Advances in Mathematics: Scientific Journal **12** (2023), no.10, 921–940
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.12.10.5

COMPARISON OF LAPLACE-ADOMIAN'S DECOMPOSITION METHOD AND LAPLACE VARIATIONAL ITERATION METHOD IN NONLINEAR BOUSSINESQ EQUATION

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ABSTRACT. In this paper, we apply two techniques Laplace Decomposition Method (LADM) and Laplace Variational Iteration Method (LVIM) to determine the analytical solution of Boussinesq Equation.

1. INTRODUCTION

A generalized Boussinesq equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = a\frac{\partial^2}{\partial x^2}\left[N(u(x,t)+bu(x,t)\right] + \frac{\partial^4}{\partial x^4}u(x,t) + f(x,t), \tag{1.1}$$

where $N(u(x,t)$ is an arbitrary sufficiently differentiable function, with the condition that $N(u(x,t) \neq 0$ to ensure nonlinearity and $f(x,t)$ is given function.

The initial conditions are given in the form of

$$\begin{cases} u(x,0) = f(x) \\ \dfrac{\partial u(x,0)}{\partial t} = g(x) \end{cases}. \tag{1.2}$$

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2020 *Mathematics Subject Classification.* 47H14, 34G20, 47J25, 65J15.
Key words and phrases. Boussinesq equation,Laplace-Adomian's decomposition method, Laplace Variational Iteration method, exact solution.
Submitted: 13.09.2023; *Accepted:* 28.09.2023; *Published:* 25.10.2023.

Equation (1.1) has been proposed as a model for the propagation of pulses along a transmission line made up of a large number of LC circuits and as a model to describe the vibrations of a dense one-dimensional network. However, in each of these studies, Rosenau remarks that (1.1) is incorrectly posed,and additional assumptions must be made about the non-linearity of $N(u)$ [11]. A classification of (1.1) is undertaken by applying both the Lie method and the unclassical method of Bluman and Cole [8]. This class of nonlinear Boussinesq equation has already been studied in some of the references ([9], [10]). Clarkson and Kruskal [10] introduced some similarity reduction of the Boussinesq equation. These reductions in symmetry are obtained by the direct method. Using this method, the equation is reduced to the first, second and fourth Painlevé equations, which involves no group theory techniques. Another important work to find exact solutions of the Boussinesq equation is studied by Clarkson [9]. He said that the solutions to this equation are obtained in two different ways: one of these, using the classical and unconventional reductions of the equations to find the corresponding ordinary differential equations, which are solvable in terms of the first, second and fourth Painlevé equations. Exact solutions are generated from these ordinary differential equations. The second way, he used further space-independent similarity reductions of the Boussinesq equation. He also generated the second and fourth Painlevé equations to find the exact solutions of the equation using these similarity reductions. In [11], Clarkson and Priestly found conditions on $N(u)$ such that it allows symmetries, in particular those beyond translational symmetries of independent variables. They used the classical Lie method, and the non-classical method, to find these symmetries. Once the symmetries of (1.1) are found, they find the associated reductions and test the differential equations, then solve the equation. However, the application methods are not entirely straightforward [11].

In this paper, the nonlinear equation (1.1) with two cases of the nonlinear term of $N(u)$ which are given in [11], homogeneous or inhomogeneous, will be treated more easily, faster and more elegantly by implementing the Laplace method of Adomian decomposition and the variational iteration method ([1], [2], [3]) rather than the traditional methods for explicit solutions. In this article we do not use any discounts or transformation to reduce the problem (1.1) to an ordinary differential equation, a system of simpler partial differential equations or any linearization, perturbation scheme. The original nonlinear equation is directly solvable preserving real physics and involving much less computation [3]. The decomposition scheme is illustrated by studying problem (1.1) to calculate approximate solution to this problem. In addition, we also illustrate the self-cancellation phenomena for problem (1.1) using the decomposition method.

2. Describing of both methods

2.1. **The Laplace transform [2].**

Let's note the laplace transform by

$$\mathcal{L}(u(x,t)) = \int_0^\infty u(x,t)e^{-st}dt. \tag{2.1}$$

From (1.1), we have:

$$\begin{cases} \mathcal{L}(\dfrac{\partial u(x,t)}{\partial t}) = s\mathcal{L}\left(u(x,t)\right) - u(x,0) \\ \mathcal{L}(\dfrac{\partial^2 u(x,t)}{\partial t^2}) = s^2\mathcal{L}\left(u(x,t)\right) - su(x,0) - \dfrac{\partial u(x,0)}{\partial t} \end{cases}. \tag{2.2}$$

Let f and g be two functions then

$$\begin{cases} f(x) * g(x) = \displaystyle\int_0^x f(x-t)g(t)dt \\ \mathcal{L}\left(f(x) * g(x)\right) = \mathcal{L}\left(f(x)\right)\mathcal{L}\left(g(x)\right) \end{cases}. \tag{2.3}$$

2.2. **Laplace-Adomian Decomposition method (LADM) ([12–15]).**

The LADM is the combination of the Laplace transform with the Adomian decomposition method (ADM). This decomposition method was first introduced by A. Suheil Khuri and has been used successfully to find the solution of differential equations. The important advantage of this method is to combine the two powerful methods to obtain the exact solutions of linear or nonlinear equations.

