ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal **12** (2023), no.10, 887–919
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.12.10.4

MULTI-COMPARTMENT VEHICLES FOR SPLIT PICK-UP AND SPLIT DELIVERY PROBLEM WITH TIME WINDOW

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ABSTRACT. In this article, we'll look at multi-compartment vehicles, which collect goods from suppliers and deliver them to different customers who place large orders. Because some products are incompatible with each other, these goods cannot be transported by single-compartment vehicles. In addition, because customers can place large orders, some suppliers and/or customers may be visited several times by different vehicles. In this work, the aim is to satisfy a group of customers while respecting the constraints linked to the capacity of each compartment and each type of product, and to ensure that each supplier is visited before the customer. Our first step is to model our problem mathematically and then solve it using an approximate method. Given its complexity, we use the genetic algorithm to solve the problem of split pick-up and delivery with time windows by multi-compartment vehicles. Our model allows us to determine a minimum distance and a minimum cost using a reasonable number of vehicles.

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2020 *Mathematics Subject Classification.* 03C98.
Key words and phrases. Genetic algorithm, Multi-compartment vehicles, split pick-up and split delivery, PDPTW.
Submitted: 14.08.2023; *Accepted:* 02.09.2023; *Published:* 19.10.2023.

1. Introduction

Road transport activities have grown considerably with the development of commercial networks, and the advent of new technologies has encouraged online shopping over the last few decades. This strong growth has resulted in complex management of transport flows and air pollution, particularly in urban areas, which has a direct impact on air quality in cities. In addition, the major issue in the distribution of goods remains the cost of transport between suppliers and customers. Responsible for collection and delivery, the main objective of transport companies is to minimize expenditure in terms of cost or service time, or the total distance covered by the fleet, while respecting the visit times defined by customers and/or suppliers. With these objectives in mind, researchers have proposed an approach based on the PDPTW (Pick-up and Delivery Problem with Time Widows) and its variants where the vehicles have a single-compartment capacity [1] However, there are difficulties with single-compartment vehicles during loading and unloading. In addition, certain products are often incompatible with each other (e.g. the distribution of petroleum products). Berte Ousmane proposed [2] the MCV-PDPTW (Multi-Compartment Vehicle Pick-up and Delivery Problem with Time Widows) to solve this problem. However, the capacity of a vehicle is limited and it cannot transport all the goods of a customer placing a large order on its own. This work can be reduced, for example, to the delivery of a supermarket which has to receive products of different incompatible types and in very large quantities. This problem leads us to consider the problem of collecting and delivering goods in which a customer or supplier may be visited at least once by different vehicles in the same time interval or not. Generally, this problem can be confronted with several difficulties namely:

- Vehicle scheduling problems,
- Problems of allocating a type of product to a compartment,
- Managing a heterogeneous fleet of vehicles,
- Exceeding vehicle capacity,
- Managing time slots,
- Producing a quality service.

Our mathematical model will take account of the problems mentioned above. The model will therefore make it possible to minimize the total cost of transport linked to the total distance travelled, minimize the number of vehicles by optimally allocating the products to the different compartments and respect the time intervals at each node. Our problem is a variant of the PDP called Split-Pickup and Split-Delivery Problem with Time Windows by multi-compartment vehicles (MCV-SPSDPTW). Several studies have been carried out on pickup and delivery problems.

Zhang and Appadoo [3] examine an MCVRP for household waste collection that takes into account ecological aspects. The authors consider alternative fuel vehicles with limited tank capacity, resulting in distance-limited rounds. They propose an adapted savings algorithm and an ACO algorithm to solve their problem.

Maroua Grid and Al [4] proposed a novel parallel combinatorial optimization method called Parallel Bees Life Algorithm (P-BLA), based on Graphics Processing Unit (GPU) to solve Dynamic Vehicle Touring Problem (DCVRP) efficiently, in terms of execution time.

Al Chami Zaher and al [5] worked on MuPDPTWPD, a variant of PDPTW and combined the Randomized Adaptive Search Procedure (GRASP) with the Hybrid Genetic Algorithm (HGA) to solve this problem by minimizing the total distance travelled. Hincapie-Potes et al [6] propose a time-window-free, single-objective static PDPT formulation that minimizes the total distance travelled by a heterogeneous fleet of vehicles. They use a gluttonous method as well as a hybrid metaheuristic and many approximate methods to solve their problem. Chen, Liu and Langevin [7] deals with a problem of perishable food distribution with time windows. The number of customers to be visited by a single vehicle is limited in order to ensure the freshness of the products. They consider an objective function composed of variable and fixed transport costs and the cost of fuel consumption. The authors solve their problem with an adaptive LNS algorithm (ALNS). Manuel Ostermeier [8] address a multi-compartment vehicle routing problem with loading and unloading costs in spice distribution where different temperature-specific product segments are transported from a sales warehouse to different retailers. To solve their problem, the authors use LNS adapted to the model. In 2021, Berte Ousmane [2] used multi-compartment vehicles to solve the pick-up and delivery problem with time windows (MCV-PDPTW). The authors use the genetic algorithm to solve their problem. Dragomir and al. [9] study a generalized pick-up and delivery problem applied to a peer-to-peer online sales context. In their problem, the seller is assumed to be mobile during the day and provides the carrier with several options defined by a collection location and a time window. The buyer also provides several options based on his movements, but can also specify an alternative person to receive the goods. The authors propose a mathematical model and a method called Multi-Start Adaptive Large Neighborhood Search (MS-ALNS) combined with local search. In this work, we will first present and model our problem, then we will propose the genetic algorithm for solving the problem because of its complexity and finally we will present the results from the simulations and interpret these results.

2. Presentation of the MCV-SPSDPTW

The Split-Pick-up and Split-Delivery Problem with Time Windows using multi-compartment vehicles is the variant studied in this article. This work solves the difficulties associated with the goods collection and delivery system and allows customers to place as many orders as they wish with the aim of being served on the same day. This problem can be represented by the following graph:

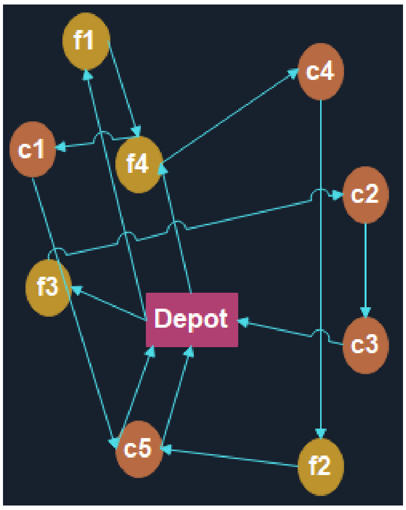

FIGURE 1. Example of a split tour

Our figure shows three (3) routes provided by three different vehicles:

1. Route 1,V_1: $Depot \to F_3 \to C_2 \to C_3 \to Depot$
2. Route 2,V_2: $Depot \to F_4 \to C_4 \to F_2 \to C_5 \to Depot$
3. Route 3,V_3: $Depot \to F_1 \to F_4 \to C_1 \to C_5 \to Depot$

Each vehicle starts its round at the depot and finishes it at the same depot. Each supplier is visited before its customer(s), as collection takes place at the supplier's premises before delivery to the customer. Supplier F4 is visited twice by compartmented vehicles V_2 and V_3. Customer C_5 is also visited twice by compartmentalized vehicles V_2 and V_3.

