ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **12** (2023), no.7, 673–683
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.12.7.3

A SINGULAR PROPERTY OF GALAMBOS COPULA

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ABSTRACT. Whatever the sample size, the Galambos copula does not change. It remains constant. This is quite singular. To demonstrate this, we'll show that the generalized Galambos copula is completely independent of sample size. Unlike the copula of Gumbel, Ali-Michael and Haq, and certainly others, whose generalization depends on sample size, the Galambos copula is special. Unlike Gumbel's, Galambos's generalized copula is not size-dependent.

1. INTRODUCTION

After publishing an initial article on the generalisation of Gumbel's law and Ali-Michael and Haq's law, we found that other private laws had been adopted. One of these is Galambos's law. Given its importance, we thought we'd extend our discussion to it. In this article, we shall show that unlike other copulas, such as Gumbel's and Ali-Michael and Haq's, for which we have proposed generalised forms, Galambos's has a particular generalised form. It does not differ from the initial copula.

So whatever the size of the sample chosen, the Galambos copula remains unchanged. Indeed, it is in our quest to propose a generalised Galambos copula

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2020 *Mathematics Subject Classification.* 62H05, 62R07 62T09, 62D20, 62D10.
Key words and phrases. copulas, Galambos copula, Gumbel copula, sample, size.
Submitted: 12.06.2023; *Accepted:* 27.06.2023; *Published:* 07.07.2023.

that we arrive at this fact, which must certainly have an appreciable statistical impact, particularly in the choice of financial models likely to provide us with more advantages.

With this new article, we intend to continue our drive to strengthen the notion of copula, particularly by taking sample size into account. This has the advantage of opening up a broad spectrum to the notion of copula.

From our point of view, in statistics, sample management is very important. It allows us to project phenomena onto a wider dimension which, ultimately, borders on the notions of physics because it concerns the quantity of the thing, and therefore the mass or weight of the thing. To ignore this would be a major failing in Statistics. So to talk about samples is to talk about their size.

It is common to consider the size of a random variable. So often, in fact, we talk about the size of the random variable. For example, we talk about univariate variables and multivariate variables. Statistical formulae are often restricted to these two situations. Rarely do we think of combining random variables to form a block or a vector, based on samples of a certain size. They can be chosen to be independent, identically distributed and to follow the same distribution. Similarly, they can be chosen as dependent and lead to copulas. Looking at the problem in this way, several combinations may be possible, leading to other structures with more or less statistical advantages.

In our opinion, this is all the more important as the management of the data of a random variable is just as important as that of the combination of random variables between them, which are more applications, unlike statistical data, which are qualitative or quantitative entities.

We therefore believe that it is essential to take account of sample size both at the level of the random variable alone and at the level of their combination.
However, the sample size must be sufficiently representative to produce convincing results. This is where our proposed law, which takes sample size into account, comes into its own.

In short, the multidimensional management of statistical data or random variables is a practice that aids good analysis and decision-making. The generalisation of copulas is therefore one of the ways of analysing these data and putting them together in order to study their dependence.

Taking sample size into account can only be an added advantage

2. Preliminaries

As a reminder, although I already mentioned it in the previous article, the notion of copula took off in 1959, when the American Sklar, wanting to solve the difficulty encountered by Maurice Fr$\acute{e}$chet, used it for the first time.

In reality, in probability theory, when the random variables $X_1, \ldots, X_n$ are independent, their joint distribution function is trivially calculated by simply multiplying the different marginal distribution functions $F_1, \ldots, F_n$ i.e. :

$$F(x_1, \ldots, x_n) = F_1(x_1) \ldots F_n(x_n).$$

However, in practice, as random variables are not always independent, as is often the case for financial risks where variables are linked to each other, researchers have invented this new function which they have named: copula.

Sklar was the first to introduce it. He proposed this distribution function which, in reality, is an entity linking the different margins of random variables in the specific case where they are dependent on each other. Since then, copulas have come a long way. They make it possible to objectively couple the marginal laws of random variables and to study their dependence, thus playing a fundamental role in the resolution of problems inherent in the dependence of random variables, particularly those encountered in the fields of finance, hydrology, biology, etc. Since the Galambos copula is discussed in this article, we'll be placing special emphasis on it.

