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STABILITY AND BOUNDEDNESS ANALYSIS OF A STATE–DEPENDENT DIFFERENTIAL EQUATIONS FOR A SYSTEM OF TWO COUPLED CIRCUITS

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ABSTRACT. Two coupled circuits are extensively used in radio electronics and communication. The problem of stability analysis of state variables describing system of two coupled circuits is very critical as unstable circuit causes damage to electrical systems. Analysis of stability and boundedness behavior of the state variables characterizing system of two coupled circuits is carried out using the Lyapunov's second method. We provide in simple form, less restrictive conditions that are implementable at the development stage and which ensure the stability and boundedness of the state variables describing system considered. For illustration, the behaviours of the system of two coupled circuits with response and its bounded output are shown.

1. INTRODUCTION

The role which Lyapunov theory plays in the analysis of stability and boundedness of differential equations, dynamical systems and models of natural phenomena cannot be disputed. Though it goes without saying that stability and boundedness is a very important problem of electrical systems, Lyapunov theory is less visible. Lyapunov's second (or direct) method allows us to predict the stability and boundedness behavior of state variables characterizing the systems of

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electrical circuits. Lyapunov functional approach remains an excellent tool in the study of dynamical systems (see also Qin et al [9]). However, the construction of these Lyapunov functionals is in indeed a general problem as there is no particular acceptable method for obtaining it. (see [2], [4], and [10]). The problem of stability and boundedness of state variables describing the system of coupled circuits are very important in radio electronics and communication. The qualitative properties of such system is more complicated to analyse. For example, Biryuk et al [1] considered a system parametrical system of two coupled circuits with external conductive connection which is represented by a system of differential equations given by

$$
\begin{aligned}
\frac{dq_1}{dt} &= -\left(G_1 + \frac{1}{R+R^{(1)}+R^{(2)}}\right)\frac{q_1}{C_1} - \frac{R+R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_1}{L_1} \\
&\quad + \frac{R^2}{R+R^1+R^2}\frac{\Phi_2}{L_2} + \frac{1}{R+R^{(1)}+R^{(2)}}\frac{q_2}{C_2} + j_1 + \frac{-\xi_1+\xi_2}{R+R^{(1)}+R^{(2)}} \\
\frac{d\Phi_1}{dt} &= \frac{R+R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{q_1}{C_1} - \left(R_1 + R^{(1)}\frac{R+R^{(2)}}{R+R(1)+R^{(2)}}\right)\frac{\Phi_1}{L_1} \\
&\quad - \frac{R^{(1)}R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_2}{L_2} + \frac{R^{(1)}}{R+R^{(1)}+R^{(2)}}\frac{q_2}{C_2} \\
&\quad + \frac{(R+R^2)\xi_1 R^{(1)}\xi_2}{R+R^{(1)}+R^{(2)}} \\
\frac{d\Phi_2}{dt} &= \frac{R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{q_1}{C_1} - \frac{R^{(1)}R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_1}{L_1} \\
&\quad - \left(R_2 + R^{(2)}\frac{R+R^{(1)}}{R+R^{(1)}+R^{(2)}}\right)\frac{\Phi_2}{L_2} \\
&\quad + \frac{R+R^{(1)}}{R+R^{(1)}+R^{(2)}}\frac{q_2}{C_2} + \frac{R^{(2)}\xi_1 + (R+R^{(1)})\xi_2}{R+R^{(1)}+R^{(2)}} \\
\frac{dq_2}{dt} &= \frac{1}{R+R^{(1)}+R^{(2)}}\frac{q_1}{C_1} - \frac{R^{(1)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_1}{L_1} + \frac{R+R^{(1)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_2}{L_2} \\
&\quad - \left(G_2 + \frac{1}{R+R^{(1)}+R^{(2)}}\right)\frac{q_2}{C_2} + j_2 + \frac{\xi_1-\xi_2}{R+R^{(1)}+R^{(2)}}.
\end{aligned} \tag{1.1}
$$