FIGURE 2. Multi-compartment vehicle

A single-compartment vehicle PDPTW problem can only transport compatible products, it may have several types of incompatible products to deliver which very often requires several vehicles to transport these products differently. The advantage of a multi-compartment vehicle in the PDPTW is that it can transport incompatible products on its own that had to be transported by several other single-compartment vehicles. This makes it possible to reduce the number of vehicles.

3. MATHEMATICAL FORMULATION OF THE PROBLEM

Our problem is characterized by the following parameters:

- N:Set of Customer, supplier and depot nodes,
- N':Set of Customer and supplier nodes,
- N_i^+ : Set of supplier nodes i,
- N_i^- : Set of customer nodes i,
- K:Set of vehicle indices,

- d_{ijk}: Euclidean distance between node i and node j travelled by vehicle k,if $d_{ijk} = \infty$ then the path between i and j does not exist,
- t_{ijk}:time taken by vehicle k to travel from node i to node j,
- $[e_i, l_i]$: time Windows of Node i,
- s_i:Stopping time at node i,
- q_{ip}:quantity of product p to be treated at node i. If $q_i > 0$, the node is a supplier; if $q_i < 0$, the node is a customer and if $q_i = 0$ then the node has been served,
- Q_k :capacity of the vehicle k,
- P: Set of the products p delivered to customers,
- W_p: the capacity of a compartment carrying product p,
- $i \in N$: index of predecessor nodes,
- $j \in N$: index of successor nodes,
- $k \in K$:index of vehicles,
- $X_{ijk} = \begin{cases} 1 & \textit{if vehicle} \quad k \quad \textit{travels from node} \quad i \quad \textit{to node} \quad j \\ 0 & \textit{else} \end{cases}$
- A_i: arrival time at node i,
- D_i: departure time at node i,
- y_{ik} : quantity present in vehicle k visiting node i,
- c_k: transport cost associated with vehicle k,
- A node (supplier or customer) can be visited several times by different vehicles,
- There is only one depot,
- Capacity constraints must be respected,
- Time constraints are rigid regarding arrival times,
- Each vehicle starts its journey from the depot and returns at the end,
- A vehicle remains stationary at a node for the time required to process the request,
- If a vehicle arrives at node i before the start date ei of its window, it waits.

The function to be minimized is given as follows

$$\textit{minimize} f = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_k d_{ij} X_{ijk}. \tag{3.1}$$

Under the constraints:

$$\sum_{i\in N}\sum_{k\in K} X_{ijk} \geq 1 \qquad \forall\, j \in N^{'}, \tag{3.2}$$

$$\sum_{j\in N}\sum_{k\in K} X_{ijk} \geq 1 \qquad \forall\, i \in N^{'}, \tag{3.3}$$

$$\sum_{j\in N} X_{0jk} = 1 \qquad \forall k \in K, \tag{3.4}$$

$$\sum_{i\in N} X_{i0k} = 1 \qquad \forall k \in K, \tag{3.5}$$

$$\sum_{i\in N} X_{iuk} - \sum_{j\in N} X_{ujk} = 0, \qquad \forall k \in K, \qquad \forall u \in N, \tag{3.6}$$

$$y_{jk} = (y_{ik} + q_i)X_{ijk}, \qquad \forall i, j \in N \qquad \forall k \in K, \tag{3.7}$$

$$y_{0k} = 0, \qquad \forall k \in K, \tag{3.8}$$

$$\sum_{i\in N} q_{ip}(\sum_{i\in N} X_{iuk}) \leq W_p, \qquad \forall u \in N \qquad \forall k \in K, \tag{3.9}$$

$$D_w \leq D_v, \qquad \forall i \in N, \quad \forall w \in N_i^+, \quad v \in N_i^-, \tag{3.10}$$

$$D_0 = 0,, \tag{3.11}$$

$$e_i \times \sum_{j\in N'} X_{ijk} \leq D_i \qquad \forall i \in N; \quad \forall k \in K, \tag{3.12}$$

$$A_j \leq l_j \times \sum_{i\in N'} X_{ijk}, \qquad \forall j \in N; \quad \forall k \in K, \tag{3.13}$$

$$A_j + s_j \leq l_j \times \sum_{i\in N'} X_{ijk}, \qquad \forall j \in N; \quad \forall k \in K, \tag{3.14}$$

$$D_i + t_{ijk} \times X_{ijk} \leq A_j \qquad \forall i, j \in N; \quad \forall k \in K. \tag{3.15}$$

, Constraint (3.1) represents the objective function which returns the fitness value. It is the minimum total cost related to the total distance travelled by all the vehicles in service [10] . Each vehicle has a set-up cost, which means that the total distance and the total cost are not necessarily proportional.

Constraints (3.2) and (3.3) ensure that each supplier or customer node can be visited several times by several vehicles in the case where customers place large orders. In this problem we require that a supplier or customer is visited only once by a single vehicle. This will enable us to manage our available fleet rationally.

Constraints (3.4) and (3.5) ensure that a vehicle leaves and returns to the depot only once. These equations fix the number of times a vehicle can be taken out and return to the depot to avoid under-turns. They guarantee that the availability of a vehicle is not exceeded.

Constraint (3.6) (fleet conservation) guarantees the continuity of a tour by a vehicle: the node visited must be left. This constraint also prevents sub-tours.

Constraints (3.7) and (3.8) ensure that the transport capacity of a vehicle is not exceeded. These constraints also ensure that all the goods have been delivered. In addition, all vehicles leave the depot with a zero quantity of goods and return to the depot empty. It should also be noted that the quantity of goods collected from each supplier must not exceed the total capacity of the vehicle.

Constraint (3.9) requires that the capacity of the compartment allocated to product p must be at least equal to the sum of customer requests for product p (Joseph, 2013) , In a compartment we only store one type of product in order to be able to guarantee food safety or hygiene for the different products ordered. Each compartment has a capacity. This constraint then requires that the quantity of the type of product dedicated to a compartment must be less than or equal to the capacity of that compartment. This will contribute to the choice of vehicles according to the number and capacity of their compartments in the heterogeneous fleet.

