm{aa}}(h)) & K(\upsilon) f_{\mathrm{aa}}(h) K(\upsilon)^T \end{bmatrix} \right| \underset{T \to \infty}{\to} 0$$

Thus, the proof of the theorem is complete. □

3. Practical Study

3.1. **Analyzing the imported and exported Energy.**

This study provides a monthly average of General Electric Company's exported Energy and its imported Energy from January 2006 through December 2015.

3.1.1. *Analyzing the imported Energy.*

Our results, based on a model of firmly fixed time series with some missing data, will be compared to those produced using the traditional approach, in which all data is recorded. We assume that the data$X_a(t), (t = (1, 2, \ldots, T]$, which is the average of the monthly imported Energy, where all observations are available of the series, is available with some missing, and write the result as $\zeta_a(t) = \beta_a(t)X_a(t), a = 1, 2, \ldots, i$, where $X_a(t), (t = 0, \pm 1, \ldots)$ is an i-vector valued time series that is firmly fixed, and $\beta_a(t)$ is a Bernoulli sequence of random variables that is stochastically independent of $X_a(t)$. Table 4.2.1 compares the results with and without missing data for the traditional situation $\beta = 1, \zeta_a(t) = X_a(t)$ with the scenario where some observations are missing in a random way, $\beta = 0$.

3.1.2. *Analyzing the Exported Energy.*

Our research results, which are based on a model of fixed-time series with some missing data, will be compared to those obtained by the traditional technique, in which all data is observed. Assuming $Y_a(t), (t = (1, 2, ..., T]$ is a fixed j-vector valued time series and $\beta_a(t)$ is a stochastically independent Bernoulli sequence of random variables, we may represent the results as $\varpi_a(t) = \beta_a(t)Y_a(t), a = 1, 2, ..., j$, where$Y_a(t), (t = 0, \pm 1, ...)$ is a monthly average of exported Energy for which all data are available of the series. The results for the standard case $\beta = 1, \varpi_a(t) = Y_a(t)$ and the case where some observations are missing at random $\beta = 0$ are compared in table 4.2.2.

3.1.3. *Analyzing of Energy Imported and Exported Using a Regression Model.*

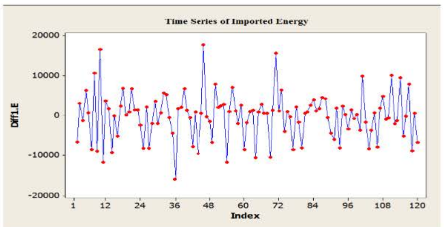
This study will compare our results; a regression model between the averages monthly imported and exported of energy, with the classical results, when all observations are available, for the two scenarios presented in the table 4.2.3.

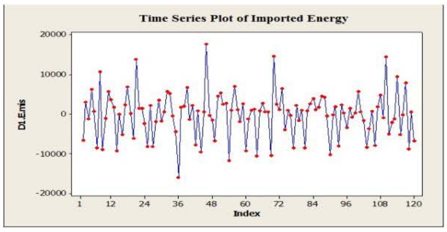
4. Conclusion

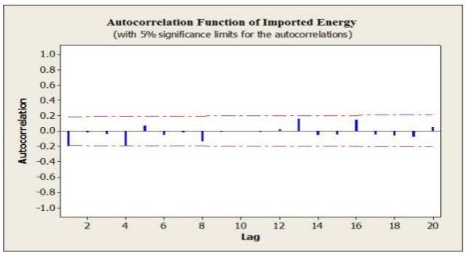
(1) The analysis of time series with missing data yielded the same results as the analysis of traditional time series.
(2) The outcomes of the studied regression model between classical time series X(t) and Y(t) were the same as in the case of missing data, in that both models satisfied the theoretical, mathematical, and least squares constraints.

