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SUPPORT THEOREM FOR THE POSITIVE RANDOM EVOLUTION EQUATION IN HÖLDER NORM

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ABSTRACT. Let's consider the stochastic differential equation

$$\begin{cases} dX_t = \sigma(X_t, V_t)dW_t + b(X_t, R_t)dt \\ X_0 = x > 0 \end{cases}$$

In this paper, we establish the support theorem for this positive random evolution equation type on $C^{\alpha,0}([0;1];\mathbb{R})$. We use the linear interpolations of W for the proof.

1. INTRODUCTION

We consider the familly of stochastic processes $X = \{X_t; 0 \leq t \leq 1\}$, with X is a solution of the Itô's differential equation

$$X_t = x + \int_0^t \sigma(X_s, V_s)dW_s + \int_0^t b(X_s, R_s)ds; \ x > 0, \tag{1.1}$$

where W is a one-dimensional standard Brownian motion on some filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$, $R = \{R_t, t \in [0; 1]\}$ is a $\mathbb{R}-$valued random variable $\mathcal{F}_t-$progressivly measurable and $V = \{V_t, t \in [0; 1]\}$ is a random process such that

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its topological support is compact subset of $C^{\alpha,0}([0;1];\mathbb{R})$, R and V satisfy some integrability conditions and W is independent of R and V. The diffusion coefficient $\sigma : \mathbb{R}\otimes\mathbb{R} \longrightarrow \mathbb{R}$ and the drift coefficient $b : \mathbb{R}\otimes\mathbb{R} \longrightarrow \mathbb{R}$ satisfy the following assumptions:

H: b is globally lipschitzian with respect to x and y, and bounded with $b(0,0) > 0$ and σ is Hölder continuous with exponent $\gamma \in \left[\frac{1}{2};1\right]$ in x, globally lipschitzian in z and $\sigma(0,0) = 0$.

H: **H'**: σ is C^2 in x, bounded together with its partial derivatives in order one and two; and the derivatives in order two σ" is locally lipschitzian.

We know that under assumption **H**, the equation (1.1) admits an unique solution X_t^ε, furthermore $X_t^\varepsilon \geq 0$ see D. Revuz and M. Yor [10] Chapter IX, Theorem 3.5.

In recent years, for $R \equiv 0$ and $V \equiv 0$ then $b(x,y) = b(x)$ and $\sigma(x,z) = \sigma(x)$, applications to finance have attracted the attention to the study of these models that are based on diffusion processes whose state space is the positive half line. Let $b(x) = \alpha(\beta - x)$ and $\sigma(x) = \rho x^\gamma$ with some constants $\alpha > 0$, $\beta > 0$ and $\gamma \in [\frac{1}{2};1]$; (1.1) is a constant elasticity of variance (CEV) model and the special case $\gamma = \frac{1}{2}$ is the Cox-Ingersoll-Ross (CIR) model.

In particular, the CIR model is used in the Heston model in order to describe the evolution of the volatility. Considering the driving noise of the volatility close to 0 is a first step in order to study the convergence of the stochastic volatility model to the classical Black and Scholes one.

For the positive diffusion type, the general Freidlin-Wentzell large deviation is studied by Baldi and Caramellino [3], but Y. Li and S. Zhang [6] have established the moderate deviation and central limit theorem. In the other hand, R.N.B. Rakotoarisoa and T.J. Rabeherimanana [9] have studied the moderate deviation with the weak convergence method. In this paper, we will proove the support theorem for the positive diffusion. For that, Stroock and Varadhan 1972 [11] have proved the support of diffusion process and given application to the maximum principle in finite dimensional state spaces and with finite dimensional Wiener processes. But A. Millet and S. Zolé [8] have established a simple proof of the support theorem for the diffusion process. Several authors have tried to extend their results for the same case but by different methods such as Ledoux and al [5] are studied the large deviation principle and support theorem via rough path. Recently; J.

Andriatahiana and al 2017 [1] have established the support theorem for random evolution equation in Hölder norm and their extend this result in Besov-Orlicz norm in 2020 [2].

Let $\alpha > 0$ and denote $C^{\alpha}([0;1];\mathbb{R})$ the set of $\alpha-$Hölder continuous functions, i.e., set of continuous functions $f : [0;1] \longrightarrow \mathbb{R}$ such that

$$\|f\|_{\alpha} = \sup_{t} f(t) + \sup_{0<|t-s|<1} \frac{|f(t)-f(s)|}{|t-s|^{\alpha}} < \infty. \tag{1.2}$$

Then, $\|.\|_{\alpha}$ is called the $\alpha-$Hölder norm.

Define the hölderian modulus of continuity of f by

$$\omega_{\alpha}(f,\delta) = \sup_{0\leq|t-s|\leq\delta} \frac{|f(t)-f(s)|}{|t-s|^{\alpha}}.$$

Let $\mathcal{H}$ denote the Cameron-Martin space and give $h \in \mathcal{H}$, $\psi \in \operatorname{supp} R$ and $\chi \in \operatorname{supp} V$ (support of the distribution of R and V). Consider the following ordinary differential equation

$$\begin{aligned} S(h,\psi,\chi)_t = x &+ \int_0^t \sigma(S(h,\psi,\chi)_s,\chi_s)\dot{h}_s ds \\ &+ \int_0^t \left[b(S(h,\psi,\chi)_s,\psi_s) - \frac{1}{2}(\nabla\sigma)\sigma(S(h,\psi,\chi)_s,\chi_s) \right] ds, \end{aligned} \tag{1.3}$$

where $x > 0$. Under assumption **H** and Lemma (3.11) of [3], (1.3) admits a unique solution $S(h,\psi)$ for $t \in [0;T]$, $T > 0$. moreover for every compact $K \subset \mathbb{R}^{+}$ and $a > 0$, there exists $b > 0$ such that $S(h,\psi,\chi) \geq b$ for every $x \in K$ and $|h| < a$, that is $S(h,\psi,\chi)$ solution of the equation (1.3) stays away from 0. We will caracterized then the support of the diffusion positive when $X_t > C$ where C is an constant positive.