Consider the functional equation

$$Au = h, \tag{2.4}$$

where A represents a differential operator of a Hilbert space H, h a function given in H and u the unknown function to be determined

By asking

$$A = L + R + N \tag{2.5}$$

equation (2.4) becomes

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = h(x,t). \tag{2.6}$$

By taking for example $L_{tt}(.) = \dfrac{\partial^2}{\partial t^2}$, $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$. By applying the Laplace transform to (2.6) we get:

$$\mathcal{L}\left[Lu(x,t)\right] + \mathcal{L}\left[Ru(x,t)\right] + \mathcal{L}\left[Nu(x,t)\right] = \mathcal{L}\left[h(x,t)\right]. \tag{2.7}$$

Using the differentiation property of the Laplace transform we get:

$$s^2\mathcal{L}\left[u(x,t)\right] - sf(x) - g(x) + \mathcal{L}\left[Ru(x,t)\right] + \mathcal{L}\left[Nu(x,t)\right] = \mathcal{L}\left[h(x,t)\right], \tag{2.8}$$

from where:

$$\begin{aligned} \mathcal{L}\left[u(x,t)\right] = \frac{f(x)}{s} + \frac{g(x)}{s^2} - \frac{1}{s^2}\mathcal{L}\left[Ru(x,t)\right] \\ - \frac{1}{s^2}\mathcal{L}\left[Nu(x,t)\right] + \frac{1}{s^2}\mathcal{L}\left[h(x,t)\right]. \end{aligned} \tag{2.9}$$

We then look for the solution $u(x,t)$ when it exists in the form of a series:

$$u(x,t) = \sum_{n=0}^{+\infty} u_n(x,t). \tag{2.10}$$

Non linear $Nu(x,t)$ operator is also noted in a series polynomials

$$Nu(x,t) = \sum_{n=0}^{+\infty} A_n(x,t), \tag{2.11}$$

where A_n are special polynomials $u_0, u_1, u_2, \cdots, u_n$ called Adomian polynomials and defined by

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{n=0}^{+\infty} \lambda^i u_i\right)\right]_{\lambda=0}, n = 0, 1, 2, \cdots, \tag{2.12}$$

where λ is a parameter used by "convenience".

By substituting (2.10) and (2.11) in (2.9), we become:

$$\begin{aligned}\sum_{n=0}^{+\infty}[\mathcal{L}u_n(x,t)] &= \frac{f(x)}{s}+\frac{g(x)}{s^2}-\frac{1}{s^2}\sum_{n=0}^{+\infty}[R\mathcal{L}u_n(x,t)] \\ &\quad -\frac{1}{s^2}\sum_{n=0}^{+\infty}[\mathcal{L}A_n(x,t)]+\frac{1}{s^2}\mathcal{L}[h(x,t)].\end{aligned} \tag{2.13}$$

We obtain the Laplace - Adomian algorithm:

$$\begin{cases}\mathcal{L}\left[u_0(x,t)\right]=\dfrac{1}{s}f(x)+\dfrac{1}{s^2}g(x)+\dfrac{1}{s^2}\mathcal{L}\left[h(x,t)\right] \\ \mathcal{L}\left[u_{n+1}(x,t)\right]=-\dfrac{1}{s^2}\mathcal{L}\left[Ru_n(x,t)\right]-\dfrac{1}{s^2}\mathcal{L}\left[A_n(x,t)\right];n\geq 0\end{cases}. \tag{2.14}$$

By applying the inverse $\mathcal{L}^{-1}$ of the Laplace transform we obtain:

$$\begin{cases}u_0(x,t)=\mathcal{L}^{-1}\left[\dfrac{1}{s}f(x)+\dfrac{1}{s^2}g(x)+\dfrac{1}{s^2}\mathcal{L}\left[h(x,t)\right]\right] \\ u_{n+1}(x,t)=-\mathcal{L}^{-1}\left[\dfrac{1}{s^2}\mathcal{L}\left[Ru_n(x,t)\right]+\dfrac{1}{s^2}\mathcal{L}\left[A_n(x,t)\right]\right];n\geq 0.\end{cases} \tag{2.15}$$

2.3. **Laplace Variational Iteration method (LVIM) [16].**

Consider the following nonlinear differential equation:

$$Lu(t)+Nu(t)=f(t), \tag{2.16}$$

where L and N denote respectively the linear and nonlinear operators and by $f(t)$ the given analytical function.

To begin with, we must first of all given the correction functional. Therefore the correction functional of the equation (2.16) is of the form

$$u_{n+1}(t)=u_n(t)+\int_0^t\lambda(\tau)\left(Lu_n(\tau)+N\tilde{u_n}(\tau)-f(\tau)\right)d\tau, \tag{2.17}$$

$\tau\in[0,\ t]$ and $n=0,1,2,\cdots$, where λ is a Lagrange multiplier which can be optimally identified by the theory of the calculus of variations, $\widetilde{u_n}$ is considered as the restricted variation which means that $\delta\,\widetilde{u_n}=0$ and $\delta f(t)=0$.

Each step of the integration for the determination of the Lagrange multiplier λ is fundamentally complicated. Moreover, integration is fundamentally convolution which motivates us to make use of the Laplace transformation.

By then applying the Laplace transformation to the equation (2.17) we have:

$$\mathcal{L}\left(u_{n+1}(t)\right)=\mathcal{L}\left(u_n(t)\right)+\mathcal{L}\left(\int_0^t \lambda(s)\left(Lu_n(s)+N\tilde{u_n}(s)-f(s)\right)ds\right). \tag{2.18}$$

In general, the Lagrange multiplier λ of the form $\lambda=\overline{\lambda}\,(t-s)$ Therefore the relation (2.18) becomes:

$$\mathcal{L}\left(u_{n+1}(t)\right)=\mathcal{L}\left(u_n(t)\right)+\mathcal{L}\left(\int_0^t \overline{\lambda}\,(t-s)\left(Lu_n(s)+N\tilde{u_n}(s)-f(s)\right)ds\right) \tag{2.19}$$

$$\Updownarrow$$

$$\mathcal{L}\left(u_{n+1}(t)\right)=\mathcal{L}\left(u_n(t)\right)+\mathcal{L}\left(\overline{\lambda}\,(t)*\left(Lu_n(t)+N\tilde{u_n}(t)-f(t)\right)\right) \tag{2.20}$$

$$\Updownarrow$$

$$\mathcal{L}\left(u_{n+1}(t)\right)=\mathcal{L}\left(u_n(t)\right)+\mathcal{L}\left(\overline{\lambda}\,(t)\right).\mathcal{L}\left(Lu_n(t)+N\tilde{u_n}(t)-f(t)\right) \tag{2.21}$$