In addition, since the Galambos copula, like the Archimedean copulas, are all part of the family of extreme value copulas, and all form part of the Archimax copulas, in order to better understand them, we will review them all.

3. Copule Archimax

See [2, 4, 5].

Definition 3.1. *A bivariate function is an Archimax copula if and only if it is of the form:*

$$pC_\rho(u, v) = \rho^{-1}[(\rho(u) + \rho(v))A(\frac{\rho(v)}{\rho(u) + \rho(v)})], \quad \forall u, v \in [0, 1],$$

with

1. *A:*$[0,1] \to [\frac{1}{2},1]$ *such that max* $(0,1-t) \leq A(t) \leq 1, \quad \forall t \in [0,1].$
2. $\rho : [0,1] \to [0,+\infty[$ *is a convex, decreasing function which verifies* $\rho(1) = 0$ *with the following convention:*

$$\rho(0) = 0, \rho(0) = \lim_{t \to 0^+} \rho(t), \text{ and } \rho^{-1}(s) = 0, \quad \forall s \geq \rho(0).$$

Remark 3.1. *Extreme value copulas and Archimedean copulas are all in the family of Archimax copulas. In fact, if we put* $\rho(t) = \ln(\frac{1}{t})$*, the copula* C_ρ *obtained is an extreme value copula.* $C_\rho(u,v) = \exp[\ln(uv)A(\frac{\ln v}{\ln(uv)})]$*. Moreover, if we put* $A(t) = 1$*, we find the form of the Archimedean copulas. We have* $C_\rho(u,v) = \rho^{-1}(\rho(u) + \rho(v))$*.*

Copule archim$\acute{e}$dienne

See [1, 2, 5, 6].

Definition 3.2. *Any archimedean copula is characterized by dependence on a generating function such that:* $C(u_1,\ldots,u_d) = (\rho^{-1})(\rho(u_1) + \cdots + \rho(u_{p)}))$ *if* $\sum_{i=1}^{d} \rho(u_i) \leq \rho(0)$*, and* $C(u_1,\ldots,u_d) = 0$*, otherwise* ρ^{-1} *is the inverse of the generator* ρ*. Basically, for a copula to be said to be Archimedean, it is necessary and sufficient that for* $d \geq 0$ *and* $\sum_{i=1}^{d} \rho(u_i) \leq \rho(0)$*,* $C(u_1,\ldots,u_d) = \rho^{-1}(\rho(u_1) + \cdots + \rho(u_d))$ *and nowhere else. Can the random variables u and v take on different values between 0 and 1.*

Remark 3.2. *Archimedean copulas are used in many applications because of their ease of handling. Furthermore, for* ρ *to be a generator of an Archimedean copula, it must be a continuous, convex and strictly decreasing function, in other words it must be of class* C^2 *so that* $\rho(1) = 0, \rho^{'}(u) \leq 0$ *and* $\rho^{''}(u) > 0$*.*

Remark 3.3. *There are actually several copulas of this type, but the best known and most widely used are the Gumbel, Clayton and Frank copulas. In reality, what differentiates them from one another is their generator.*

3.1. **Extreme value copulas.**

Definition 3.3. *Copulas of this type are those which verify the relationship:* $C(u_1^k, u_2^k) = C^k(u_1,u_2)$ *for all k positive.*

Remark 3.4. *To construct a distribution of two-dimensional extreme values, all we need to do is couple the margins derived from the theory.*

Corollary 3.1. *Given $X_1, \dots, X_n$, n random variables whose respective distribution functions are: $F_1(x_1), \dots, F_n(x_n)$ and let H be the distribution function of the vector $(X_1, \dots, X_n)$, there exists a function C such that:*

$$C(u_1, \dots, u_n) = H(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)), \quad \forall (u_1, \dots, u_n) \in I^n.$$