Constraints (3.10) and (3.11) ensure that precedence is respected. As the suppliers and their respective customers have been identified, our principle is that the goods should be collected from the suppliers and delivered to the customers. These constraints require each supplier to be visited by a vehicle before its customer.

Constraints (3.12), (3.13), (3.14) and (3.15) ensure that the time windows are respected. Each customer or supplier has the right to define a time interval in which they would like to be visited.

Constraint (3.12) requires that the vehicle departure time at the customer or supplier node must be greater than the lower limit of the time window defined by that customer or supplier. This is to avoid long waits or delays at each node. Constraint (3.13) requires the arrival time at a customer or supplier node to be less than the upper limit of the time window for that node.

Constraint (3.14), like the previous constraint, requires that the service time increased by the time of arrival at each node is less than the upper limit of the time window for that node. This prevents delays at each customer or supplier node.

Finally, constraint (3.15) stipulates that the departure time from the predecessor node and the journey time between the predecessor node and the successor node must not have any impact on the arrival time at the next node.

4. Complexity of the SPSDPTW

The split pick-up and split delivery problem with time window (MCV-SPSDPTW) that we solve reduces polynomial to the PDPTW which is NP-hard [2] so our MCV-SPSDPTW problem is also NP-hard.

5. The choice of metaheuristics

We have shown that MCV-SPSDPTW is a difficult problem to solve. Exact methods can only solve to optimality on small instances. So, according to [11] , it makes sense to turn to approximate methods known as heuristics because they are robust and fast for large instances. These approximate methods perform well on instances of any size and allow us to obtain very good solutions in a short space of time. We will therefore implement a meta-heuristic based on the genetic algorithm to solve our MCV-SPSDPTW problem.

6. Solving the MCV-SPSDPTW using the genetic algorithm

Genetic algorithms are optimization method inspired by the techniques of genetics and natural evolution, i.e. selection, crossover, mutation. Based on Darwin's work, John Holland [12] had the ingenious idea of adapting these genetic algorithms to optimization problems. The main fundamental elements of genetic algorithms are: assessment of an individual's level of adaptation, selection, crossover and mutation. Our objective in this work is to minimize the transport cost of our problem, which is characterized by:

- A single depot,
- all the vehicles we will be using are multi-compartment,
- Each supplier can be associated with several customers,
- Each customer can place orders with several suppliers,
- All customers must be served,
- Each supplier or customer can be visited several times by different types of vehicle,
- Each vehicle visits a supplier or customer once and only once,
- A minimum of two vehicles is required to complete the tour, due to the specific nature of our problem.

6.1. **Coding.** All vehicles have an order of passage at each supplier or customer node. The nodes served by the first vehicles will no longer be visited and each vehicle leaves the depot and returns at the end of its round. Orders are recorded in a chromosome.

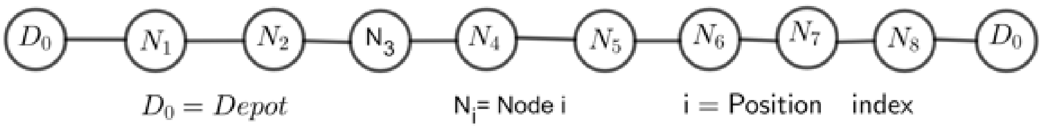

FIGURE 3. Coding example

6.2. **Objective function.** The objective function always returns a value called fitness. It evaluates the individual's performance. In our work, fitness is the value returned by the function:

$$\textit{minimize} f = \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_k d_{ij} X_{ijk},$$

which is nothing more than the total cost relative to the total distance travelled by the vehicles.

6.3. **Cross-over.** This is a genetic operation whereby two parent chromosomes give rise to two child chromosomes. Through cross-breeding, the child chromosomes inherit genes from the parents. The aim of cross-breeding is to enrich the diversity of the population The crossing can be single or multiple.

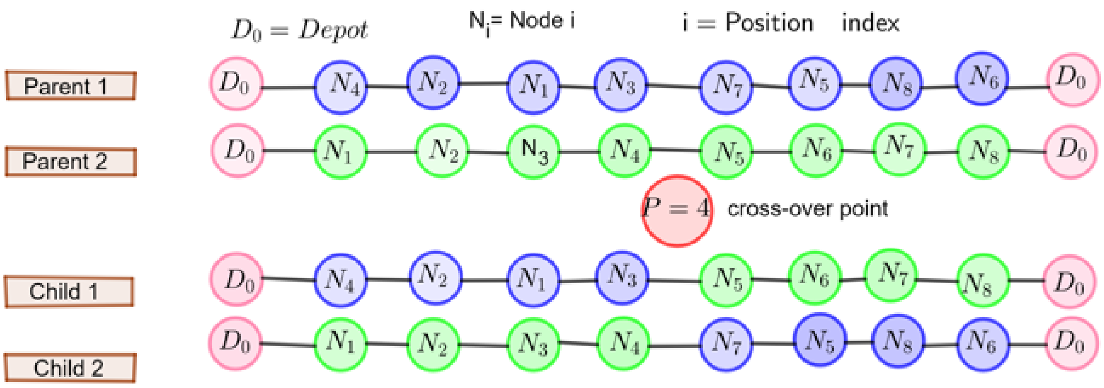

FIGURE 4. Example of crossing-over

6.4. **Mutation.** The mutation operation consists of randomly exchanging two or more genes on the same chromosome, giving rise to a new individual. In our problem, mutation is used to correct the precedence and time window constraints.

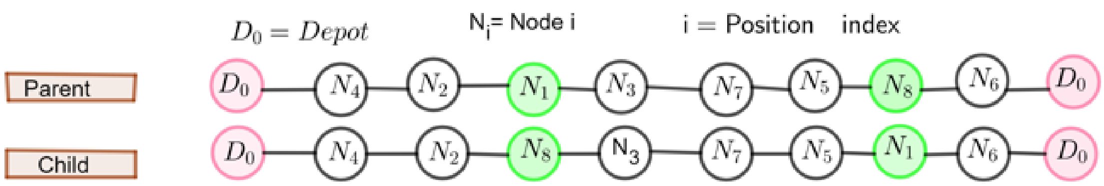

FIGURE 5. Example of mutation

6.5. **Selection.** To solve our problem, we opt for selection by ranking, which consists of sorting individuals according to their fitness values. Ranked in this way, we set a certain number of chromosomes that will participate in reproduction.

6.6. **Generating the initial population.** Our initial population is created to serve as the basis for future populations. The choice of this population is very important, as it allows a more or less rapid convergence towards the global optimum.