We will give the proof of the characterization of the support of $\mathbb{P} \circ X^{-1}$ as the closure of $\{S(h,\psi,\chi); h \in \mathcal{H}, \psi \in \operatorname{supp} R, \chi \in \operatorname{supp} V\}$. To proove this result, we use the approximation theorem of the stochastic system adapted linear interpolation of ω^n of ω and Millet result, see [8]. Thus, we check the convergence in probability of $\|S(\omega^n,\psi,\chi) - X(\omega)\|_{\alpha}$ and $\|X(\omega-\omega^n+h) - S(h,\psi,\chi)\|_{\alpha}$ to zero in L^2. Since the low of transformation T_n of ω is absolutely continuous with respect to $\mathbb{P}$, the second convergence yields that the support $\mathbb{P} \circ X^{-1}$ contain

$\overline{\{S(h,\psi,\chi); h\in\mathcal{H}, \psi\in\operatorname{supp} R, \chi\in\operatorname{supp} V\}}$ but the first one implies the inverse inclusion in usual way.

The rest of this paper is organised as follow; the following section introduce some preliminaries. In the last section, we will give our main result and characterized the support for positive

2. General Results and Approximations

In this section, we state criteria of convergence in Holder norms and a general theorem characterizing of the law of the Weiner functional wich is useful to our results. The following proposition is a consequence of the Garcia-Rodemich-Rumsey lemma

Proposition 2.1.
(i) Let $\{Y^n(t)\}$ a sequence of $\mathbb{R}-$valued processes such that
A1: For every $p\in[1,\infty)$ there exists C such that

$$\sup_n \mathbb{E}\left(|Y^n(t)-Y^n(s)|^{2p}\right)\leq C|t-s|^p$$

for every $s,t\in[0;1]$. Then, for every $\lambda>0$ and $\beta<\frac{p-1}{2p}$ there exists $C>0$ such that

$$\sup_n \mathbb{P}\left(\sup_{|s-t|<1, s\neq t}\frac{|Y^n(t)-Y^n(s)|}{|t-s|^\beta}>\lambda\right)\leq C\lambda^{-2p}. \tag{2.1}$$

(ii) Let $(Y^n(t))_{t\in[0;1]}$ be a sequence of $\mathbb{R}-$valued processes satisfying A1 with the following assumption A2:
A2: for any $\varepsilon>0$

$$\lim_{n\longrightarrow\infty}\mathbb{P}\left(\sup_{0\leq i\leq 2^n}|Y(i2^{-n})|>\varepsilon\right)=0.$$

Then, for any $\alpha\in[0;1/2]$, one has that

$$\lim_n \mathbb{P}(\|Y^n\|_\alpha>\varepsilon)=0. \tag{2.2}$$

The following proposition state sufficient conditions for inclusions on the support of the law of the measurable map $F:\Omega\longrightarrow E$, where $(E,\|.\|)$ is a separable Banach space. The proof is straightforward.

Proposition 2.2. *Let $F : \Omega \longrightarrow E$ be measurable. Let $\zeta_1 : \mathcal{H} \longrightarrow E$ be a measurable map, and let $H_n : \Omega \longrightarrow \mathcal{H}$ be a sequence of variables such that for any $\varepsilon > 0$,*

$$\lim_n \left(\|F(\omega) - \zeta_1(H_n(\omega))\| > \varepsilon\right) = 0, \tag{2.3}$$

then

$$\operatorname{supp}(F \circ \mathbb{P}^{-1}) \subset \overline{\{\zeta_1(h); h \in \mathcal{H}\}}. \tag{2.4}$$

Let $\zeta_2 : \mathcal{H} \longrightarrow E$ be a map and for fixed h let $T_n^h : \Omega \longrightarrow \Omega$ be a sequence of measurable transformations such that $\mathbb{P} \circ (T_n^h)^{-1} \ll \mathbb{P}$, and for any $\varepsilon > 0$,

$$\lim_n \Big(\|F(T_n^h(\omega)) - \zeta_2(h))\| < \varepsilon\Big) = 0, \tag{2.5}$$

then

$$\operatorname{supp}(F \circ \mathbb{P}^{-1}) \supset \overline{\{\zeta_2(h); h \in \mathcal{H}\}}. \tag{2.6}$$

Given an integer $n > 0$; define $\mathcal{D}_n = \{i2^{-n}; 0 \leq i \leq 2^n\}$ the set of n-dyadic points. For $t \in [0;1]$, $\frac{k}{2^n} < t < \frac{k+1}{2^n}$, set

$$\tilde{t}_n = \frac{k}{2^n} \; and \; \bar{t}_n = \frac{k-1}{2^n} \vee 0 \tag{2.7}$$

and let W^n be the adapted linear interpolation of W defined by

$$W_t^n = W_{\bar{t}_n} + 2^n(t - \tilde{t}_n)(W_{\tilde{t}_n} - W_{\bar{t}_n}). \tag{2.8}$$

We consider the map $\zeta_1 = \zeta_2 = S(.)$, $H_n(\omega) = \omega_n$, and $T_n^h(\omega) = \omega - \omega^n + h$. Girsanov's theorem implies that $\mathbb{P} \circ (T_n^h)^{-1}$ is absolutely continuous with respect to $\mathbb{P}$. Let $X^n(\omega) = X(\omega - \omega^n + h)$. Fix $\alpha < \frac{1}{2}$ and let $\beta \in]0; \frac{1}{2}[$, $X - x$, $X \circ T_h^n - x$, $S_.(\omega^n, \psi, \chi) - x$ and $S_.(h, \psi, \chi) - x$ a.s. belong to $C^\alpha([0;1];\mathbb{R})$ and have initial value 0, using Ciesielski [4], it is easy to see that they also belong to the separable Banach space H_0^α of $C^\alpha([0;1];\mathbb{R})$ defined by:

$$H_0^\alpha = \{f \in C^\alpha([0;1];\mathbb{R});\; f(0) = 0,\; |f(t) - f(s)| < \circ|t-s|^\alpha \; as \; |t-s| \longrightarrow 0\}$$