Corollary 3.2. *In the bivariate case, for F and G the marginal distribution functions linked respectively to X and Y, and H the joint distribution of F and G, there exists a copula C such that:*

$$C(u, v) = H(F^{-1}(u), G^{-1}(v)), \quad \forall u, v \in I^2.$$

4. Galambos copula

Definition 4.1. *Let (X,Y) be a pair of random variables. The Galambos law is defined as follows*

$$C(u, v) = uv \exp\{(-\ln u)^{-\theta} + (-\ln v)^{-\theta})^{-\frac{1}{\theta}}]\}, \quad \forall \theta > 0.$$

The following proposition proposes an approach to construct the Galambos copula

Proposition 4.1. *From the definition, we know that Galambos's law is defined as follows $C(u, v) = uv \exp\{(-\ln u)^{-\theta} + (-\ln v)^{-\theta})^{-\frac{1}{\theta}}]\}, \forall \theta > 0$.*

Proof. Let (X,Y) be a pair of random variables whose joint function H is defined by:

$$H(x, y) = \exp -[(x + y) - (x^{-\theta} + y^{-\theta})^{-\frac{1}{\theta}}] \quad \forall (x, y) \in [0, +\infty[\times[0, +\infty[, \theta > 0,$$

and has marginals: $F_1(x) = \exp(-x)$ and $F_2(y) = \exp(-y)$. Since

$$\begin{aligned} F_1(x) &= \lim_{y \to +\infty} H(x, y), \\ &= \lim \exp -[(x + y) - (x^{-\theta} + y^{-\theta})^{-\frac{1}{\theta}}] \rightsquigarrow \lim \exp -[(x + y) - (y^{-\theta})^{-\frac{1}{\theta}}] \\ &= \lim \exp(-(x + y - y)) = e^{-x} \end{aligned}$$

From before $F_2(y) = \lim_{x \to +\infty} H(x, y)$. Hence $F_2(y) = e^{-y}$. So, we have $F_1^{-1}(u) = -\ln u$ and $F_2^{-1}(v) = -\ln v; \forall (u, v) \in I^2$. Using the corollary of Sklar's theorem, we get:

$$
\begin{aligned}
C(u,v) = H(F_1^{-1}(u), F_2^{-1}(v)) &= H(-\ln u, -\ln v), \quad \forall (u,v) \in I^2 \\
&= \exp -[(-\ln u - \ln v) - ((-\ln u)^{-\theta} + (-\ln v)^{-\theta})^{-\frac{1}{\theta}}] \\
&= \exp -[(-\ln uv) - ((-\ln u)^{-\theta} + (-\ln v)^{-\theta})^{-\frac{1}{\theta}}] \\
&= \exp -[(-\ln uv) - ((-\ln u)^{-\theta} + (-\ln v)^{-\theta}] \\
&= \exp(\ln uv) + ((-\ln u)^{-\theta} + (-\ln v)^{-\theta})^{-\frac{1}{\theta}}].
\end{aligned}
$$

From the fact that: $\exp(a+b) = (\exp a)(\exp b)$ we get:

$$C(u,v) = \exp(\ln uv) \exp((-\ln u)^{-\theta} + (-\ln v)^{-\theta})^{-\frac{1}{\theta}}].$$

From this we get:

$$C(u,v) = uv \exp[(-\ln u)^{-\theta} + (-\ln v)^{-\theta})^{-\frac{1}{\theta}}] \quad \forall \theta > 0.$$

This $\exp$ ression defines the Galambos copula. □

Proposition 4.2. *Let (X,Y) be a random vector of* $[0,+\infty[\times[0,+\infty[$. *Let* $H_{\theta,n}$ *be the application defined by:*

$$(x,y) \to [\frac{1}{exp\{-[(x+y) - (x^{-\theta} + y^{-\theta})^{-\frac{1}{\theta}}])\}}]^n, \quad \forall (x,y) \in [0,+\infty[\times[0,+\infty[.$$

Hence we have: $H_{\theta,n}(x,y) = [\frac{1}{exp\{-[(x+y)-(x^{-\theta}+y^{-\theta})^{-\frac{1}{\theta}}]\}}]^n$. *This function fully satisfies the conditions of a distribution function. The generalised Galambos copula is defined in the same way as the unit copula, which is very special. Hence:* $C_{n,\theta}(u,v) = uv\exp\{[(\ln u)^{-\theta} + (\ln v)^{-\theta})]^{-\frac{1}{\theta}}\}$, $\forall n \in \mathbb{N}$.