To determine the optimal value of $\overline{\lambda}\,(t-s)$, we will first take the following variation $u_n(t)$. So

$$\frac{\delta}{\delta u_n}\mathcal{L}\left(u_{n+1}(t)\right)=\frac{\delta}{\delta u_n}\mathcal{L}\left(u_n(t)\right)+\frac{\delta}{\delta u_n}\mathcal{L}\left(\overline{\lambda}\,(t)\right).\mathcal{L}\left(Lu_n(t)+N\tilde{u_n}(t)-f(t)\right), \tag{2.22}$$

which give

$$\frac{\delta}{\delta u_n}\mathcal{L}\left(u_{n+1}(t)\right)=\frac{\delta}{\delta u_n}\mathcal{L}\left(u_n(t)\right)+\frac{\delta}{\delta u_n}\mathcal{L}\left(\overline{\lambda}\,(t)\right).\mathcal{L}\left(Lu_n(t)\right) \tag{2.23}$$

because $\delta\overline{u}_n=0$ et $\delta f(t)=0$

In the case where the linear operator L is defined by: $L=\frac{d}{dt}\,(.)$ We will have

$$\mathcal{L}\left(Lu_n(t)=s\mathcal{L}\left(Lu_n(t)\right)-u_0(t),\right. \tag{2.24}$$

and so

$$\delta\mathcal{L}\left(Lu_n(t)=s\mathcal{L}\left(L\delta u_n(t)\right)-\delta u_0(t),\quad \delta u_0(t)=0,\right. \tag{2.25}$$

$$\delta\mathcal{L}\left(Lu_n(t)=s\mathcal{L}\left(L\delta u_n(t)\right).\right. \tag{2.26}$$

Thereby,

$$\delta\mathcal{L}\left(u_{n+1}(t)\right)=\delta\mathcal{L}\left(u_n(t)\right)+\mathcal{L}\left(\overline{\lambda}\,(t)\right).\mathcal{L}\left(L\delta u_n(t)\right), \tag{2.27}$$

that is

$$\mathcal{L}\left(\delta u_{n+1}(t)\right) = \left(1 + \mathcal{L}\left(\overline{\lambda}\left(t\right)\right)\right) \mathcal{L}\left(L\delta u_n(t)\right). \tag{2.28}$$

The extreme condition of u_{n+1} requires that $\delta u_{n+1} = 0$, which implies that $\mathcal{L}\left(\delta u_{n+1}(t)\right) = 0$ therefore,

$$\left(1 + s\mathcal{L}\left(\overline{\lambda}\left(t\right)\right)\right) \mathcal{L}\left(L\delta u_n(t)\right) = 0, \quad L\left(\delta u_n(t)\right) \neq 0, \tag{2.29}$$

so

$$1 + s\mathcal{L}\left(\overline{\lambda}\left(t\right)\right) = 0 \Longrightarrow \mathcal{L}\left(\overline{\lambda}\left(t\right)\right) = -\frac{1}{s}. \tag{2.30}$$

From this value of $\mathcal{L}\left(\overline{\lambda}\left(t\right)\right)$, we have the iteration formulation

$$\mathcal{L}\left(u_{n+1}(t)\right) = \mathcal{L}\left(u_n(t)\right) - \frac{1}{s}\mathcal{L}\left(L\left(u_n(t)\right) + N\left(u(t)\right) - f(t)\right) \tag{2.31}$$

$$\Updownarrow$$

$$u_{n+1}(t) = u_n(t) - \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}\left(L\left(u_n(t)\right) + N\left(u(t)\right) - f(t)\right)\right). \tag{2.32}$$

3. Test example

In this section, we present some examples with analytical solution to show the efficiency of method described in previons section for solving equation (1.1)

3.1. **Example 1.**

Consider a nonlinear Boussinesq equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = -2\frac{\partial^2}{\partial x^2}\left(N(u(x,t)) + 3u(x,t)\right) + \frac{\partial^4}{\partial x^4}u(x,t), \tag{3.1}$$

with initial conditions

$$\begin{cases} u(x,0) = e^x \\ \dfrac{\partial u(x,0)}{\partial t} = 2e^x \end{cases}, \tag{3.2}$$

where

$$N(u(x,t)) = \ln\left(u(x,t)\right).$$

3.1.1. ***The Laplace Decomposition Method.***

Applying the Laplace transform (denoted by $\mathcal{L}$) we have

$$\begin{aligned}\mathcal{L}\left(u(x,t)\right) &= \frac{1}{s}u(x,0) + \frac{1}{s^2}\frac{\partial u(x,0)}{\partial t} - \frac{2}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(N(u(x,t))\right)\right) \\ &- \frac{6}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(u(x,t)\right)\right) + \frac{1}{s^2}\mathcal{L}\left(\frac{\partial^4}{\partial x^4}u(x,t)\right).\end{aligned} \tag{3.3}$$

Using initial conditions Eqs (3.3) becomes

$$\begin{aligned}\mathcal{L}\left(u(x,t)\right) &= \left(\frac{1}{s} + \frac{2}{s^2}\right)e^x - \left(\frac{2}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(N(u(x,t))\right)\right)\right) \\ &- \left(\frac{6}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(u(x,t)\right)\right)\right) + \left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^4}{\partial x^4}u(x,t)\right)\right).\end{aligned} \tag{3.4}$$

Applying inverse Laplace transform we get

$$\begin{aligned}u(x,t) &= \left(2t+1\right)e^x - \mathcal{L}^{-1}\left(\frac{2}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(N(u(x,t))\right)\right)\right) \\ &- \mathcal{L}^{-1}\left(\frac{6}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(u(x,t)\right)\right)\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^4}{\partial x^4}u(x,t)\right)\right).\end{aligned} \tag{3.5}$$

Since the principle of the method consists in giving the solution in the form of an infinite series as defined in (2.10) and the nonlinear term $N\left(u(x,t)\right)$ as defined in (2.11). By replacing the relations (2.10) and (2.11) in (3.5) we have:

$$\begin{aligned}\sum_{n=0}^{\infty}u_n(x,t) &= \left(2t+1\right)e^x + \sum_{n=0}^{\infty}\left(-\mathcal{L}^{-1}\left(\frac{2}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(A_n(x,t)\right)\right)\right)\right. \\ &\left.-\mathcal{L}^{-1}\left(\frac{6}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(u_n(x,t)\right)\right)\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^4}{\partial x^4}u_n(x,t)\right)\right)\right).\end{aligned} \tag{3.6}$$