6.7. **Operating principle of the genetic algorithm.** From a random generation of the initial population obtained by means of a genetic operator called mutation at a certain frequency. To go from one generation to the next, the three operations of crossing, mutation and selection are repeated until the stop condition is reached. The figure below shows how the genetic algorithm works.

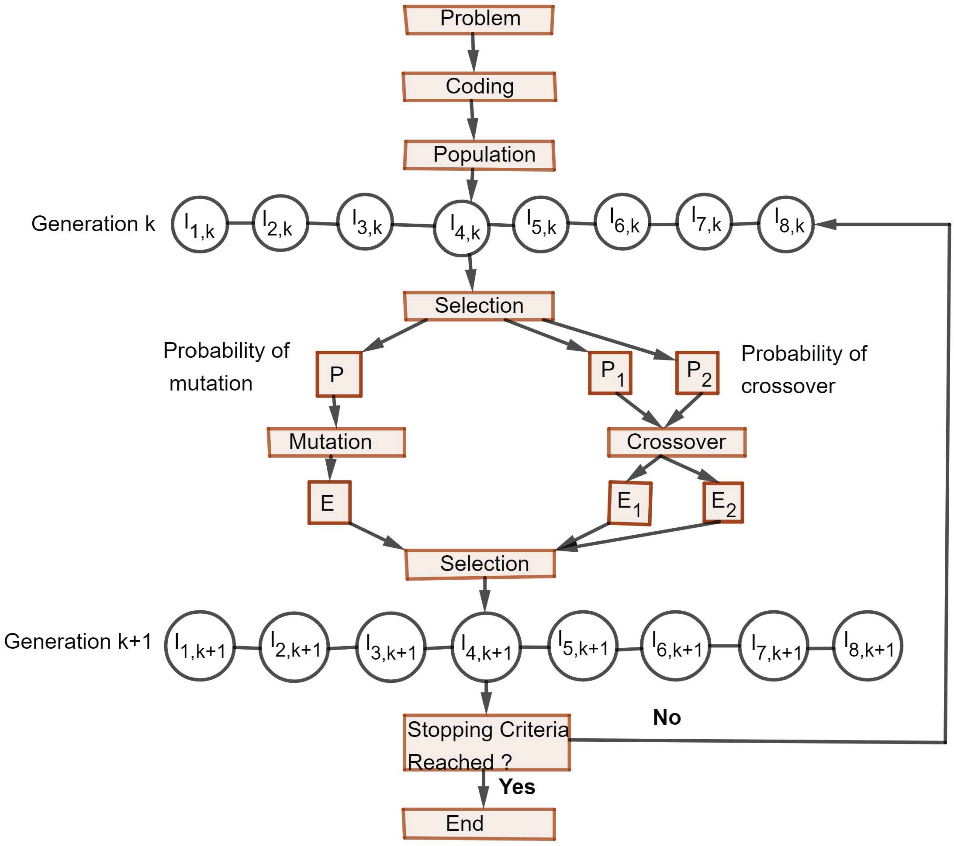

FIGURE 6. Operation of the genetic algorithm

6.8. **Algebraic resolution of the MCV-SPSDPTW.** Let's consider two cars with different capacities, such as the first vehicle, V_1, with a capacity of 120 and three (3) compartments with a capacity of 40 each, and the second vehicle, V_2, with a capacity of 200 and two (2) compartments with a capacity of 100 each. Consider the supplier/customer pairs $(F_1; C_1); (F_3; C_5); (F_1; C_2); (F_2; C_5); (F_1; C_3); (F_2; C_1); (F_2; C_4)$ and $(F_3; C_1)$. The different orders from different customers are recorded in the table below.

TABLE 1. Table of the different orders

Suppliers	Quantity of products	Customers C1	C2	C3	C4	C5	Total
Supplier F1	q_{1F1}	10	10	20			120
	q_{2F1}		40				
	q_{3F1}	20		20			
Supplier F3	q_{1F3}	100					200
	q_{2F3}					100	
Supplier F2	q_{1F2}				60	20	220
	q_{2F2}	100			20	20	
Total		230	50	40	80	140	540

The objective in this example is to organize a tour which respects the precedence constraints, the vehicle capacity constraints and the compartment and time window constraints of our model. From the analysis of the table, we can see from customer C_1's order that we can't do the tour with a single vehicle in a single round; similarly, we can't organize a two-round tour with a single vehicle because customer C_1's time window may not be respected. This means that we have to use two vehicles for this tour, with each vehicle having to make a single turn and return to the depot. Depending on the number of compartments and their capacities, we organize the tour as follows:

1. Vehicle V_1 will visit the following supplier/customer pairs $(F_1; C_1)$; $(F_2; C_4)$; $(F_1; C_3)$; $(F_2; C_5)$ and $(F_1; C_2)$.
2. Vehicle V_2 will visit the following supplier/customer pairs $(F_3; C_1)$; $(F_3; C_5)$ and $(F_2; C_1)$.

6.8.1. *Respecting the precedence constraint.* The following solution is not feasible because the precedence constraints are not respected.

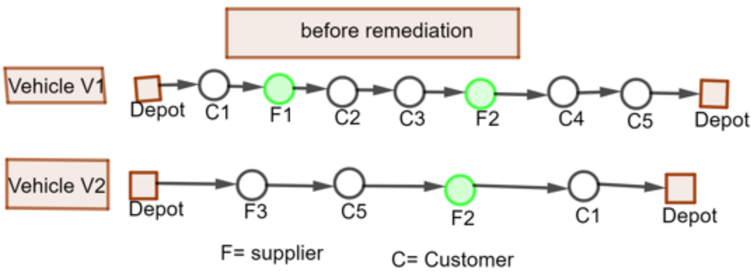

FIGURE 7. Precedence constraint

After correcting the constraints, we obtain:

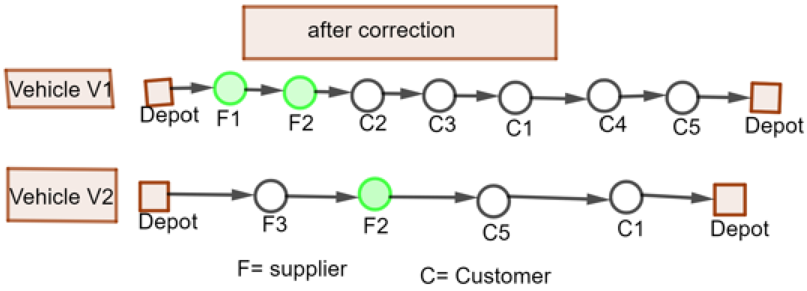

FIGURE 8. Correction of precedence constraints

In connection with the table of the various controls recorded in the table (1), it should be noted that the capacity constraint is simultaneously corrected.