Proof. Let (X,Y) be a random vector (or pair of random variables). Let $H_{\theta,n}$ be the application defined by:

$$(x,y) \to [\frac{1}{\exp\{-[(x+y) - (x^{-\theta} + y^{-\theta})^{-\frac{1}{\theta}}]\}}]^n, \quad \forall (x,y) \in [0,+\infty[\times[0,+\infty[.$$

Hence the distribution function: $H_{\theta,n}(x,y) = [\frac{1}{\exp\{-[(x+y)-(x^{-\theta}+y^{-\theta})^{-\frac{1}{\theta}}]\}}]^n$. As defined, H is absolutely continuous on $\mathbb{R}\times\mathbb{R}$, which allows us to speak of a copula.

The marginal functions are given by:

$$\begin{aligned}
F_{1,n}(x) &= \lim_{y\to+\infty} H_{\theta,n}(x,y)\\
&= \lim_{y\to+\infty}[\frac{1}{\exp\{-[(x+y)-(x^{-\theta}+y^{-\theta})^{-\frac{1}{\theta}}]\}}]^n\\
&= \lim_{y\to+\infty}[\frac{1}{\exp\{-[(x+y)-(y^{-\theta})^{-\frac{1}{\theta}}]\}}]^n\\
&= \lim_{y\to+\infty}[\frac{1}{\exp\{-(x+y-y)\}}]^n = e^{nx}\\
F_{2,n}(x) &= \lim_{x\to+\infty} H_{\theta,n}(x,y)\\
&= \lim_{x\to+\infty}[\frac{1}{\exp\{-[(x+y)-(x^{-\theta}+y^{-\theta})^{-\frac{1}{\theta}}]\}}]^n\\
&= \lim_{x\to+\infty}[\frac{1}{\exp\{-[(x+y)-(x^{-\theta})^{-\frac{1}{\theta}}]\}}]^n\\
&= \lim_{x\to+\infty}|\frac{1}{\exp\{-(y+x-x)\}}]^n = e^{ny}.
\end{aligned}$$

Assuming $u = F_{1,n}(x)$ and $v = F_{2,n}(y)$, the reciprocal functions are:$F_{1,n}^{-1}(u)$ and $F_{2,n}^{-1}(v)$, $F_{1,n}^{-1}(u) = \ln u^{\frac{1}{n}}$ and $F_{2,n}^{-1}(v) = \ln v^{\frac{1}{n}}$.

From the corollary of Sklar's theorem, we get:

$$\begin{aligned}
C_{n,\theta}(u,v) &= H_{n,\theta}(F_{1,n}^{-1}(u), F_{2,n}^{-1}(v))\\
&= [\frac{1}{\exp\{-[(\ln u^{\frac{1}{n}}+\ln v^{\frac{1}{n}})-[(\ln u^{\frac{1}{n}})^{-\theta}+(\ln v^{\frac{1}{n}})^{-\theta}]^{-\frac{1}{\theta}}]\}}]^n\\
&= [\frac{1}{\exp\{-[(\ln u^{\frac{1}{n}}v^{\frac{1}{n}})-|(\frac{1}{n})^{-\theta}(\ln u)^{-\theta}+(\frac{1}{n})^{-\theta}(\ln v)^{-\theta}]^{-\frac{1}{\theta}}\}}]^n\\
&= [\frac{1}{\exp\{-[\ln(uv)^{\frac{1}{n}}-(\frac{1}{n})[((\ln u)^{-\theta}+(\ln v)^{-\theta}])]^{-\frac{1}{\theta}}]\}}]^n\\
&= [\frac{1}{\exp\{\ln(uv)^{-\frac{1}{n}}+\frac{[(\ln u)^{-\theta}+(\ln v)^{-\theta}]^{-\frac{1}{\theta}}}{n}\}}]^n.
\end{aligned}$$