The recursive relation is defined by

$$\begin{cases} u_0(x,t) = \left(2t+1\right)e^x \\ u_n(x,t) = -v^{-1}\left(\frac{2}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(A_n(x,t)\right)\right)\right) \\ \qquad - \mathcal{L}^{-1}\left(\frac{6}{s^2}\mathcal{L}\left(\frac{\partial^2}{\partial x^2}\left(u_n(x,t)\right)\right)\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^4}{\partial x^4}u_n(x,t)\right)\right). \end{cases} \tag{3.7}$$

For the nonlinear term

$$\begin{cases} A_0(x,t) = \ln\left(u_0(x,t)\right) \\ A_1(x,t) = \dfrac{u_1(x,t)}{u_0(x,t)} \\ A_2(x,t) = \dfrac{u_2(x,t)}{u_0(x,t)} - \dfrac{1}{2}\dfrac{u_1^2(x,t)}{u_0^2(x,t)} \\ \qquad \vdots \end{cases} . \tag{3.8}$$

From the algorithm (3.7), the iterations are

$$\begin{cases} A_0(x,t) = \ln\left((2t+1)\right) + x \\ u_1(x,t) = -\dfrac{5}{3}t^3e^x - \dfrac{5}{2}t^2e^x \\ A_1(x,t) = -5\dfrac{t^2\left(2t+3\right)}{12t+6} \\ u_2(x,t) = \left(\dfrac{5}{12}t^5 + \dfrac{25}{24}t^4\right)e^x = \dfrac{50t^5}{5!}e^x + \dfrac{25t^4}{4!}e^x \\ A_2(x,t) = -\dfrac{5}{36}\dfrac{t^4\left(4t^2+12t+15\right)}{\left(2t+1\right)^2} \\ u_3(x,t) = -\dfrac{125t^6}{6!}e^x - \dfrac{250t^7}{7!}e^x \\ \qquad \vdots \end{cases} , \tag{3.9}$$

and we obtain, the series form of the approximate solution as

$$\begin{aligned} &u(x,t) \\ &= u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots \\ &= 2te^x + e^x + -\frac{5}{3}t^3e^x - \frac{5}{2}t^2e^x + \frac{50t^5}{5!}e^x + \frac{25t^4}{4!}e^x + -\frac{125t^6}{6!}e^x - \frac{250t^7}{7!}e^x + \cdots \\ &= \left(1 - \frac{5}{2}t^2 + \frac{25t^4}{4!} - \frac{125t^6}{6!} + \cdots\right)e^x + \left(2t - \frac{5}{3}t^3 + \frac{50t^5}{5!} - \frac{250t^7}{7!} + \cdots\right)e^x \\ &= \left(1 - \frac{5}{2}t^2 + \frac{25t^4}{24} - \frac{25}{144}t^6 + \cdots\right)e^x + \left(2t - \frac{5}{3}t^3 + \frac{5}{12}t^5 - \frac{25}{504}t^7 + \cdots\right)e^x \\ &u(x,t) \\ &= \cos\left(\sqrt{5}t\right)e^x + \frac{2\sqrt{5}}{5}\sin\left(\sqrt{5}t\right)e^x. \end{aligned} \tag{3.10}$$

That is

$$u(x,t) = e^{x}\left(\cos\left(\sqrt{5}t\right) + \frac{2\sqrt{5}}{5}\sin\left(\sqrt{5}t\right)\right). \tag{3.11}$$

3.1.2. ***The LVIM***.

The Laplace variational iteration correction functional will be constructed in the following manner:

$$\begin{aligned} \mathcal{L}\left(u_{n+1}(x,t)\right) = \mathcal{L}\left(u_n(x,t)\right) + \mathcal{L}\Bigg(\int_0^x \overline{\lambda}(t-s)\Bigg(\frac{\partial^2 u_n(x,s)}{\partial s^2} \\ +2\frac{\partial^2}{\partial x^2}\left(N(\overline{u}_n(x,s)+3\overline{u_n}(x,s)\right) - \frac{\partial^4}{\partial x^4}\overline{u_n}(x,s)\Bigg)\,ds\Bigg), \end{aligned} \tag{3.12}$$

or equivalently, upon applying the properties of Laplace transform, we have

$$\begin{aligned} \mathcal{L}\left(u_{n+1}(x,t)\right) &= \mathcal{L}\left(u_n(x,t)\right) + \mathcal{L}\Bigg(\overline{\lambda}(t) * \Bigg(\frac{\partial^2 u_n(x,t)}{\partial t^2} \\ &\quad +2\frac{\partial^2}{\partial x^2}\left(N\left(\overline{u}_n(x,t)+3\overline{u}_n(x,t)\right) - \frac{\partial^4}{\partial x^4}\overline{u}_n(x,t)\right)\Bigg)\Bigg) \\ &= \mathcal{L}\left(u_n(x,t)\right) + \mathcal{L}\left(\overline{\lambda}(t)\right).\mathcal{L}\Bigg(\frac{\partial^2 u_n(x,t)}{\partial t^2} \\ &\quad + 2\frac{\partial^2}{\partial x^2}\left(N\left(\overline{u}_n(x,t)+3\overline{u}_n(x,t)\right) - \frac{\partial^4}{\partial x^4}\overline{u}_n(x,t)\right)\Bigg). \end{aligned} \tag{3.13}$$

Tanking the variation with respect to $u_n(x,t)$ and making the above correction functional stationary, noting that $\delta\overline{u}_n(x,t) = 0$, we have

$$\begin{aligned} \mathcal{L}\left(\delta u_{n+1}(x,t)\right) = \mathcal{L}\left(\delta u_n(x,t)\right) + \mathcal{L}\left(\overline{\lambda}(t)\right).\Big(s^2\mathcal{L}\left(\delta u_n(x,t)\right) \\ -s\delta u_n(x,0) - \frac{\partial \delta u_n(x,0)}{\partial t}\Big). \end{aligned} \tag{3.14}$$