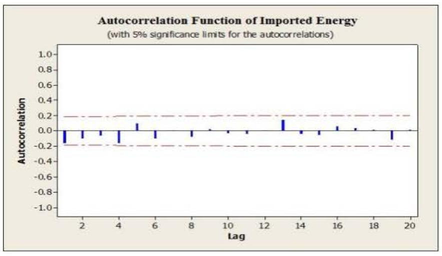
5. FIGURES

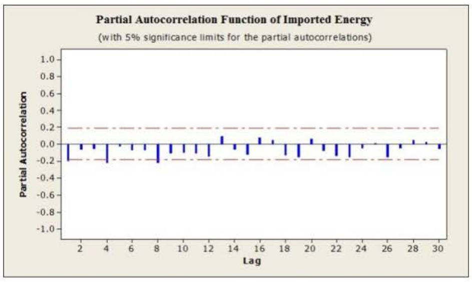
Table 4.1.1. Comparing of the outcomes with and without missing data of the imported Energy

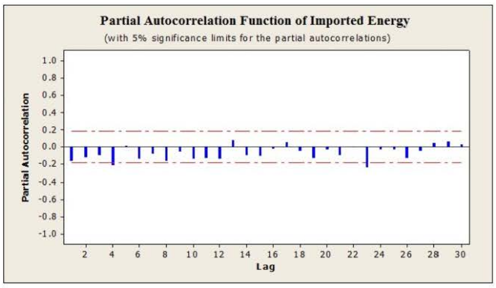
<table>
<tr><td>without missing data</td><td>with missing data</td></tr>
<tr><td>

The monthly average imported energy</td><td>

The monthly average imported energy</td></tr>
<tr><td>

ACF of the monthly average imported Energy</td><td>

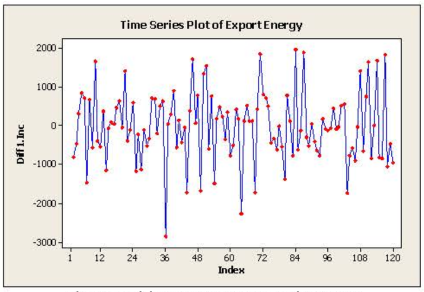
ACF of the monthly average imported Energy</td></tr>
<tr><td>

PACF of the monthly average imported Energy</td><td>

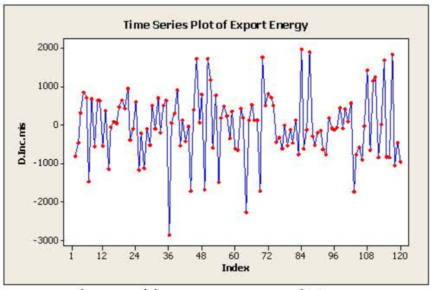
PACF of the monthly average imported Energy</td></tr>
<tr><td>ARIMA Model: imported Energy without missing data
ARIMA(1,1,1)
Final Estimates of Parameters
Type Co-ef SE Co-ef T P
AR 1 0.6234 0.0821 8.19 0.000
MA 1 0.8981 0.0100 47.94 0.000
Residuals: SS = 4021137281 , MS = 27791227, DF = 116
Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 22 36 48
Chi-Square 8.75 20.0 31.2 44.96
DF 9 21 33 45
P-Value 0.450 0.605 0.501 0.432</td><td>ARIMA Model: imported Energy with missing data
ARIMA(1,1,1)
Final Estimates of Parameters
Type Co-ef SE Co-ef T P
AR 1 0.5486 0.0798 7.80 0.000
MA 1 0.9014 0.0100 73.15 0.000
Residuals: SS = 3901621742 , MS = 27576233, DF = 116
Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48
Chi-Square 5.98 18.5 29.4 35.5
DF 9 21 33 45
P-Value 0.721 0.803 0.628 0.794</td></tr>
</table>

Table 4.1.2. Comparing of the outcomes with and without missing data of the Exported Energy