Since $\exp(a+b) = \exp(a)\exp(b)$ we have:

$$[\frac{1}{\exp\ln(uv)^{-\frac{1}{n}}\exp\{\frac{[(\ln u)^{-\theta}+(\ln v)^{-\theta}]^{-\frac{1}{\theta}}]}{n}\}}]^n = [\frac{(uv)^{\frac{1}{n}}}{\exp\{[\frac{[(\ln u)^{-\theta}+(\ln v)^{-\theta}]^{-\frac{1}{\theta}}]}{n}\}}]^n$$
$$= \frac{uv}{[\exp\{(\frac{1}{n})[((\ln u)^{-\theta}+(\ln v)^{-\theta}]^{\frac{1}{\theta}}\}]^n}.$$

Since $(e^m)^n = e^{nm}$ then:

$$\frac{uv}{\exp\{(n\frac{1}{n})[(\ln u)^{-\theta}+(\ln v)^{-\theta}]^{\frac{1}{\theta}}\}} = uv\exp\{[(\ln u)^{-\theta}+(\ln v)^{-\theta}]^{-\frac{1}{\theta}}\}.$$

This defines our proposal for a generalised Galambos's law which, as we can see, does not depend on the size of the sample,

$$C_{n,\theta}(u,v) = C(u,v) = uv\exp\{-[(\ln u)^{-\theta}+(\ln v)^{-\theta}]^{-\frac{1}{\theta}}\}.$$

□

4.1. **Another generalised Galambos construction approach.** Since this other approach depends on extreme values, we make the following proposal in order to make it a preliminary gateway.

Proposition 4.3. *Let $(X_1,\dots,X_n)$ be a random vector whose random variables are independent and identically distributed and follow the same F distribution. Let F_i be the marginal distribution functions where F is the joint distribution function. If F is a Galambos distribution then the copula induced by F, called the Galambos copula, is also an extreme value copula.*

Proof. We know that for Galambos's law to be a law of extreme values, the necessary and sufficient condition for any copula to be of extreme values is that it is max-stable, i.e. that it satisfies the condition: $C(u_1^k,u_2^k) = C^k(u_1,u_2)$ for all k positive. According to the above, the Galambos copula is defined by: $C(u,v) = uv\exp\{[(-\ln u)^{-\theta}+(-\ln v)^{-\theta})^{-\frac{1}{\theta}}]\}, \forall\theta>0$, or $C(u,v) = uv\exp\{-[(\ln u)^{-\theta}+(\ln v)^{-\theta})^{-\frac{1}{\theta}}]\}, \forall\theta>0$. Also, for it to be an extreme value copula it is necessary and sufficient that:

$C(u^k, v^k) = C^k(u, v)$,

$$\begin{aligned} C(u^k, v^k) &= u^k v^k \exp\{[(-\ln u^k)^{-\theta} + (-\ln v^k)^{-\theta}]^{-\frac{1}{\theta}}\} \\ &= u^k v^k \exp\{[(-k\ln u)^{-\theta} + (-k\ln v)^{-\theta}]^{-\frac{1}{\theta}} \\ &= u^k v^k \exp\{(k^{-\theta}[(-\ln u)^{-\theta} + (-\ln v)^{-\theta}])^{-\frac{1}{\theta}}\} \\ &= u^k v^k \exp\{k[(-\ln u)^{-\theta} + (-\ln v)^{-\theta}]^{-\frac{1}{\theta}}\} \\ &= u^k v^k [exp\{[(-\ln u)^{-\theta} + (-\ln u)^{-\theta}])^{-\frac{1}{\theta}}\}]^k \\ &= [uv\exp\{[(-\ln u)^{-\theta} + (-\ln v)^{-\theta})^{-\frac{1}{\theta}}]\}]^k = C^k(u, v). \end{aligned}$$

Hence the Galambos copula is indeed an extreme value copula. □

We propose an approach for constructing the generalized Galambos copula from extreme values.