Taking $\delta u_n(x,0) = \dfrac{\partial \delta u_n(x,0)}{\partial t} = 0$ we have

$$\mathcal{L}\left(\delta u_{n+1}(x,t)\right) = \left(1 + s^2\mathcal{L}\left(\overline{\lambda}(t)\right)\right)\mathcal{L}\left(\delta u_n(x,t)\right). \tag{3.15}$$

This implies that

$$1 + s^2\mathcal{L}\left(\overline{\lambda}(t)\right) = 0, \tag{3.16}$$

that is

$$\mathcal{L}\left(\overline{\lambda}(t)\right) = -\frac{1}{s^2}. \tag{3.17}$$

Substituting equation (3.17) into (3.13) yields the iteration scheme:

$$\begin{aligned}\mathcal{L}\left(u_{n+1}(x,t)\right) = \mathcal{L}\left(u_n(x,t)\right) - \frac{1}{s^2}\mathcal{L}\left(\frac{\partial^2 u_n(x,t)}{\partial t^2} + 2\frac{\partial^2}{\partial x^2}\left(N\left(u_n(x,t)\right.\right.\right.\\ \left.\left.+3u_n(x,t)\right) - \frac{\partial^4}{\partial x^4}u_n(x,t)\right).\end{aligned} \tag{3.18}$$

Applying inverse Laplace transform we get

$$\begin{aligned}u_{n+1}(x,t) = u_n(x,t) - \mathcal{L}^{-1}\left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^2 u_n(x,t)}{\partial t^2} + 2\frac{\partial^2}{\partial x^2}\left(N\left(u_n(x,t)\right.\right.\right.\right.\\ \left.\left.\left.+3u_n(x,t)\right) - \frac{\partial^4}{\partial x^4}u_n(x,t)\right)\right).\end{aligned} \tag{3.19}$$

We can use the initials conditions to select $u_0(x,t) = (2t+1)\,e^x$.Using this selection into the correction functional gives the following successive approximations

$$\begin{cases} u_0(x,t) = e^x + 2te^x \\ u_1(x,t) = \left(1 - \frac{5}{2}t^2\right)e^x + \left(2t - \frac{5}{3}t^3\right)e^x \\ u_2(x,t) = \left(1 - \frac{5}{2}t^2 + \frac{25}{24}t^4\right)e^x + \left(2t - \frac{5}{3}t^3 + \frac{5}{12}t^5\right)e^x \\ u_3(x,t) = \left(1 - \frac{5}{2}t^2 + \frac{25}{24}t^4 - \frac{25}{144}t^6\right)e^x + \left(2t - \frac{5}{3}t^3 + \frac{5}{12}t^5 - \frac{25}{504}t^7\right)e^x \\ \quad\vdots \\ u_n(x,t) = \left(1 - \frac{5}{2}t^2 + \frac{25}{24}t^4 - \frac{25}{144}t^6 + \cdots\right)e^x \\ \qquad\qquad + \left(2t - \frac{5}{3}t^3 + \frac{5}{12}t^5 - \frac{25}{504}t^7 + \cdots\right)e^x \end{cases} \tag{3.20}$$

And so on for other approximations. The LVIM admis the use of

$$u(x,t) = \lim_{n\longrightarrow+\infty} u_n(x,t). \tag{3.21}$$

This gives the following approximation solution

$$
\begin{aligned}
u(x,t) &= \left(1 - \frac{5}{2}t^2 + \frac{25}{24}t^4 - \frac{25}{144}t^6 + \cdots\right)e^x \\
&\quad + \left(2t - \frac{5}{3}t^3 + \frac{5}{12}t^5 - \frac{25}{504}t^7 + \cdots\right)e^x \\
&= e^x\left(\cos\left(\sqrt{5}t\right)\right) + e^x\left(\frac{2\sqrt{5}}{5}\sin\left(\sqrt{5}t\right)\right).
\end{aligned} \tag{3.22}
$$

Therefore, we obtain

$$
u(x,t) = e^x\left(\cos\left(\sqrt{5}t\right) + \frac{2\sqrt{5}}{5}\sin\left(\sqrt{5}t\right)\right), \tag{3.23}
$$

which is the exact solution this problem

3.1.3. ***Partial Conclusion.***

We find that we have the same result in both approaches.

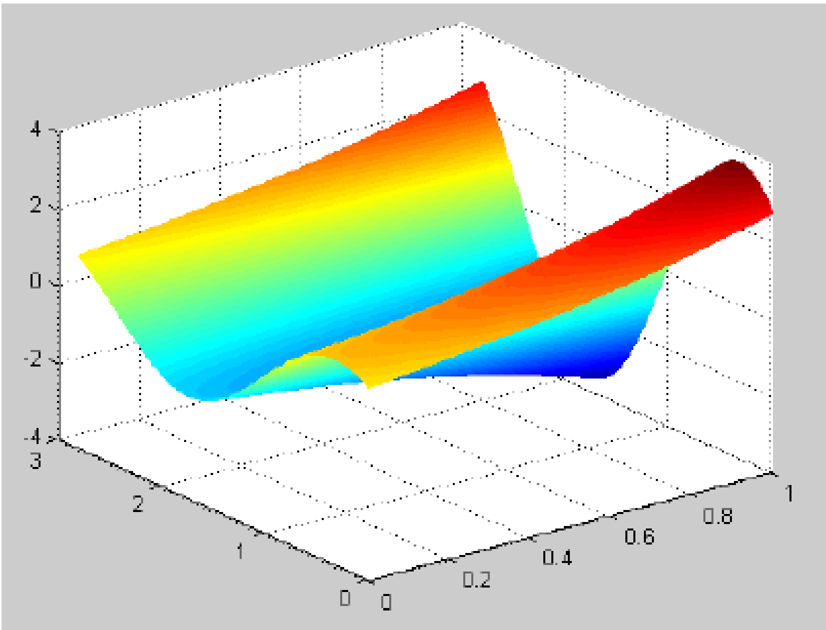

FIGURE 1. *Numerical simulation of the solution of example 1 for* $x \in [0,1]$ *and* $t \in \left[0, \frac{2\pi}{\sqrt{5}}\right]$.