6.8.2. *Respecting the time window.* Before correction:

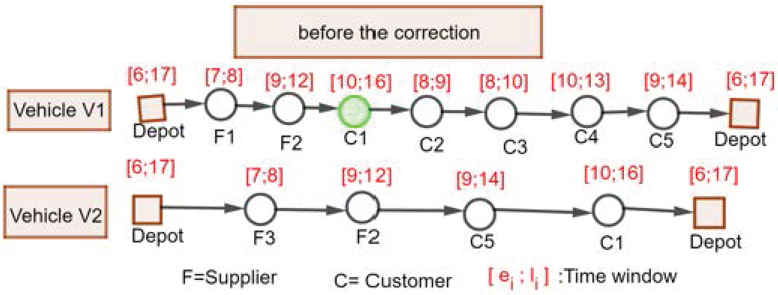

FIGURE 9. Time window

After correction we get:

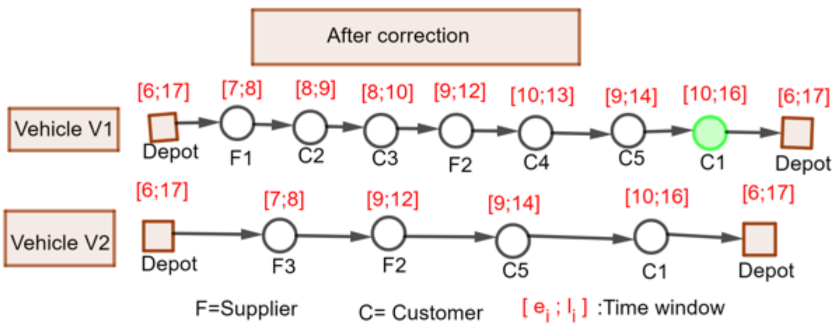

FIGURE 10. Time window corrected

After correcting the precedence constraints, the capacity constraints of the vehicles and their compartments, the compatibility constraints of the products stored in the different compartments and the time constraints, we obtain the following feasible solution.

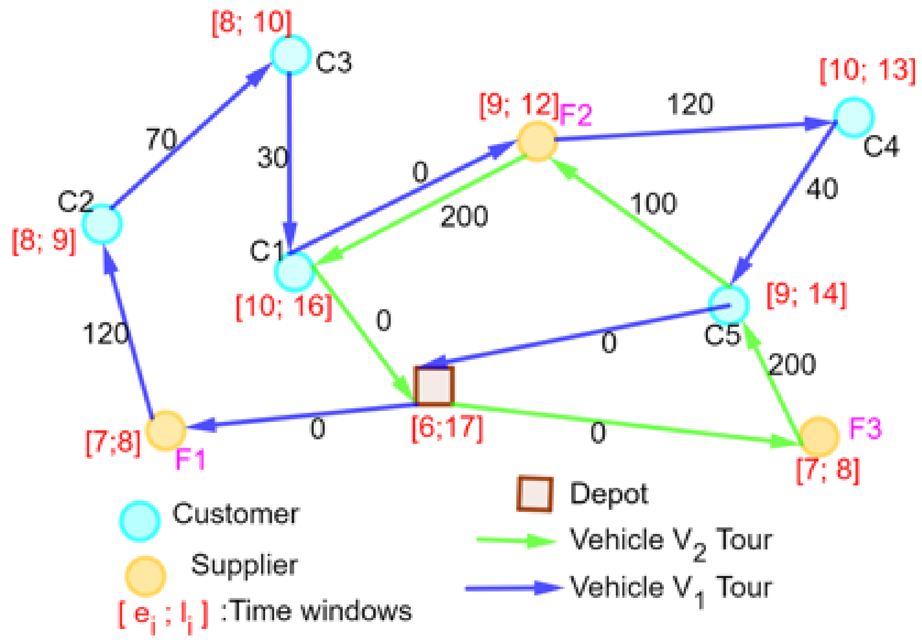

FIGURE 11. Solution of SPSDPTW

To obtain a minimal solution, we will have to explore all feasible solutions, and given the size of our nodes, we plan to use the genetic algorithm.

7. INSTANCES AND RESULTS

The problem we are solving is the MCV-SPSDPTW (Multi-Compartment Vehicle for a Split-Pick-up and Split-Delivery Problem with Time Windows),where customers can place as many orders as they like and have them delivered within the same timeframe. To do this, we have a fleet of heterogeneous vehicles such that each vehicle has several compartments of the same size. By applying the genetic algorithm, this programme enables vehicles coming from the depot to pick up goods from suppliers and deliver them to the various customers, respecting the time and compartment constraints, and return to the depot at the end of the round. In our work, the chromosome is made up of: a depot, suppliers and customers. Our program starts a vehicle that leaves the depot, picks up a supplier and checks whether there are any other suppliers with whom it has customers in common. Once the precedence conditions have been checked, the vehicle loads the goods in an orderly fashion into the various compartments as far as it can before delivering to the various customers. This process is repeated until all the nodes have been selected by the vehicles, so that all the orders are collected from the various suppliers. The route of each vehicle is established and all customers are delivered before returning to the depot. As we are dealing with a split problem, for optimum use of the vehicles in our fleet, we engage the vehicles in descending order of the number of compartments in each vehicle.

7.1. **Results.** Following our various simulations, we present in this table the optimal solutions for four different instances in terms of the number of nodes. In each instance, we present two solutions with different numbers of vehicles used, minimum cost and minimum total distance.

Table 2: Table of instance1 results (100 nodes)

Instan-ce	Numbers of nodes	Minimum distance	Minimum cost	Numbers of vehicles	Features Vehicles	Traversed routes
					V16: 130 comparti-ments	V16: D1 -F19 -F1 -F42 -F44 -F29 -F13 -F12 -F20 -F36-F32 -C21 -F46 -C20 -F14 -C3 -F39 -F25 -F45 -C40 -F3 -F7 -F31 -C32 -F47 -F23 -F22 -F10 -C47 -F50 -C29 -F8 -C30 -C31 -F33 -C7 -F24 -C16 -F4 -C8 -F35 -F2 -C2 -C15 -F28 -F49