Proposition 4.4. *([1, 2]) Let $(X_1, \dots, X_n)$ and $(Y_1, \dots, Y_n)$ be two random vectors whose random variables are independent and identically distributed and follow the F and G distributions respectively.*

Let $(X_1, Y_1), \dots, (X_n, Y_n)$ be a sequence of pairs of (X,Y) made up of random variables which follow the Galambos distribution which we denote $H(x, y)$.
$H(x, y)$ is therefore the joint distribution function associated with the margins F and G.

Let $H_n(x, y)$ be the joint distribution function associated with the margins $F_n(x)$ and $G_n(y)$, respectively the margins of $(X_1, ..X_n)$ and $(Y_1, \dots, Y_n)$. Given the independence of the random variables, we will have $H_n(x, y) = [F(x)G(y)]^n = [H(x, y)]^n = \exp n\{-[(x + y) - (x^{-\theta} + y^{-\theta})^{-\frac{1}{\theta}]}\}$. According to the corollary of Sklar's theorem, the copula associated with $H_n(x, y)$ will therefore be:

$$C_n(u, v) = H_n(F_n^{-1}(u), G_n^{-1}(v)).$$

Let's determine the margins $F_n(x)$ and $G_n(y)$, $F_n(x) = \lim_{y\to+\infty} H_n(x, y) = e^{-nx}$, and $G_n(y) = \lim_{x\to+\infty} H_n(x, y) = e^{-ny}$. The quantiles will therefore be $F_n^{-1}(u) = \ln u^{-\frac{1}{n}}$

and $G_n^{-1}(v) = \ln v^{-\frac{1}{n}}$,

$$\begin{aligned}
C_{n,\theta}(u,v) &= H_n(F_n^{-1}(u), G_n^{-1}(v)) \\
&= \exp n\{-[((\ln u^{-\frac{1}{n}}) + (\ln v^{-\frac{1}{n}})) - ((\ln u^{-\frac{1}{n}})^{-\theta} + (\ln v^{-\frac{1}{n}})^{-\theta})^{-\frac{1}{\theta}]} \\
&= \exp n\{-((\ln u^{-\frac{1}{n}}) - (\ln v^{-\frac{1}{n}})) + ((\ln u^{-\frac{1}{n}})^{-\theta} + (\ln v^{-\frac{1}{n}})^{-\theta})^{-\frac{1}{\theta}]}\} \\
&= \exp n(-((\ln u^{-\frac{1}{n}})) \cdot \exp n(-((\ln v^{-\frac{1}{n}})) \\
&\quad \cdot \exp n\{((\ln u^{-\frac{1}{n}})^{-\theta} + (\ln v^{-\frac{1}{n}})^{-\theta})^{-\frac{1}{\theta}]}\} \\
&= uv \cdot \exp n\{([(\frac{1}{n})^{-\theta}]^{-\frac{1}{\theta}}((\ln u)^{-\theta} + (\ln v)^{-\theta})^{-\frac{1}{\theta}]}\} \\
&= uv \cdot \exp\{((\ln u)^{-\theta} + (\ln v)^{-\theta})^{-\frac{1}{\theta}]}\} = C_\theta(u,v).
\end{aligned}$$

5. Conclusion

Like the Fréchet-Hoeffding bounds $W(u,v) = max(u+v-1,0)$ and $M(u,v) = min(u,v)$ which have the singularity of being the smallest and largest copulas respectively. No copula can lie outside the field formed by these two copulas. We have highlighted a singularity which gives the Galambos copula certain advantages.

Intuitively, we think that the most important thing would be that it could represent a sort of centre of gravity for copulas. And this fact can have a major impact on the choice of this model, because the weights are balanced and the financial and other risks are reduced. So what could be more advantageous than choosing this model in finance or in any other field where risk forecasting is more than necessary.

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