Thus, by Proposition 2.2, the equality

$$\mathbb{P} \circ X^{-1} = \overline{\{S(h, \psi, \chi); h \in \mathcal{H}, \psi \in \operatorname{supp} R, \chi \in \operatorname{supp} V\}}$$

will follow from the following convergence results for every $\varepsilon > 0$,

$$\lim_n (\|X(\omega) - S(\omega^n, \psi, \chi)\|_\alpha > \varepsilon) = 0, \tag{2.9}$$

$$\lim_n(\|X(\omega-\omega^n+h)-S(h,\psi,\chi)\|_\alpha<\varepsilon)=0. \tag{2.10}$$

Approximation of stochastic integrals of Riemann sums implies that $X^n(\omega) := X(\omega-\omega^n+h)$ is solution of the following stochastic differential equation (SDE in short)

$$\begin{aligned} X_t^n = x &+ \int_0^t \sigma(X_s^n,V_s)dW_s + \int_0^t \sigma(X_s^n,V_s)\dot{\omega}_s^n ds \\ &+ \int_0^t \sigma(X_s^n,V_s)\dot{h}_s ds + \int_0^t b(X_s^n,R_s)ds, \end{aligned} \tag{2.11}$$

while $S(\omega^n,\psi,\chi)$ satisfies

$$\begin{aligned} S(\omega^n,\psi,\chi)_t &= S(\omega^n,\psi,\chi)_0 + \textstyle\int_0^t \sigma(S(\omega^n,\psi,\chi)_s,\chi_s)\dot{\omega}_s^n ds \\ &+ \textstyle\int_0^t [b(S(\omega^n,\psi,\chi)_s,\psi_s) - \frac{1}{2}(\bigtriangledown\sigma)\sigma(S(\omega^n,\psi,\chi)_s,\chi_s)]ds. \end{aligned} \tag{2.12}$$

Thus both processus X^n and $S(\omega^n,\psi)$ are particular of a diffusion $Y_.^n$ solution of the following SDE

$$\begin{aligned} Y_t^n &= x + \textstyle\int_0^t F(Y_s^n,V_s)dW_s + \int_0^t G(Y_s^n,V_s)\dot{\omega}_s^n ds + \int_0^t h(Y_s^n,V_s)\dot{h}_s ds \\ &+ \textstyle\int_0^t I(Y_s^n,V_s)ds + \int_0^t B(Y_s^n,R_s)ds, \end{aligned} \tag{2.13}$$

and

$$\begin{aligned} \bar{Y}_t^n &= x + \textstyle\int_0^t F(\bar{Y}_s^n,\chi_s)dW_s + \int_0^t G(\bar{Y}_s^n,\chi_s)\dot{\omega}_s^n ds + \int_0^t h(\bar{Y}_s^n,\chi_s)\dot{h}_s ds \\ &+ \textstyle\int_0^t I(\bar{Y}_s^n,\chi_s)ds + \int_0^t B(\bar{Y}_s^n,\phi_s)ds, \end{aligned} \tag{2.14}$$

where the coefficients F, G, H, I and B satisfy the condition: **H**": F, G, H, I, B : $\mathbb{R}\otimes\mathbb{R} \longrightarrow \mathbb{R}$ globally lipschitzian functions and G is a class C^2 with bounded derivatives.

Given the functions F, G, H, I and B and let Z the solution of the following SDE

$$\begin{aligned} Z_t = x &+ \int_0^t [F(Z_s,V_s)+G(Z_s;V_s)]dW_s + \int_0^t H(Z_s,V_s)\dot{h}_s ds \\ &+ \int_0^t B(Z_s,R_s)ds + \int_0^t \bigtriangledown G(Z_s,V_s)[F(Z_s,V_s)+\frac{1}{2}G(Z_s,V_s)]ds. \end{aligned} \tag{2.15}$$

For $\alpha\in[0;1/2[$, (2.9) and (2.10) are particular case of the following convergence for $\delta>0$

$$\lim_n(\|Y^n-Z\|_\alpha>\delta)=0, \tag{2.16}$$

and

$$\lim_n (\|\bar{Y}^n - Z\|_\alpha > \delta) = 0. \tag{2.17}$$

Indeed $F = 0,\ G = \sigma,\ H = 0,\ I = -\frac{1}{2}(\bigtriangledown\sigma)\sigma$, and $B = b$ we obtain (2.9) while (2.10) yields as $F = \sigma,\ G = -\sigma,\ H = \sigma$, $I = 0$ and $B = b$. It is well known as for $s, t \in [0; 1]$ and $p \in [0; \infty)$

$$\mathbb{E}(|Z_t - Z_s|^{2p}) \leq C|t - s|^p. \tag{2.18}$$

Thus, by Proposition 2.1, it suffices to chek that for any $s, t \in [0; 1]$ and $p \in [0; \infty)$

$$\sup_n \mathbb{E}\left(|Y_t^n - Y_s^n|^{2p}\right) \leq C|t - s|^p, \tag{2.19}$$

$$\sup_n \mathbb{E}\left(|\bar{Y}_t^n - \bar{Y}_s^n|^{2p}\right) \leq C|t - s|^p, \tag{2.20}$$

and

$$\lim_n \mathbb{E}\left(\sup_{0\leq i\leq 2^n} |Y_{i2^{-n}}^n - Z_{i2^{-n}}|^2\right) = 0, \tag{2.21}$$

$$\lim_n \mathbb{E}\left(\sup_{0\leq i\leq 2^n} |\bar{Y}_{i2^{-n}}^n - Z_{i2^{-n}}|^2\right) = 0. \tag{2.22}$$

3. MAIN RESULT AND DISCUSSION

We begin this section with our main result.