						-C11 -F30 -F18 -F15 -C22 -C48 -F26 -F17 -C27 -F27 -C45 -F5 -C41 -F48 -C17 -C28 -F11 -C10 -C46 -C1 -C12 -F43 -C34 -C49 -F34 -C13 -F38 -C43 -F21 -C38 -F9 -C37 -C44 -C42 -C14 -C6 -C18 -C5 -C19 -F41 -C4 -C26 -C50 -C24 -C33 -C23 -C35 -D1
	100	33614	3215314	3	V15: 110 compartiments	V15: D1 -F36 -F32 -F37 -F16 -C40 -F4 -F2 -F49 -C11 -F18 -C22 -C39 -F17 -C27 -C13 -C44 -C24 -C23 -D1
Instance 1					V14: 100 compartiments	V14: D1 -F49 -F40 -F6 -F18 -F15 -C9 -C22 -C27 -C36 -C25 -C24 -D1
	100	31571	2752865	4	V16: 130 compartiments	V16: D1 -F38 -F4 -F31 -F10 -C5 -F49 -F36 -C45 -F29 -C40 -C27 -F39 -C10-F15 -C11 -F45 -F14 -F32 -C44 -F13 -F30 -C38 -F34 -C37 -F41 -C42 -C47 -F40 -C2 -F12 -F18 -C46 -C3 -F21 -C35 -C1 -C36 -F28 -C31 -C25 -F43 -C33 -F16 -C19 -C24 -C48 -C22 -D1

					V15: 110 comparti-ments	V15: D1 -F49 -F33 -F48 -F8 -F19 -F3 -F46 -F6 -F11 -F5 -F9 -F1 -C30 -F7 -F2 -F36 -F22 -F37 -F25 -F35 -C40 -F26 -F23 -F50 -F20 -C28 -C50 -F17 -C21 -F42 -C41 -F47 -C27 -C10 -C17 -C6 -C26 -C18 -C9 -C7 -C20 -C29 -C13 -C43 -C39 -C23 -C14 -C34 -C4 -C12 -C8 -C15 -C16 -D1
					V14: 100 comparti-ments	V14: D1 -F36 -F27 -F29 -F22 -F44 -F24 -C49 -F37 -F25 -C32 -C40 -F23 -F50 -F20 -C17 -C26 -C18 -C47 -C20 -C13 -C39 -C15 -C16 -C22 -D1
					V13: 90 comparti-ments	V13 : D1 - F20 - C20 - D1

Table 3: Table of instance 2 results (130 nodes)

Insta-nce	Numbers of nodes	Minimum distance	Minimum cost	Numbers of vehicles	Features Vehicles	Traversed routes
					V16: 130 comparti-ments	V16 : D1 - F60 - F41 - F7 - F13 - F50 - F19 - F52 - F11 - F35 - C21 - F44 - C32 - F56 - F6 - F65 - F61

						- F27 - F23 - F9 - F53 - F57 - F24 - C59 - F40 - F5 - C8 - F30 - F39 - F15 - F2 - C16 - F64 - F25 - F17 - C60 - F48 - F59 - F33 - C36 - F22 - C25 - C56 - F12 - C2 - C28 - F1 - F32 - F26 - F51 - F46 - C35 - F54 - C37 - F21 - C41 - F38 - C52 - F47 - C51 - C9 - C14 - C38 - F43 - C63 - C49 - F55 - C43 - F4 - C61 - F34 - C44 - F45 - C62 - F28 - C64 - C13 - F8 - C31 - C33 - F37 - C23 - C57 - F3 - C29 - F62 - F29 - C55 - C18 - C10 - C4 - F18 - C11 - C46 - C3 - C48 - C1 - C19 - C7 - C26 - F63 - F31 - C54 - C6 - C30 - C42 - F16 - C47 - C24 - C45 - C12 - C17 - C65 - C20 - C22 - C53 - C39 - C50 - D1
	130	49 600	4560002	4	V15: 110 comparti-ments	V15 : D1 - F39 - F15 - F36 - F2 - F42 - F64 - F58 - F25 - F20 - C25 - F10 - C40 - F32 - F26 - F51 - F46 - C37 - F38 - C52 - C38 - F4 - C5

Instance 2						- C44 - C13 - C34 - F29 - F14 - C58 - C10 - F18 - C11 - C1 - C30 - C47 - C24 - C15 - C65 - C20 - C22 - C50 - D1
					V14: 100 compartiments	V14: D1 - F25 - F20 - F59 - F49 - C13 - C57 - F18 - C27 - C15 - C20 - C22 - D1
					V13: 90 compartiments	V13: D1 - F20 - F17 - F59 - F49 - F33 - F22 - F10 - F12 - C43 - C23 - C57 - C18 - C10 - F18 - C1 - C27 - C20 - C22 - D1
					V16: 130 compartiments	V16 : D1 - F6 - F25 - F51 - F39 - F1 - F18 - F4 - F23 - F38 - F42 - F41 - C37 - F22 - F17 - F61 - C18 - F56 - F24 - C23 - F58 - F59 - C22 - F15 - F35 - C9 - F33 - C16 - C17 - F27 - F16 - C60 - F49 - F26 - F14 - C25 - F9 - F48 - C28 - F13 - C13 - F20 - C11 - F3 - C20 - F47 - F55 - F62 - C55 - C24 - F52 - F10 - C54 - F64 - C29 - F32 - C2 - F54 - C27 - F37 - C61

						- F45 - C52 - C38 - F12 - C43 - F36 - C3 - F11 - C40 - F46 - C31 - C5 - C4 - F29 - C47 - C12 - C1 - F7 - C57 - C50 - F21 - C8 - C49- F44 - C10 - F31 - C39 - F28 - C48 - F50 - C6 - C26 - C45 - C19 - C41 - C14 - C58 - C15 - C65 - C32 - C30 - C64 - C34 - C44 - C62 - C35 - D1
	130	46 931	4294615	4	V15: 110 compartiments	V15 : D1 - F4 - F23 - F38 - F42 - F19 - F5 - C21 - C17 - F20 - C11 - F3 - F43 - C20 - F62 - C54 - F64 - F32 - F57 - C33 - C52 - C38 - C59 - C12 - C10 - C34 - C44 - D1
					V14: 100 compartiments	V14 : D1 - F42 - F41 - F63 - F53 - F65 - C53 - F34 - F2 - F30 - F19 - F5 - F61 - F40 - F8 - C51 - C21 - C16 - C13 - F20 - C20 - C55 - C42 - C63 - C7 - C36 - C46 - C10 - C34 - C35 - D1
Instance 2					V13: 90 compartiments	V13 : D1 - F40 - F24 - F60 - F8 - C16 - F20 - C20 - C56 - C7 - C36 - D1

Table 4: Table of instance 3 results (150 nodes)