3.2. Example 2:

Consider the nonlinear Boussinesq equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = -2\frac{\partial^2}{\partial x^2}\left(u^3(x,t) - \frac{1}{2}u(x,t)\right) + \frac{\partial^4}{\partial x^4}u(x,t) \tag{3.24}$$

subject to the initial conditions:

$$\begin{cases} u(x,0) = \dfrac{1}{x} \\ \dfrac{\partial u(x,0)}{\partial t} = -\dfrac{1}{x^2} \end{cases}. \tag{3.25}$$

3.2.1. ***The LADM***.

The equation (3.24) can be written

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial^2 u(x,t)}{\partial x^2} + \frac{\partial^4 u(x,t)}{\partial x^4} - 2\frac{\partial^2 N(u(x,t))}{\partial x^2} \tag{3.26}$$

with

$$N(u(x,t)) = u^3(x,t). \tag{3.27}$$

Apply the Laplace transform with respect to the variable t of equation (3.24) we obtain

$$\begin{aligned} \mathcal{L}\left(u(x,t)\right) = &\left(\frac{1}{s}\frac{1}{x} - \frac{1}{s^2}\frac{1}{x^2}\right) + \left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)\right) + \left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^4 u(x,t)}{\partial x^4}\right)\right) \\ &- \frac{2}{s^2}\mathcal{L}\left(\frac{\partial^2 N(u(x,t))}{\partial x^2}\right). \end{aligned} \tag{3.28}$$

Applying the inverse Laplace transform gives us

$$\begin{aligned} u(x,t) = &\frac{1}{x} - \frac{t}{x^2} + \mathcal{L}^{-1}\left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^4 u(x,t)}{\partial x^4}\right)\right) \\ &- \mathcal{L}^{-1}\left(\frac{2}{s^2}\mathcal{L}\left(\frac{\partial^2 N(u(x,t))}{\partial x^2}\right)\right). \end{aligned} \tag{3.29}$$

By substituting (2.10) and (2.11) in (3.29), we become

$$\sum_{n=0}^{+\infty} u_n(x,t) = \frac{1}{x} - \frac{t}{x^2} + \sum_{n=0}^{+\infty}\mathcal{L}^{-1}\left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^2 u_n(x,t)}{\partial x^2}\right)\right)$$

$$+\sum_{n=0}^{+\infty}\mathcal{L}^{-1}\left(\frac{1}{s^2}\mathcal{L}\left(\frac{\partial^4 u_n(x,t)}{\partial x^4}\right)\right)-\int_{n=0}^{+\infty}\mathcal{L}^{-1}\left(\frac{2}{s^2}\mathcal{L}\left(\frac{\partial^2 A_n(x,t)}{\partial x^2}\right)\right). \tag{3.30}$$

Now, (3.30) gives the following algorithm:

$$\begin{cases} u_0(x,t)=\dfrac{1}{x} \\ u_1(x,t)=-\dfrac{t}{x^2}+\mathcal{L}^{-1}\left(\dfrac{1}{s^2}\mathcal{L}\left(\dfrac{\partial^2 u_0(x,t)}{\partial x^2}\right)\right) \\ +\mathcal{L}^{-1}\left(\dfrac{1}{s^2}\mathcal{L}\left(\dfrac{\partial^4 u_0(x,t)}{\partial x^4}\right)\right)-\mathcal{L}^{-1}\left(\dfrac{2}{s^2}\mathcal{L}\left(\dfrac{\partial^2 A_0(x,t)}{\partial x^2}\right)\right) \\ u_{n+1}(x,t)=\mathcal{L}^{-1}\left(\dfrac{1}{s^2}\mathcal{L}\left(\dfrac{\partial^2 u_n(x,t)}{\partial x^2}\right)\right) \\ +\mathcal{L}^{-1}\left(\dfrac{1}{s^2}\mathcal{L}\left(\dfrac{\partial^4 u_n(x,t)}{\partial x^4}\right)\right)-\mathcal{L}^{-1}\left(\dfrac{2}{s^2}\mathcal{L}\left(\dfrac{\partial^2 A_n(x,t)}{\partial x^2}\right)\right). \end{cases} \tag{3.31}$$

For the nonlinear term

$$\begin{cases} A_0=u_0^3 \\ A_1=3u_0^2u_1 \\ A_2=3u_0^2u_2+3u_0u_1^2 \\ A_3=u_1^3+3u_0^2u_3+6u_0u_1u_2. \end{cases} \tag{3.32}$$

From the algorithm (3.31), the iterations are:

$$\begin{aligned} u_0(x,t)&=\frac{1}{x}, \quad A_0=\frac{1}{x^3} \\ u_1(x,t)&=-\frac{t}{x^2}+\frac{t^2}{x^3}, \quad A_1=\frac{3t^2}{x^5}-\frac{3t}{x^4} \\ u_2(x,t)&=-\frac{t^3}{x^4}+\frac{t^4}{x^5}+15\frac{t^4}{x^7}, \quad A_2=3\frac{t^2}{x^5}-9\frac{t^3}{x^6}+6\frac{t^4}{x^7}+45\frac{t^4}{x^9} \\ u_3(x,t)&=-\frac{t^5}{x^6}+\frac{t^6}{x^7}-15\frac{t^4}{x^7}-\frac{21}{5}\frac{t^5}{x^8}+\frac{308}{5}\frac{t^6}{x^9}+2250\frac{t^6}{x^{11}} \\ A_3&=-\frac{1}{5}\frac{t^3}{x^{13}}\left(-50t^3x^4-1374t^3x^2-33\,750t^3+90t^2x^5\right. \\ &\quad\left.+513t^2x^3-45tx^6+225tx^4+5x^7\right) \end{aligned} \tag{3.33}$$

$$
\begin{aligned}
u_4(x,t) = &-\frac{t^7}{x^8} + \frac{t^8}{x^9} + \frac{21}{5}\frac{t^5}{x^8} - \frac{308}{5}\frac{t^6}{x^9} - 2250\frac{t^6}{x^{11}} - \frac{612}{35}\frac{t^7}{x^{10}} \\
&- \frac{1782}{7}\frac{t^7}{x^{12}} + \frac{1098}{7}\frac{t^8}{x^{11}} + \frac{597\,663}{35}\frac{t^8}{x^{13}} + 921\,375\frac{t^8}{x^{15}}
\end{aligned}
\tag{3.34}
$$