Theorem 3.1. *Let σ and b be functions such that conditions **H** and **H'** are in place, and let X the solution of the equation* (1.1).*Then, for any $\alpha \in [0; 1/2[$ the support of the probability $P \circ X^{-1}$ is the cloussure of the set $\{S(h, \psi); h \in \mathcal{H}\}$ such that $S(h)$ is given by equation* (1.3)

Proof. According to the Proposition 2.2, it is sufficient to prove that estimations (2.3) and (2.5) are trues. For that, the equation (2.3) is a consequence of (2.9) but (2.10) yields the (2.5) one. Furthermore, since (2.9) and (2.10) are particular case of (2.16) and (2.17), to complete the proof, by Proposition 2.1, it suffices to check that (2.19), (2.20),(2.21) and (2.22) are fulfils. □

Proposition 3.1. *Let (X) be an random process and let* $\phi \in \operatorname{supp} X$. *Then, we have*

$$\lim_n \mathbb{E}\left(\sup_{0\leq i\leq 2^n} |\phi_{i2^{-n}} - X_{i2^{-n}}| = 0 \right). \tag{3.1}$$

Proof. We know that $P(X \in \operatorname{supp} X) = 1$, there exist $\varphi \in \operatorname{supp} X$ such that $\varphi = X(t)$ for any $t \in [0;1]$. Then, $\mathbb{E}(\sup_{0\leq i\leq 2^n} |\phi_{i2^{-n}} - \varphi_{i2^{-n}}|^2) \longrightarrow 0$ as n large enough complete the proof. By following result of [7] $\sup_{0\leq i\leq 2^n} |\phi_{i2^{-n}} - \varphi_{i2^{-n}}| \longrightarrow 0$, the desired proposition follows □

We use the following theorem which is a one version of the Azréla-Ascoli theorem (see Mellouk [7])

Theorem 3.2. *A set* $A \subset C^{\alpha,0}([0;1],\mathbb{R})$ *has a compact closure in* $A \subset C^{\alpha}([0;1],\mathbb{R})$*if and only if the following two conditions holds*

$$\sup_{f\in A} \|f\|_\alpha < \infty$$

and

$$\lim_{\delta\downarrow 0} \sup_{f\in A} \omega_\alpha(f,\delta) = 0.$$

The following results are consequence of this theorem, since $\operatorname{supp} Z$ is a compact suset of $C^{\alpha,0}([0;1],\mathbb{R})$, we have

$$\lim_n \sup_{t\in[0;1]} |\chi_s - \chi_{\bar{s_n}}| = 0 \tag{3.2}$$

and by triangular inequalitytogether with the Proposition3.1, we obtain

$$\lim_n \sup_{t\in[0;1]} |V_s - V_{\bar{s_n}}| = 0. \tag{3.3}$$

Proposition 3.2. *Let F, G,H,I and B be the functions such that conditions **H**" is satisfied and let* (Y_t^n) *(resp* $\bar{Y}_t^n$*) be the respective solutions of* (2.13) *(resp* (2.14)*). Then, given* $p \in [0;\infty)$*, there exists an constant C suth that for* $s,t \in [0;1]$ *we have*

$$\sup_n \mathbb{E}\left(|Y_t - Y_s|^{2p}\right) \leq C|t-s|^p$$

and

$$\sup_n \mathbb{E}\left(|\bar{Y}_t - \bar{Y}_s|^{2p}\right) \leq C|t-s|^p.$$

Proof. We use the same argument for the proof of the two inequalities in this proposition. Fix $p \in [0;\infty)$ and $s,t \in \mathbb{R}$,then for every $n \geq 1$

$$\mathbb{E}\left(|Y_t - Y_s|^{2p}\right) \leq C(I_1 + I_2 + I_3 + +I_4 I_5) \tag{3.4}$$

with

$$\begin{aligned}
I_1 &= \mathbb{E}\left(|\textstyle\int_s^t F(Y_u^n, V_u)dW_u|^{2p}\right),\\
I_2 &= \mathbb{E}\left(|\textstyle\int_s^t G(Y_s^n, V_u)\dot{\omega}_s^n ds|^{2^p}\right),\\
I_3 &= \mathbb{E}\left(|\textstyle\int_s^t H(Y_u^n, V_u)\dot{h}_u du|^{2p}\right),\\
I_4 &= \mathbb{E}\left(|\textstyle\int_s^t I(Y_u^n, V_u)du|^{2p}\right),\\
I_5 &= \mathbb{E}\left(|\textstyle\int_s^t B(Y_u^n, R_u)du|^{2p}\right).
\end{aligned}$$

Burkholder's inequality on I_1, from Schwartz and Hölde's inequalities, we have

$$I_1 + I_3 + I_4 + I_5 \leq C|t-s|^p,$$

and for the I_2, we can write that

$$I_2 \leq I_2^1(n) + I_2^2(n),$$

where

$$\begin{aligned}
I_2^1(n) &= \mathbb{E}\left(|\textstyle\int_s^t G(Y_{\bar{u}_n}^n, V_{\bar{u}_n})\dot{\omega}_u^n du|^{2^p}\right),\\
I_2^2(n) &= \mathbb{E}\left(|\textstyle\int_s^t [G(Y_u^n, V_u) - G(Y_{\bar{u}_n}^n, V_{\bar{u}_n})]\dot{\omega}_u^n du|^{2^p}\right).
\end{aligned}$$

Clearly, we have $\sup_n I_2^1(n) \leq C|t-s|^p$. By Hölder's inequality for the conjugate exponents $a > 1$ and $b > 1$, we have

$$\begin{aligned}
I_2^2(n) &\leq |t-s|^{2p+1} \textstyle\int_s^t \left[\mathbb{E}\left(|G(Y_u^n, V_u) - G(Y_{\bar{u}_n}^n, V_{\bar{u}_n})|^{2pa}\right)\right]^{\frac{1}{a}} \left[\mathbb{E}\left(|\dot{\omega}_u^n|^{2pb}\right)\right]^{\frac{1}{b}} du\\
&\leq C|t-s|^{2p+1} 2^{np} \textstyle\int_s^t \left[\mathbb{E}\left(|Y_u^n - Y_{\bar{u}_n}^n|^{2pa} + |V_u - V_{\bar{u}_n}|^{2pa}\right)\right]^{\frac{1}{a}} du.
\end{aligned}$$

By the equation (2.18),there exist an constant C such that $\sup_n \mathbb{E}|V_u - V_{\bar{u}_n}|^{2p} \leq C2^{-np}$ Thus, the proof is reduced to check that this estimate follows in the particular case $s = \bar{u}_n$ et $t = u$. These arguments imply that

$$\sup_s \mathbb{E}\Big(\Big|\int_{\bar{s}_n}^s \Big[F(Y_u^n, V_u)dW_u + H(Y_u^n, V_u)\dot{h}_u + B(Y_u^n, R_u)du\Big]\Big|^{2p}\Big) \leq C2^{-np}.$$