Instan- - ce	Numbers of nodes	Minimum distance	Minimum cost	Numbers of vehicles	Features Vehicles	Traversed routes
					V16: 130 comparti- ments	V16 : D1 - F53 - F49 - F60 - F6 - F44 - C56 - F26 - F19 - F23 - F68 - F73 - C9 - F8 - F47 - F1 - F35 - F74 - F10 - F29 - F50 - F30 - F71 - F63 - F11 - C63 - F64 - F21 - C7 - F3 - F75 - C26 - F20 - F54 - F31 - F65 - C52 - F55 - F9 - C73 - F5 - C4 - F45 - F51 - C53 - F37 - C31 - F36 - F67 - C32 - F15 - C10 - F14 - C68 - F12 - F13 - C6 - F61 - C65 - C67 - C14 - F42 - C5 - F17 - C20 - C27 - F18 - C39 - F43 - C62 - C3 - C34 - C23 - C66 - F57 - C45 - F2 - F66 - C2 - C25 - C47 - C50 - C29 - C74 - C41 - F69 - C13 - F52 - C64 - F58 - C12 - F39 - C61 - F56 - C37 - C58 - C70 - C75 - C1 - C46 - C60 - F38 - C38 - C22 - C51 -

						C21 - C40 - C33 - C17 - C55 - C72 - C59 - C19 - D1
	150	57301	5007924	5	V15: 110 compartiments	V15 : D1 - F63 - F64 - F21 - F3 - F41 - F20 - F54 - F33 - F65 - F46 - C52 - F9 - F5 - F32 - C53 - C16 - F36 - F67 - F25 - C10 - C35 - F13 - F59 - C6 - F61 - F16 - C14 - C15 - F72 - F7 - C20 - F18 - F43 - C62 - C43 - F57 - F2 - F66 - C44 - F48 - C2 - C30 - C74 - C8 - C13 - C12 - C57 - C75 - C28 - C22 - C69 - C51 - C24 - C40 - C33 - C55 - C59 - C19 - D1
					V14: 100 compartiments	V14 : D1 - F20 - F31 - F33 - F65 - F46 - F32 - F70 - F24 - F45 - F28 - F4 - F51 - C71 - F27 - C16 - C31 - F36 - F67 - F25 - C11 - F16 - C65 - C48 - F7 - C20 - C43 - C45 - C44 - C30 - C74 - C8 - C49 - C22 - C51 - C24 - C40 - D1

						V13: 90 comparti-ments	V13 : D1 - F4 - F51 - F27 - F62 - F40 - C54 - F22 - F36 - F67 - F34 - F25 - C36 - F15 - C11 - C65 - C42 - C18 - C25 - C74 - C49 - C22 - C24 - C40 - D1
Instan-ce 3						V12 : 80 comparti-ments	V12: D1 - F34 - F25 - F15 - C15 - C42 - C13 - C22 - C24 - D1
						V16: 130 comparti-ments	V16 : D1 - F21 - F28 - F60 - C48 - F20 - F69 - F14 - F1 - F55 - F50 - F75 - F17 - F15 - F56 - F23 - C25 - F30 - C19 - F24 - F10 - C61 - F33 - C56 - F44 - F72 - F5 - F51 - F59 - C60 - F48 - F34 - C72 - F68 - C20 - F37 - C57 - F13 - F39 - F35 - F27 - F7 - F16 - C37 - F46 - C23 - F58 - F54 - C39 - F41 - F63 - C66 - C2 - C35 - F18 - C8 - F57 - C58 - F40 - C59 - F3 - F9 - F2 - F43 - C14 - F61 - C41 - C16 - C69 - F11 - C26 - C65 - C53 - C22 - C55 - F12 - C43

						- C3 - C42 - F67 - C10 - F73 - C13 - F49 - C33 - C74 - C12 - C62 - F6 - C4 - C28 - C30 - C68 - C6 - C9 - F62 - C54 - C24 - C1 - C49 - F52 - C73 - C36 - C64 - C27 - C46 - C32 - C17 - D1
Instance 3	150	57403	4918402	5	V15: 110 compartiments	V15 : D1 - F56 - F22 - F23 - F30 - F24 - F10 - F5 - F51 - F26 - F59 - C60 - F48 - F4 - F42 - F25 - F36 - F38 - F34 - F8 - C18 - F45 - F65 - F68 - C51 - F53 - C57 - C5 - C31 - C34 - C50 - F18 - C63 - F57 - C59 - F2 - C11 - F43 - C16 - C65 - C38 - C7 - C22 - C42 - C10 - C40 - C13 - C33 - C28 - C15 - C73 - C46 - C17 - D1
					V14: 100 compartiments	V14 : D1 - F4 - F42 - F25 - F36 - F74 - F47 - F71 - F64 - C29 - F19 - F70 - F66 - C52 - C67 - C34 - C70 - F18 - F40 - C75 - F2 - C11 - C22 - C21 - C40 - C13 - C71 - C15 - C36 - D1

					V13: 90 compartiments	V13 : D1 - F64 - F38 - F19 - F34 - F8 - F70 - F65 - F29 - F68 - C51 - F53 - F32 - C52 - C44 - F18 - C63 - F57 - C59 - F2 - C38 - C7 - C22 - C21 - C47 - C42 - C13 - C71 - C73 - D1
					V12 : 80 compartiments	V12 : D1 - F29 - F68 - F53 - F32 - F31 - F66 - C44 - C63 - F57 - F40 - C59 - C75 - F2 - C47 - C13 - C73 - C36 - C45 - D1

Table 5: Table of instance 4 results (180 nodes)

Instance	Numbers of nodes	Minimum distance	Minimum cost	Numbers of vehicles	Features vehicles	Traversed routes
					V16: 130 compartiments	V16 : D1 - F47 - F79 - F62 - F12 - F73 - F8 - F72 - F4 - F76 - F23 - C29 - F67 - C3 - C11 - F3 - F54 - F7 - F35 - F42 - C69 - F85 - F68 - F13 - F5 - F56 - C8 - F87 - F14 - C41 - F55 - C54 - F40 - F31 - F38 - C38 - F58 - C12 - F20 - C61 - F52 - F10 - F6 - C2 - F83 - F59 - F80 - F51 - C87 - F16 - C7 - F39 - C81 - F17 - F60

						- C58 - F46 - F11 - F18 - F32 - C1 - C86 - F78 - C17 - F30 - F86 - F81 - C10 - F84 - C80 - F1 - C68 - C45 - C79 - F19 - F57 - C88 - F9 - F28 - C30 - C14 - C46 - F64 - C73 - C21 - C56 - F77 - C65 - C37 - C62 - C83 - F26 - C57 - F44 - C90 - C48 - C24 - C89 - C6 - F82 - C84 - C23 - C34 - F69 - C9 - F45 - F27 - C32 - C85 - C4 - C52 - C49 - C20 - C31 - C22 - F90 - C82 - C76 - C16 - C5 - C72 - C50 - C74 - C44 - C59 - C36 - F2 - C60 - C64 - C13 - D1
Instance 4	180	76302	6505886	10	V15: 110 compartiments	V15 : D1 - F23 - F24 - F74 - F67 - F88 - F5 - F70 - F56 - F65 - F89 - F55 - F50 - C51 - F63 - F21 - F66 - F37 - F33 - F36 - F61 - F41 - C78 - F53 - C43 - F38 - F34 - C38 - F58 - C61 - F10 - F49 - C67 - F48 - F39 - F60 - C58 - F46 - C27