$$\vdots$$

Afterwards

$$
\begin{aligned}
&u(x,t) \\
&= u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots \\
&= \left(\frac{1}{x} - \frac{t}{x^2} + \frac{t^2}{x^3} - \frac{t^3}{x^4} + \frac{t^4}{x^5} - \frac{t^5}{x^6} + \frac{t^6}{x^7} - \frac{t^7}{x^8} + \frac{t^8}{x^9} + \cdots\right) \\
&+ 15\frac{t^4}{x^7} - 15\frac{t^4}{x^7} - \frac{21}{5}\frac{t^5}{x^8} + \frac{21}{5}\frac{t^5}{x^8} + \frac{308}{5}\frac{t^6}{x^9} - \frac{308}{5}\frac{t^6}{x^9} \\
&+ 2250\frac{t^6}{x^{11}} - 2250\frac{t^6}{x^{11}} - \frac{612}{35}\frac{t^7}{x^{10}} - \frac{1782}{7}\frac{t^7}{x^{12}} + \frac{1098}{7}\frac{t^8}{x^{11}} \\
&+ \frac{597663}{35}\frac{t^8}{x^{13}} + 921\,375\frac{t^8}{x^{15}} + \cdots = \frac{1}{x+t} \\
&+ \underbrace{-\frac{612}{35}\frac{t^7}{x^{10}} - \frac{1782}{7}\frac{t^7}{x^{12}} + \frac{1098}{7}\frac{t^8}{x^{11}} + \frac{597\,663}{35}\frac{t^8}{x^{13}} + 921\,375\frac{t^8}{x^{15}} + \cdots}_{\text{small terms}}.
\end{aligned}
\tag{3.35}
$$

That is

$$u(x,t) = \frac{1}{x+t} - \text{small terms}. \tag{3.36}$$

From where

$$u(x,t) = \frac{1}{x+t}, \tag{3.37}$$

which is the exact solution of (3.24) and (3.25).

3.2.2. ***The Laplace Variational Iteration Method.***

The correction functional for (3.24) reads

$$
\begin{aligned}
u_{n+1}(x,t) = u_n(x,t) + \int_0^x \overline{\lambda}(t-s)\Bigg(&\frac{\partial^2 u_n(x,s)}{\partial s^2} \\
&+2\frac{\partial^2}{\partial x^2}\left(\overline{u}_n^3(x,s) - \frac{1}{2}\overline{u}_n(x,s)\right) - \frac{\partial^4}{\partial x^4}\overline{u}_n(x,s)\Bigg)\, ds.
\end{aligned}
\tag{3.38}
$$

By applying the Laplace transformation to (3.38) we have:

$$
\begin{aligned}
\mathcal{L}\left(u_{n+1}(x,t)\right) &= \mathcal{L}\left(u_n(x,t)\right) + \mathcal{L}\left(\int_0^x \overline{\lambda}(t-s)\left(\frac{\partial^2 u_n(x,s)}{\partial s^2}\right.\right.\\
&\left.\left. +2\frac{\partial^2}{\partial x^2}\left(\overline{u}_n^3(x,s) - \frac{1}{2}\overline{u}_n(x,s)\right) - \frac{\partial^4}{\partial x^4}\overline{u}_n(x,s)\right) ds\right)\\
&= \mathcal{L}\left(u_n(x,t)\right) + \mathcal{L}\left(\overline{\lambda}(t) * \left(\frac{\partial^2 u_n(x,t)}{\partial t^2}\right.\right.\\
&\left.\left. +2\frac{\partial^2}{\partial x^2}\left(\overline{u}_n^3(x,t) - \frac{1}{2}\overline{u}_n(x,t)\right) - \frac{\partial^4}{\partial x^4}\overline{u}_n(x,t)\right)\right)\\
&= \mathcal{L}\left(u_n(x,t)\right) + \mathcal{L}\left(\overline{\lambda}(t)\right).\mathcal{L}\left(\frac{\partial^2 u_n(x,t)}{\partial t^2} + 2\frac{\partial^2}{\partial x^2}\left(\overline{u}_n^3(x,t)\right.\right.\\
&\left.\left. -\frac{1}{2}\overline{u}_n(x,t)\right) - \frac{\partial^4}{\partial x^4}\overline{u}_n(x,t)\right).
\end{aligned}
\tag{3.39}
$$

By taking the variation relative to u_n on a:

$$
\begin{aligned}
\delta\mathcal{L}\left(u_{n+1}(x,t)\right) &= \delta\mathcal{L}\left(u_n(x,t)\right) + \mathcal{L}\left(\overline{\lambda}(t)\right).\mathcal{L}\left(\delta\frac{\partial^2 u_n(x,t)}{\partial t^2}\right.\\
&\left. +2\frac{\partial^2}{\partial x^2}\left(\delta\overline{u}_n^3(x,t) - \frac{1}{2}\delta\overline{u}_n(x,t)\right) - \frac{\partial^4}{\partial x^4}\delta\overline{u}_n(x,t)\right).
\end{aligned}
\tag{3.40}
$$

with $\delta\overline{u}(x,t) = 0$ we have

$$
\begin{aligned}
\mathcal{L}\left(\delta u_{n+1}(x,t)\right) &= \mathcal{L}\left(\delta u_n(x,t)\right) + \mathcal{L}\left(\overline{\lambda}(t)\right).\left(s^2\mathcal{L}\left(\delta u_n(x,t)\right)\right.\\
&\left. -s\delta u_n(x,0) - \delta\frac{\partial u_n(x,0)}{\partial t}\right)
\end{aligned}
\tag{3.41}
$$

$$\Updownarrow$$

$$
\mathcal{L}\left(\delta u_{n+1}(x,t)\right) = \left(1 + s^2\mathcal{L}\left(\overline{\lambda}(t)\right)\right)\mathcal{L}\left(\delta u_n(x,t)\right)
\tag{3.42}
$$

because $\delta u_n(x,0) = \delta\frac{\partial u_n(x,0)}{\partial t} = 0$.