They introduced a suitable Lyapunov function and obtained sufficient conditions for the stability of system (1.1), a natural process, a homogenous equation in (1.1) where the system is not connected to any external connection or element. This is an interesting result since (1.1) is rather a non-changing system without

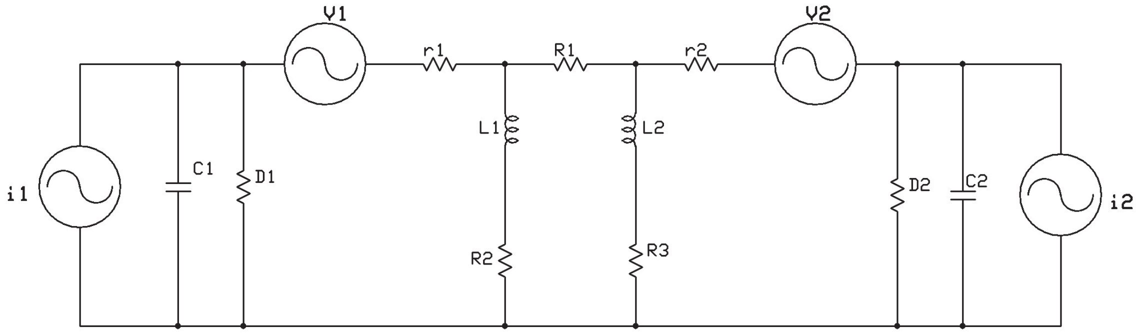

FIGURE 1. *Two coupled circuits*

response. The case where the system (1.1) is connected to external connection or element was left open. The proposed Lyapunov functional in [1] does not posses a functional relationship to the original system equation (1.1) and the effect of the heterogenous equation on the system of two coupled circuits was not considered which raised the present analysis where a Lyapunov functional generation technique is constructed which posses a functional relationship to the same original system considered with external connection or element where the effect of a continuously changing system over time with response is also analysed. Works on the application of Lyapunov functional approach to the analysis of the critical variables describing electrical circuits system are scarcely few. For example, very recently, Olutimo et al [6] obtained sufficient conditions for the boundedness of state variables for a loaded DC servo motor system. Lenka [3] established new stability conditions for certain class of non-autonomous fractional order systems using time varying Lyapunov functions. Also, Olutimo and Omoko [5] obtained the stability and boundedness of state variables of system of RLC circuit as well as Olutimo and Adams [7] who gave sufficient conditions for the stability and boundedness of system of delay differential equations.

$$\frac{dq_1}{dt} = -\left(D_1 + \frac{1}{R_1 + r_1 + r_2}\right)\frac{q_1}{C_1} - \frac{R_1 + r_2}{R_1 + r_1 + r_2}\frac{\phi_1}{L_1} + \frac{r_2}{R_1 + r_1 + r_2}\frac{\phi_2}{L_2}$$
$$+ \frac{1}{R_1 + r_1 + r_2}\frac{q_2}{C_2} + i_1 + \frac{-V_1 + V_2}{R_1 + r_1 + r_2}$$