Therefore, we should checking that for every $p \in [0;\infty)$

$$\sup_s \mathbb{E}\left[\left|\int_{\bar{s}_u}^s G(Y_u^n, V_u)\dot{\omega}_u^n du\right|^{2p}\right] \leq C2^{-np}.$$

Clearly

$$
\begin{aligned}
\mathbb{E}\left[\left|\int_{\bar{s}_u}^{s} G(Y_u^n,V_u)\dot{\omega}_u^n du\right|^{2p}\right] &\leq C\mathbb{E}\left(\left(2^n\int_{\bar{s}_n}^{\tilde{s}_n}|G(Y_u^n,V_u)du|^{2p}\right)|W_{\bar{u}_n}-W_{\bar{u}_n-2^{-n}\vee 0}|^{2p}\right)\\
&\quad +C\mathbb{E}\left(\left(2^n\int_{\tilde{s}_n}^{s}|G(Y_u^n,V_u)du|^{2p}\right)|W_{\tilde{u}_n}-W_{\bar{u}_n}|^{2p}\right)\\
&\leq C\Big[\mathbb{E}\left(|W_{\bar{u}_n}-W_{\bar{u}_n-2^{-n}\vee 0}|^{2p}\right)+\mathbb{E}\left(|W_{\tilde{u}_n}-W_{\bar{u}_n}|^{2p}\right)\Big]\\
. \qquad &\leq C2^{-np}.
\end{aligned}
$$

This inequality implies $\sup_n I_2^2(n) \leq C|t-s|^p$. The proof of the proposition is complete. □

Proposition 3.3. *Assume F, G,H,I and B the functions such that condition **H**" is satisfied and that the solution $(Y_.^n)$ of* (2.13) *satisfies* (2.19). *Let $(Z_.)$ be a solution of* (2.15). *Then, we obtain*

$$\lim_n \mathbb{E}\left(\sup_{0\leq i\leq 2^n}|Y_{i2^{-n}}^n - Z_{i2^{-n}}|^2\right)=0.$$

Proposition 3.4. *Assume F, G,H,I and B the functions such that condition **H**" is satisfied and that the solution $(\bar{Y}_.^n)$ of* (2.13) *satisfies* (2.19). *Let $(Z_.)$ be a solution of* (2.15). *Then, we obtain*

$$\lim_n \mathbb{E}\left(\sup_{0\leq i\leq 2^n}|\bar{Y}_{i2^{-n}}^n - Z_{i2^{-n}}|^2\right)=0.$$

We need the following lemmas for the proof of the Proposition 3.3, we omit their demonstrations wich can be found in [8].

Lemma 3.1. *Suppose that $(Y_.^n)$ is a sequence of processes such that* (2.19) *holds. Let f be an globally Lipschitz function; then*

$$\lim_n \mathbb{E}\left(\sup_{0\leq k\leq 2^n}\left|\int_0^{k2^{-n}} f(Y_{\bar{s}_n}^n)\left[\dot{\omega}_s^n ds - dW_s\right]\right|^2\right)=0. \tag{3.5}$$

Lemma 3.2. *Let $(J_t^n)_{t\in[0;1]}$ be a sequence of measurable processes such that there exists $p\in[1;\infty)$, $C>0$ and an sequence $\alpha(n)$ such that*

$$\lim_n \alpha(n)=0 \text{ and } \sup_t \mathbb{E}|J_t^n|^{2p}\leq \alpha(n)2^{-np}.$$

Then

$$\lim_n \mathbb{E}\left(\sup_{0\le k\le 2^n}\left|\int_0^{k2^{-n}} |J_s^n \dot{\omega}_s^n| ds\right|^2\right) = 0. \tag{3.6}$$

Proof of Proposition 3.3. Let $n > 1$ and let us take $t = k2^{-n}$, then

$$\begin{aligned}
Y_t^n - Z_t = &\int_0^t [(F+G)(Y_{\bar{s}_n}^n, V_{\bar{s}_n}) - (F+G)(Z_{\bar{s}_n}, V_{\bar{s}_n})] dW_s + \int_0^t [H(Y_{\bar{s}_n}^n, V_{\bar{s}_n}) \\
&- H(Z_{\bar{s}_n}, V_{\bar{s}_n})] \dot{h}_s ds + \int_0^t [I(Y_{\bar{s}_n}^n, V_{\bar{s}_n}) - I(Z_{\bar{s}_n}, V_{\bar{s}_n})] ds \\
&+ \int_0^t \left(\left[B(Y_{\bar{s}_n}, R_s) + ((\bigtriangledown G)F + \frac{1}{2}(\bigtriangledown G)G)(Y_{\bar{s}_n}^n, V_{\bar{s}_n})\right]\right. \\
&\left. - \left[B(Z_{\bar{s}_n}, R_s) + ((\bigtriangledown G)F + \frac{1}{2}(\bigtriangledown G)G)(Z_{\bar{s}_n}, V_{\bar{s}_n})\right]\right) ds + \sum_{j=1}^6 J_j^n(t),
\end{aligned}$$