By taking the stationary correction functional one obtains

$$
1 + s^2\mathcal{L}\left(\overline{\lambda}(t)\right) = 0.
\tag{3.43}
$$

That is

$$
\mathcal{L}\left(\overline{\lambda}(t)\right) = -\frac{1}{s^2}.
\tag{3.44}
$$

Substituting this value of the Lagrangian multiplier into functional (3.38) gives the iteration

$$\begin{aligned}\mathcal{L}\left(u_{n+1}(x,t)\right) = \mathcal{L}\left(u_n(x,t)\right) - \frac{1}{s^2}\mathcal{L}\Bigg(\frac{\partial^2 u_n(x,t)}{\partial t^2} \\ +2\frac{\partial^2}{\partial x^2}\left(u_n^3(x,t) - \frac{1}{2}u_n(x,t)\right) - \frac{\partial^4}{\partial x^4}u_n(x,t)\Bigg)\end{aligned} \tag{3.45}$$

$$\Updownarrow$$

$$\begin{aligned}u_{n+1}(x,t) = u_n(x,t) - \mathcal{L}^{-1}\Bigg(\frac{1}{s^2}\mathcal{L}\Bigg(\frac{\partial^2 u_n(x,t)}{\partial t^2} \\ +2\frac{\partial^2}{\partial x^2}\left(u_n^3(x,t) - \frac{1}{2}u_n(x,t)\right) - \frac{\partial^4}{\partial x^4}u_n(x,t)\Bigg)\Bigg).\end{aligned} \tag{3.46}$$

The given initial values admit the use

$$u_0(x,t) = \frac{1}{x} - \frac{1}{x^2}t. \tag{3.47}$$

Using (3.46) we obtain the following successive approximations:

$$\begin{cases} u_1(x,t) = \frac{1}{x} - \frac{1}{x^2}t + \frac{1}{x^3}t^2 - \frac{1}{x^4}t^3 + \text{small terms} \\ u_2(x,t) = \frac{1}{x} - \frac{1}{x^2}t + \frac{1}{x^3}t^2 - \frac{1}{x^4}t^3 + \frac{1}{x^5}t^4 - \frac{1}{x^6}t^5 + \text{small terms} \\ u_3(x,t) = \frac{1}{x} - \frac{1}{x^2}t + \frac{1}{x^3}t^2 - \frac{1}{x^4}t^3 + \frac{1}{x^5}t^4 - \frac{1}{x^6}t^5 + \frac{t^6}{x^7} - \frac{t^8}{x^8} \\ \qquad + \frac{t^8}{x^9} + \text{small terms} \\ \quad \vdots \\ u_n(x,t) = \frac{1}{x} - \frac{1}{x^2}t + \frac{1}{x^3}t^2 - \frac{1}{x^4}t^3 + \frac{1}{x^5}t^4 - \frac{1}{x^6}t^5 + \frac{t^6}{x^7} - \frac{t^7}{x^8} + \frac{t^8}{x^9} + \cdots \end{cases}, \tag{3.48}$$

and in a closed form by

$$u(x,t) = \frac{1}{x+t}. \tag{3.49}$$

3.2.3. ***Partial Conclusion.***

We end up with the same solutions using the Laplace Adomian method and the Laplace variational iteration method

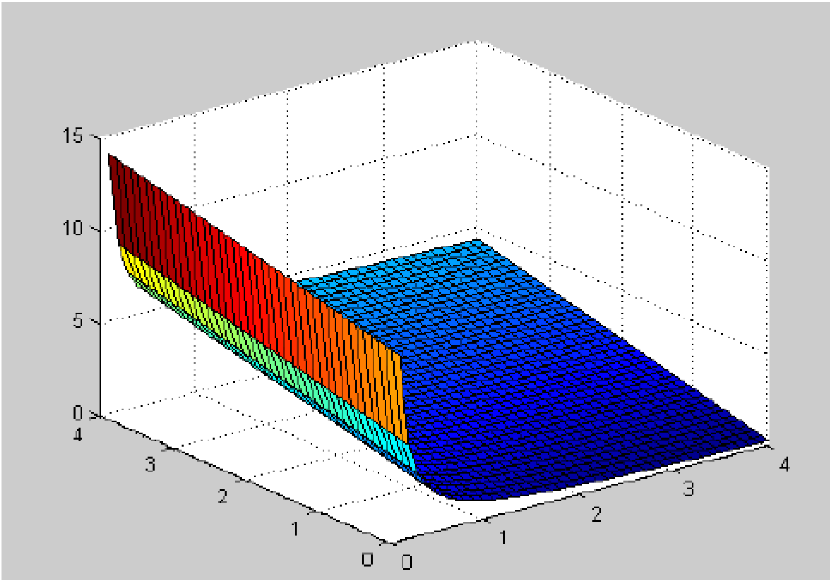

FIGURE 2. *Numerical simulation of the solution of example 2 for* $x \in [0, 4]$ *and* $t \in [0, 4]$

4. CONCLUSION

The main objective of this work is to conduct a comparative study between the LADM method and the LVIM method. Both methods are powerful and efficient methods that both give higher accuracy approximations and closed form solutions if they exist. An important conclusion can be drawn here. The LVIM method gives several successive approximations using iteration of the correction functional. However, the LADM method provides the components of the exact solution where these components must follow the summation given in (2.10). Furthermore, the LVIM method requires the evaluation of the Lagrange multiplier, while LADM requires the evaluation of the A domian polynomials, which mostly require tedious algebraic calculations. It is interesting to note that, unlike the successive approximations obtained by LVIM, the A domian polynomials are not taken into account, successive obtained by LVIM, LADM provides the solution in successive components that will be summed to obtain the serial solution. Most importantly, LVIM reduces the computational volume by not requiring the A domian polynomials, making the iteration straightforward and simple. However, LADM requires the use of A domian polynomials for nonlinear terms, which requires more extensive work. For nonlinear equations that appear frequently to express nonlinear phenomena, the LVIM method facilitates the computational work and gives the solution quickly compared to the LADM method.

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