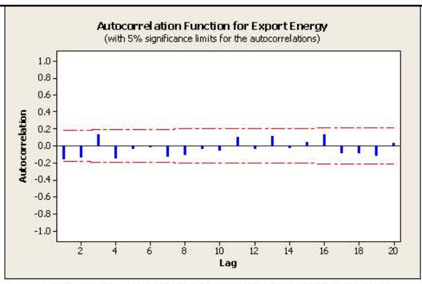
without missing data	with missing data



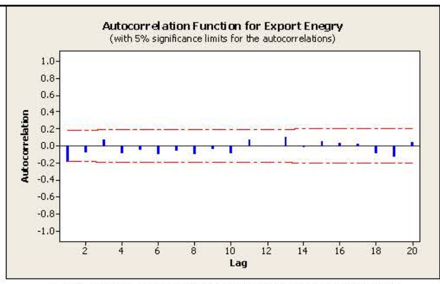
The monthly average exported Energy



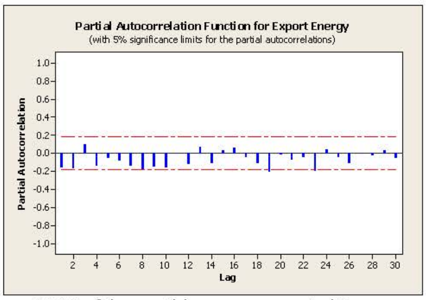
The monthly average exported Energy



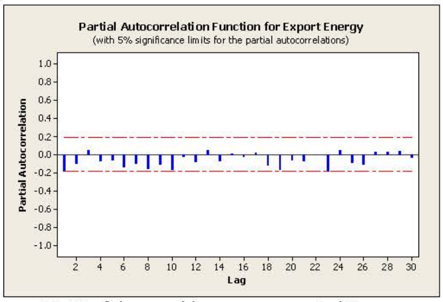
ACF of the monthly average exported Energy



ACF of the monthly average exported Energy



PACF of the monthly average exported Energy



PACF of the monthly average exported Energy

ARIMA Model: The exported Energy with missing data

ARIMA(1,1,1)

Final Estimates of Parameters

Type	Co-ef	SE Co-ef	T	P
AR 1	0.7536	0.0615	8.98	0.000
AM 2	0.9014	0.0151	47.47	0.000

Residuals: SS = 89431624, MS = 638140, DF = 116

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	8.1	19.95	25.89	44.8
DF	9	21	33	45
P-Value	0.515	0.522	0.720	0.622

ARIMA Model: The exported Energy without missing data

ARIMA(1,1,1)

Final Estimates of Parameters

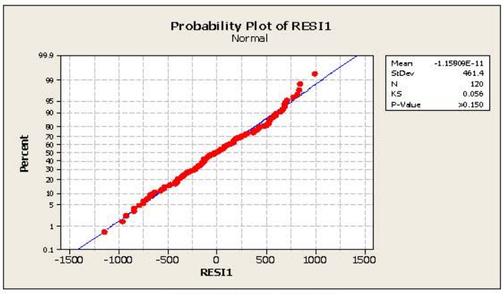
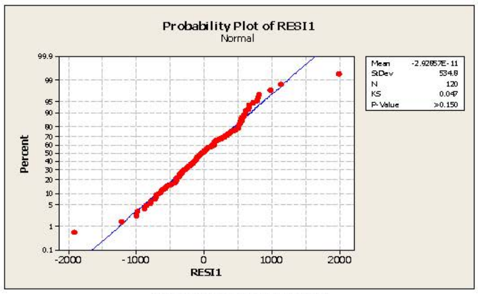
Type	Co-ef	SE Co-ef	T	P
AR 1	0.7656	0.0589	98.69	0.000
AM 1	0.8902	0.0093	67.98	0.000

Residuals: SS = 90135160, MS = 682338, DF = 116

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	12.1	25.98	35.89	51.1
DF	9	21	33	45
P-Value	0.202	0.205	0.317	0.258

Table 4.1.3. Comparing of the outcomes with and without missing data of the regression analysis

Without missing data	With missing data
The regression model is Exported Energy = 3260 + 0.190 imported Energy Predictor Co-ef SE Co-ef T P Constant 3260 1242 2.49 0.013 imported Energy 0.1901 0.003589 30.29 0.000 S = 459.317 R-Sq = 88.5% R-Sq= 88.4% Analysis of Variance Source DF SS MS F P Regression 1 186005060 186005060 917.73 0.000 Residual Error 118 24520185 223663 Total 119 210525245 Durbin-watson statistic = 1.69868	The regression model is Exported Energy = 2413 + 0.189 imported Energy Predictor Co-ef SE Co-ef T P Constant 2413 1374 1.49 0.038 imported Energy 0.189 0.006423 24.83 0.000 S = 645.097 R-Sq = 82.8% R-Sq(adj) = 82.7% Analysis of Variance Source DF SS MS F P Regression 1 167904350 178004361 608.05 0.000 Residual Error 118 29140020 297304 Total 119 197044370 Durbin-watson statistic = 1.57860
 Plot fpr the Residuals	 Plot fpr the Residuals

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