where

$$\begin{aligned}
J_1^n(t) &= \int_0^t \left(F(Y_s^n, V_s) - F(Y_{\bar{s}_n}^n, V_{\bar{s}_n}) - [F+G](Z_s, V_s) + [F+G](Z_{\bar{s}_n, \bar{s}_n})\right) dW_s, \\
J_2^n(t) &= \int_0^t \left(H(Y_s^n, V_s) - H(Y_{\bar{s}_n}^n, V_{\bar{s}_n}) - H(Z_s, V_s) + H(Z_{\bar{s}_n}, V_{\bar{s}_n})\right) \dot{h}_s ds, \\
J_3^n(t) &= \int_0^t \left(I(Y_s^n, V_s) - I(Y_{\bar{s}_n}^n, V_{\bar{s}_n}) - I(Z_s, V_s) + I(Z_{\bar{s}_n}, V_{\bar{s}_n})\right) ds, \\
J_4^n(t) &= \int_0^t \left(B(Y_s^n, R_s) - B(Y_{\bar{s}_n}^n, R_s) - \left[B(Z_s, R_s) + (\bigtriangledown G)F + \frac{1}{2}(\bigtriangledown G)G\right](Z_s, V_s)\right. \\
&\quad \left. + B(Z_{\bar{s}_n}, R_s)\left[(\bigtriangledown G)F + \frac{1}{2}(\bigtriangledown G)G\right](Z_{\bar{s}_n}, V_{\bar{s}_n})\right) ds, \\
J_5^n(t) &= \int_0^t G(Y_{\bar{s}_n}^n, V_{\bar{s}_n}) \left[\dot{\omega}_s^n ds - dW_s\right], \\
J_6^n(t) &= \int_0^t \left[G(Y_s^n, V_s) - G(Y_{\bar{s}_n}^n, V_{\bar{s}_n})\right] \dot{\omega}_s^n ds - \int_0^t \left[(\bigtriangledown G)F + \frac{1}{2}(\bigtriangledown G)G\right](Y_{\bar{s}_n}^n, V_{\bar{s}_n}) ds.
\end{aligned}$$

Gronwall lemma applied to the function φ_n defined by

$$\varphi_n(t) = \mathbb{E}\left(\sup_{i2^{-n}\le t} |Y_{i2^{-n}}^n - Z_{i2^{-n}}|^2\right),$$

implies that

$$\mathbb{E}\left(\sup_{0\le i\le 2^n}|Y^n_{i2^{-n}}-Z_{i2^{-n}}|^2\right)\le C\sum_{j=1}^{5}\mathbb{E}\left(\sup_{0\le i\le 2^n}|J^n_j(i2^{-n})|^2\right).$$

Burkholder's inequality and Proposition 3.2 imply that $\mathbb{E}\left(\sup_t |J_1^n(t)|^2\right)\le C2^{-n}$, since (2.15) is a particular case of (2.13). Scwartz's inequality yields $\mathbb{E}\left(\sup_t |J_2^n(t)|^2\right)\le C\|h\|^2_{\mathcal{H}}2^{-n}$ and $\mathbb{E}\left(\sup_t |J_3^n(t)|^2\right)\le C2^{-n}$. Since the functions $(\bigtriangledown G)F$ and $(\bigtriangledown G)G$ are lipschitzians, the Proposition 3.2 implies that $\mathbb{E}\left(\sup_t |J_4^n(t)|^2\right)<\infty$. Lemma 3.1 yields $\mathbb{E}\left(\sup_{0\le k\le 2^n}|J_5^n(k2^{-n})|^2\right)=0$. Therefore, the proof is reduced to check that

$$\begin{aligned}\lim_n \mathbb{E}\Bigg(\sup_{0\le k\le 2^n}\Bigg|\int_0^t &\left[G(Y^n_s,V_s)-G(Y^n_{\bar{s}_n},V_{\bar{s}_n})\right]\dot{\omega}^n_s ds\\ -\int_0^t &\left[(\bigtriangledown G)F+\frac{1}{2}(\bigtriangledown G)G\right](Y^n_{\bar{s}_n},V_{\bar{s}_n})ds\Bigg|^2\Bigg)=0.\end{aligned}\tag{3.7}$$

By Taylor's formula, we have

$$\left|G(Y^n_t,V_t)-G(Y^n_{\bar{t}_n},V_{\bar{t}_n})-\bigtriangledown G(Y^n_{\bar{t}_n})[Y^n_t-Y^n_{\bar{t}_n}]\right|<C|Y^n_t-Y^n_{\bar{t}_n}|^2$$

Let

$$\begin{aligned}\Phi_n(s)=&\int_{\bar{s}_n}^s\left[F(Y^n_u,V_u)-F(Y^n_{\bar{u}_n},V_{\bar{u}_n})\right]dW_u+\int_{\bar{s}_n}^s\left[G(Y^n_u,V_u)-G(Y^n_{\bar{u}_n},V_{\bar{u}_n})\right]\dot{\omega}^n_u du\\ &+\int_{\bar{s}_n}^s H(Y^n_u,V_u)\dot{h}_u du+\int_{\bar{s}_n}^s B(Y^n_u,R_u)du,\end{aligned}$$

then

$$\mathbb{E}\left(\sup_{1\le k\le 2^n}|J^n_6(t)|^2\right)\le C\sum_{j=1}^{6}K^n_j(t),$$

where

$$K_1^n(t)=\mathbb{E}\left(\sup_{1\le k\le 2^n}\left|\int_0^{k2^{-n}}\left(|Y^n_s-Y^n_{\bar{s}_n}|+|V_s-V_{\bar{s}_n}|\right)^2|\dot{\omega}^n_s|ds\right|^2\right),$$

$$K_2^n(t)=\mathbb{E}\left(\sup_{1\le k\le 2^n}\left|\int_0^{k2^{-n}}|\bigtriangledown G(Y^n_s,V_s)\Phi_n(s)||\dot{\omega}^n_s|ds\right|^2\right),$$

$$K_3^n(t) = \mathbb{E}\left(\sup_{1\le k\le 2^n}\left|\int_0^{k2^{-n}} (\nabla G)F(Y^n_{\bar{s}_n}, V_{\bar{s}_n})\left(\int_{\bar{s}_n}^{\tilde{s}_n} dW_u\right)\dot{\omega}^n_s ds \right.\right.$$
$$\left.\left. - \int_0^{k2^{-n}} (\nabla G)F(Y^n_{\bar{s}_n}, V_{\bar{s}_n}) ds\right|^2\right),$$

$$K_4^n(t) = \mathbb{E}\left(\sup_{1\le k\le 2^n}\left|\int_0^{k2^{-n}} (\nabla G)F(Y^n_{\bar{s}_n}, V_{\bar{s}_n})\left(\int_{\tilde{s}_n}^{s} dW_u\right)\dot{\omega}^n_s ds\right|^2\right),$$

$$K_5^n(t) = \mathbb{E}\left(\sup_{1\le k\le 2^n}\left|\int_0^{k2^{-n}} (\nabla G)G(Y^n_{\bar{s}_n}, V_{\bar{s}_n})\left(\int_{\tilde{s}_n}^{\bar{s}_n} \dot{\omega}^n_u du\right)\dot{\omega}^n_s ds\right|^2\right),$$

$$K_6^n(t) = \mathbb{E}\left(\sup_{1\le k\le 2^n}\left|\int_0^{k2^{-n}} (\nabla G)G(Y^n_{\bar{s}_n}, V_{\bar{s}_n})\left(\int_{\tilde{s}_n}^{s} \dot{\omega}^n_u du\right)\dot{\omega}^n_s ds \right.\right.$$
$$\left.\left. - \frac{1}{2}\int_0^{k2^{-n}} (\nabla G)G(Y^n_{\bar{s}_n}, V_{\bar{s}_n}) ds\right|^2\right).$$