						- C17 - C40 - F81 - C35 - C10 - F1 - F29 - C53 - F57 - C30 - F64 - C39 - C56 - C42 - C37 - C71 - C63 - C47 - C85 - C52 - C75 - C20 - C55 - C19 - C77 - C26 - C16 - C5 - C28 - C74 - C59 - C60 - D1
					V14: 100 compartiments	V14 : D1 - F61 - F41 - F53 - F22 - F38 - F34 - C38 - F58 - F20 - F6 - F16 - F60 - C58 - F18 - C35 - F64 - C56 - C42 - C24 - C63 - C9 - C52 - C20 - C55 - C22 - C18 - D1
					V13: 90 compartiments	V13 : D1 - F20 - F10 - F75 - F49 - C66 - F39 - C27 - F32 - F86 - C10 - C80 - F64 - C37 - C52 - C20 - C44 - D1
					V12 : 80 compartiments	V12: D1 - F49 - F6 - F71 - F48 - F16 - F43 - F39 - F60 - F46 - F15 - C33 - F18 - F25 - C27 - C30 - C56 - C70 - C15 - C37 - C24 - C9 - C22 - C25 - C28 - D1
					V11: 70 compartiments	V11: D1 - F18 - C22 - D1

					V9: 50 compartiments	V9 : D1 - F18 - C22 - D1
					V10: 40 compartiments	V10 : D1 - F18 - F25 - C15 - C22 - D1
					V4: 30 compartiments	V4 : D1 - F25 - C22 - C13 - D1
					V2: 20 compartiments	V2 : D1 - F25 - C13 - D1
					V16: 130 compartiments	V16 : D1 - F31 - F23 - F71 - F15 - F89 - F13 - F70 - F36 - F7 - F86 - F63 - F84 - F69 - F82 - F30 - C17 - F64 - F55 - F5 - F74 - F72 - C52 - F11 - F77 - C40 - F17 - F8 - F59 - F52 - F39 - F90 - F62 - C7 - C82 - F37 - F73 - F26 - F9 - C69 - F53 - C4 - F66 - F25 - C75 - C89 - F48 - C84 - F10 - F45 - C80 - F22 - F18 - F54 - F16 - C62 - F27 - C53 - F49 - C23 - F80 - C67 - C22 - C18 - F38 - C68 - F67 - C27 - F60 - F19 - C2 - F75 - F79 - C87 - F35 - C14 - F12

						- C49 - C28 - C61 - F6 - C31 - C72 - C5 - C24 - C3 - C54 - F65 - C39 - C46 - C66 - F46 - C10 - C51 - F4 - C71 - F83 - C37 - F76 - F42 - C56 - F81 - C45 - C70 - C63 - C41 - C38 - C77 - C25 - F78 - C1 - C50 - C76 - C8 - C9 - C83 - C90 - F20 - C86 - C21 - C64 - C30 - F47 - C11 - C85 - C20 - C88 - C74 - C29 - C57 - C34 - D1
Instance 4	180	68545	5 940 612	6	V15: 110 compartiments	V15 : D1 - F64 - F55 - F5 - F29 - F40 - F28 - C52 - C48 - F62 - F2 - F26 - F9 - F53 - F87 - F66 - F25 - F51 - F50 - C75 - F61 - F48 - C26 - C36 - F33 - F22 - F43 - F18 - F27 - F58 - C22 - F32 - C18 - F38 - C47 - C58 - C14 - C49 - C28 - C61 - C15 - C43 - C65 - C6 - C54 - C10 - F4 - C44 - F83 - C13 - F76 - F42 - C63 - C79 - C38 - C50 - C83 - C90 - C33 - F20 - C11 - C20 - C34 - C55 - C16 - D1

					V14: 100 compartiments	V14 : D1 - F25 - F51 - F50 - F61 - F88 - F10 - F57 - F45 - C26 - F3 - F22 - F43 - F41 - F18 - F16 - F14 - F27 - F1 - F68 - C22 - F32 - F56 - C18 - F38 - C14 - C49 - C31 - C5 - C78 - C24 - C73 - C65 - C35 - C59 - C10 - C44 - C13 - C60 - C38 - C1 - C33 - C12 - C55 - D1
					V13: 90 compartiments	V13 : D1 - F18 - F16 - C22 - F38 - F67 - C24 - C38 - C74 - D1
Instance 4	180				V12: 80 compartiments	V12: D1 - F16 - F14 - F27 - F44 - F49 - F1 - F24 - F58 - C32 - F21 - C27 - C58 - C14 - C49 - C24 - C19 - C1 - C20 - C16 - D1
					V11: 70 compartiments	V11: D1 - F58 - F68 - F34 - F21 - F85 - C81 - C58 - C73 - C42 - C19 - D1

7.2. **Analysis and interpretation of results.**

- In instance 1 two different solutions are presented With three vehicles: the first solution offers us a high total distance tour at a high cost whereas the second solution uses four vehicles to produce a minimum total distance tour with a minimum service cost. We explain this by the cost of putting the vehicles into service, the distance covered by the different vehicles and the compartment constraints. The disparity in the number of vehicles is also explained by the position of customers with large orders and their respective suppliers.

- In instance 2, both solutions use the same number of vehicles. The first solution gives us a long tour at a high cost while the second solution provides us with a short distance tour at a relatively low cost. We explain this by the different routes of each vehicle in the tour.
- As in the previous instance, both solutions in instance 3 used the same number of vehicles. The first solution gives us a small cost with a large distance and the second solution produces the opposite effect. Given that the vehicles have different service costs, we explain this by the order in which the vehicles pass through the supplier and customer nodes for large orders. The results of instance 2 and instance 3 show that the minimum total cost of the tour is not proportional to the minimum total distance covered by all the vehicles in service.
- The last instance presents two solutions whose characteristics are opposite to those of the first instance. In both proposed solutions, the minimum total cost is dependent on the total distance and the number of vehicles used. We explain this by the fact that customers place very large orders for goods and the increase in the number of orders.

8. Conclusion

In this article we have established the mathematical model of the MCV-SPSDPTW. Taking into account its NP-hard complexity, we have numerically solved our problem using the genetic algorithm, which gives us very good optimal solutions with minimal service cost and minimal total distance.

Acknowledgment

We would like to thank the LAMMA research laboratory of the University of Lomé and its various research teams for the good collaboration. Our special thanks go to ESATIC for facilitating our research stay by hosting us at the African School of ICT: LASTIC, which we remain very grateful to.

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