Proposition 3.2 and Lemma 3.2 imply that $\lim_n K_1^n(t) = 0$. Let $\hat{K}^n(s) = \nabla G(Y^n_s, V_s)\Phi(s)$, then Proposition 3.2 and Hölder's inequality imply that if $a > 1$ and $b > 1$ are conjugate exponents and for any $p \in [1;\infty)$ and $s \in [0;1]$,

$$\mathbb{E}\left(|\hat{K}^n(s)|^{2p}\right) \le \left[\mathbb{E}\left(|\nabla G(Y^n_s, V_s)|^{2pa}\right)\right]^{\frac{1}{a}}\left[\mathbb{E}|\Phi_n(s)|^{2pb}\right]^{\frac{1}{b}} \le C\left[\mathbb{E}|\Phi_n(s)|^{2pb}\right]^{\frac{1}{b}}.$$

Therefore, in order to apply Lemma 3.2, it suffices to check that $\sup_n\left(\mathbb{E}|\Phi_n(s)|^{2p}\right) \le \alpha(n)2^{-np}$ with $\lim_n \alpha(n) = 0$. Burkholdre's and Hölder's inequality and Proposition 3.2 yield

$$\begin{aligned}
\mathbb{E}\left(|\Phi_n(s)|^{2p}\right) \le\ & C\mathbb{E}\Big[\Big|\textstyle\int_{\bar{s}_n}^{s}\left(|Y^n_s - Y^n_{\bar{s}_n}| + |V_s - V_{\bar{s}_n}|\right)^2 du\Big|^p \\
& +2^n\left(\textstyle\int_{\bar{s}_n}^{\tilde{s}_n}\left(|Y^n_s - Y^n_{\bar{s}_n}| + |V_s - V_{\bar{s}_n}|\right)^{2p} du\right)|W_{\bar{s}_n} - W_{\bar{s}_n - 2^{-n}\vee 0}|^{2p} \\
& +2^n\left(\textstyle\int_{\tilde{s}_n}^{s}\left(|Y^n_s - Y^n_{\bar{s}_n}| + |V_s - V_{\bar{s}_n}|\right)^{2p} du\right)|W_{\tilde{s}_n} - W_{\bar{s}_n -}|^{2p} \\
& +\left(\sup_n\left\{\left(\textstyle\int_I |\dot{h}_u|^2 du\right)^p; \lambda(I) \le 2^{1-n}\right\}2^{-(n-1)(p-1)} + 2^{-n(2p-1)}\right) \\
& \times \textstyle\int_{\bar{s}_n}^{s}(1 + |Y^n_u|^{2p})du\Big] \\
\le\ & C2^{-np}\alpha(n)
\end{aligned}$$

where $\alpha(n) = 2^{-np} + \sup\{(\int_I |\dot{h}_u|^2 du)^p; \lambda(I) \leq 2^{1-n}\}$, wich tends to zero when n tends to ∞. Thus Lemma 3.2 implies that $\lim_n K_2^n = 0$, and

$$\begin{aligned}
K_3^n &= \mathbb{E}\left(\sup_{1\leq k\leq 2^n}\left|\sum_{i=0}^{(k-2)\vee 0}(\nabla G)F(Y^n_{(i-1)2^{-n}\vee 0}, V_{(i-1)2^{-n}\vee 0})\right.\right.\\
&\qquad\left.\left.\left[(W_{(i-1)2^{-n}} - W_{i2^{-n}})^2 - 2^{-n}\right]\right|^2\right)\\
&\leq \sum_{i=0}^{(2^n-2)}\mathbb{E}\left((\nabla G)F(Y^n_{(i-1)2^{-n}\vee 0}, V_{(i-1)2^{-n}\vee 0})^2\right)\mathbb{E}\left[|W_{(i-1)2^{-n}} - W_{i2^{-n}}|^2\right]\\
&\leq C2^n2^{-2n} \longrightarrow 0 \; as \; n \longrightarrow \infty
\end{aligned}$$

then, $\lim_n K_3^n = 0$. A similar computation yields

$$\begin{aligned}
K_6^n &= \mathbb{E}\left(\sup_{2\leq k\leq 2^n}\left|\sum_{i=0}^{k-2}(\nabla G)G(Y^n_{i2^{-n}}, V_{i2^{-n}})\right.\right.\\
&\qquad\left.\left.\left[\left[2^{2n}\int_{(i+1)2^{-n}}^{(i+2)2^{-n}}\int_{(i+1)2^{-n}}^{s} du\, ds\right]\left[(W_{(i-1)2^{-n}} - W_{i2^{-n}})^2 - 2^{-n-1}\right]\right]\right|^2\right)\\
&\leq C2^n2^{-2n} \to_{n\to\infty} 0
\end{aligned}$$

Finally, by Doob's inequality and Proposition 3.2, we have

$$\begin{aligned}
K_4^n &= \mathbb{E}\left(\left|\int_0^1 2^n(\tilde{s}_n + 2^{-n} - s)(\nabla G)F(Y^n_{\tilde{s}_n}, V_{\tilde{s}_n})[W_{\tilde{s}_n} - W_{\bar{s}_n}]dW_s\right|^2\right)\\
&\leq C\int_0^1 \mathbb{E}\left(|(\nabla G)F(Y^n_{\tilde{s}_n}, V_{\tilde{s}_n})|^2\right)\mathbb{E}(|W_{\tilde{s}_n} - W_{\bar{s}_n}|^2)ds\\
&\leq C2^{-n};
\end{aligned}$$

and for conjugate exponents $a > 0$ and $b > 0$

$$\begin{aligned}
K_5^n &= \mathbb{E}\left(\left|\int_0^1(\nabla G)G(Y^n_{\bar{s}_n}, V_{\bar{s}_n})\left(\int_{\bar{s}_n}^{\tilde{s}_n}\dot{\omega}^n_s ds\right)dw_s\right|^2\right)\\
&\leq C\int_0^1 \mathbb{E}\left(|(\nabla G)F(Y^n_{\bar{s}_n}, V_{\bar{s}_n})|^{2a}\right)^{\frac{1}{a}}\mathbb{E}\left(|W_{\bar{s}_n} - W_{(\bar{s}_n - 2^{-n})\vee 0}|^{2b}\right)^{\frac{1}{b}} ds\\
&\leq C2^{-n}.
\end{aligned}$$

The proof of $\lim_n\left(\sup_{1\leq k\leq 2^n} K_5^n(2^{-n})\right) = 0$ is complete, and hence the desired proposition follows. □

Proof of Proposition 3.4. For $t = i2^{-n}$ with $0 \leq i \leq 2^n$

$$
\begin{aligned}
\bar{Y}^n_t - Z_t = &\int_0^t [(F+G)(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) - (F+G)(Z_{\bar{s}_n}, V_{\bar{s}_n})] dW_s + \int_0^t [H(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) \\
&- H(Z_{\bar{s}_n}, V_{\bar{s}_n})] \dot{h}_s ds + \int_0^t [I(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) - I(Z_{\bar{s}_n}, V_{\bar{s}_n})] ds \\
&+ \int_0^t \left(\left[B(\bar{Y}_{\bar{s}_n}, \psi_s) + ((\triangledown G)F + \frac{1}{2}(\triangledown G)G)(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) \right] \right. \\
&- \left. \left[B(Z_{\bar{s}_n}, R_s) + ((\triangledown G)F + \frac{1}{2}(\triangledown G)G)(Z_{\bar{s}_n}, V_{\bar{s}_n}) \right] \right) ds \\
&+ \sum_{j=1}^{6} \bar{J}^n_j(t),
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{J}^n_1(t) &= \int_0^t \left(F(\bar{Y}^n_s, \chi_s) - F(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) - [F+G](Z_s, V_s) + [F+G](Z_{\bar{s}_n}, V_{\bar{s}_n}) \right) dW_s, \\
\bar{J}^n_2(t) &= \int_0^t \left(H(\bar{Y}^n_s, \chi_s) - H(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) - H(Z_s, V_s) + H(Z_{\bar{s}_n}, V_{\bar{s}_n}) \right) \dot{h}_s ds, \\
\bar{J}^n_3(t) &= \int_0^t \left(I(\bar{Y}^n_s, \chi_s) - I(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) - I(Z_s, V_s) + I(Z_{\bar{s}_n}, V_{\bar{s}_n}) \right) ds, \\
\bar{J}^n_4(t) &= \int_0^t \left(B(\bar{Y}^n_s, \psi_s) - B(\bar{Y}^n_{\bar{s}_n}, \psi_s) - \left[B(Z_s, R_s) + (\triangledown G)F + \frac{1}{2}(\triangledown G)G \right] (Z_s, V_s) \right. \\
&\quad \left. + B(Z_{\bar{s}_n}, R_s) \left[(\triangledown G)F + \frac{1}{2}(\triangledown G)G \right] (Z_{\bar{s}_n}, V_{\bar{s}_n}) \right) ds, \\
\bar{J}^n_5(t) &= \int_0^t G(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) \left[\dot{\omega}^n_s ds - dW_s \right], \\
\bar{J}^n_6(t) &= \int_0^t \left[G(\bar{Y}^n_s, \chi_s) - G(\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) \right] \dot{\omega}^n_s ds - \int_0^t \left[(\triangledown G)F + \frac{1}{2}(\triangledown G)G \right] (\bar{Y}^n_{\bar{s}_n}, \chi_{\bar{s}_n}) ds.
\end{aligned}
$$

By Burkholder and Schwrtz inequalities, we have

$$
|\bar{Y}^n_t - Z_t|^2 \leq C \left[\int_0^t \left(|\bar{Y}^n_{\bar{s}_n} - Z_{\bar{s}_n}|^2 + |\chi_{\bar{s}_n} - V_{\bar{s}_n}|^2 + |\psi_s - R_s|^2 \right) ds + \sum_{k=1}^{6} \bar{K}^n_t \right].
$$

Proposition 3.1 and Gronwall inequality apply to $\alpha(t) = \mathbb{E}\left(\sup_{i2^{-n} \leq t} |\bar{Y}^n_{i2^{-n}} - Z_{i2^{-n}}|^2 \right)$ imply

$$\mathbb{E}\left(\sup_{0\leq i\leq 2^n}|\bar{Y}^n_{i2^{-n}} - Z_{i2^{-n}}|\right) \leq C\sum_{k=1}^{6}\mathbb{E}\left(\sup_{0\leq i\leq 2^n}|\bar{J}^n_k(i2^{-n})|^2\right). \tag{3.8}$$

Furthermore, the rest of the demonstration is to prove that $\mathbb{E}(\sup_{i2^{-n}}\bar{J}^n_j(t)) \longrightarrow 0$ as n tend to ∞ for $j = 1$ to 6. Thus, by Burkholder inequality, equations (3.2) and (3.3), we have

$$\mathbb{E}(\sup_t|\bar{J}^n_1(t)|^2) \leq C2^{-n}. \tag{3.9}$$

With the equations (3.2) and (3.3) and using the same argument of the proof of the moments estimate $J^n_j(t)$ for $j = 2$ to 6, we obtain

$$\sum_{j=2}^{6}\bar{J}^n_j(t) \leq C2^{-n}. \tag{3.10}$$

The inequalities (3.8),(3.9) and (3.10) complete the proof as n tend to ∞. □

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