$$
\begin{aligned}
\frac{d\phi_1}{dt} &= \frac{R_1+r_2}{R_1+r_1+r_2}\frac{q_1}{C_1} - \left(R_2 + r_1\frac{R_1+r_2}{R_1+r_1+r_2}\right)\frac{\phi_1}{L_1} \\
&\quad - \frac{r_1r_2}{R_1+r_1+r_2}\frac{\phi_2}{L_2} + \frac{r_1}{R_1+r_1+r_2}\frac{q_2}{C_2} + \frac{(R_1+r_2)V_1 - r_1V_2}{R_1+r_1+r_2} \\
\frac{d\phi_2}{dt} &= \frac{r_2}{R_1+r_1+r_2}\frac{q_1}{C_1} - \frac{r_1r_2}{R_1+r_1+r_2}\frac{\phi_1}{L_1} - \left(R_3 + r_2\frac{R_1+r_1}{R_1+r_1+r_2}\right)\frac{\phi_2}{L_2} \\
&\quad + \frac{R_1+r_1}{R_1+r_1+r_2}\frac{q_2}{C_2} + \frac{r_2V_1 - (R_1+r_1)V_2}{R_1+r_1+r_2} \\
\frac{dq_2}{dt} &= \frac{1}{R_1+r_1+r_2}\frac{q_1}{C_1} - \frac{r_1}{R_1+r_1+r_2}\frac{\phi_1}{L_1} + \frac{R_1+r_1}{R_1+r_1+r_2}\frac{\phi_2}{L_2} \\
&\quad - \left(D_2 + \frac{1}{R_1+r_1+r_2}\right)\frac{q_2}{C_2} + i_2 + \frac{V_1-V_2}{R_1+r_1+r_2}.
\end{aligned}
\tag{1.2}
$$

The above heterogeneous equation can be represented as a linear first order vector differential equation below:

$$
\frac{d}{dt}(X) = A(t) + F(t), \tag{1.3}
$$

where $X = X(q_1(t), q_2(t), \phi_1(t), \phi_2(t))$ is a unknown function of t, $A(t)$ is a 4×4 non-symmetric matrix depending only $L_1, L_2, C_1, C_2, R_1, R_2, R_3, r_1, r_2$ and

$$
F(t) = \begin{pmatrix} i_1 + \frac{-V_1+V_2}{R_1+r_1+r_2} \\ \frac{(R_1+r_2)V_1 - r_1V_2}{R_1+r_1+r_2} \\ \frac{r_2V_1-(R_1+r_1)V_2}{R_1+r_1+r_2} \\ i_2 + \frac{V_1-V_2}{R_1+r_1+r_2} \end{pmatrix}
$$

is a free column vector.

Our motivation comes from the paper by Biryuk et al [1]. With respect to our observation in the literature, no work based on [1] was found. The aim of this paper is to obtain sufficient conditions for the asymptotic stability and boundedness of state variables characterizing system (1.2) where $F(t) = 0$ and $F(t) \neq 0$ due to external conductive element respectively. The essence of the latter is that the net voltage $V_1 - V_2$ across the system generates small back emf which in turn generate large current when connected to external conductive connection or element. This indeed becomes critical depending on the behavior of the state variables. Also, an explanatory illustration and geometric arguments are given on the behavior of the state variables of system (1.2). Our results improve and are different from the result of Biryuk et al [1].

The expectation of every physical problem modelled into system differential equations depend largely on the state variables describing the system. According to Peng and Pileggi [8], state variables are the minimum set of variables that fully describe the system circuit and its response to any given set of inputs. By this, it shows that the behavior of state-determined system like Figure 1 completely characterized by the stability and boundedness of set charges q_1, q_2, capacitors C_1, C_2, inductances L_1, L_2 and magnetic fluxes ϕ_1, ϕ_2. Now, we consider a state-characterized system in Figure 1 represented by a system of differential equations in (1.2), where q_1, q_2, ϕ_1, ϕ_2 are real valued state variables representing the energy stored in the system of coupled circuits. These four independent energy storages in the coupled circuit system are the current source which store energy in the capacitors and the inductors which store energy in a magnetic field. The first time derivatives of the state variables exist, V_1, V_2 are input voltages and feed current i_1, i_2 are all finite but undefined. The voltage sources have internal resistances r_1, r_2. R_1, R_2, R_3 are active resistances and D_1, D_2 are conductivities. We supposed also that circuit elements (capacitance, resistance, inductance and conductance) are always positive. Lyapunov second method will be employed as a basic tool to verify the results established in this work. The Lyapunov's second method lies in constructing a scalar function $U(q_1, q_2, \phi_1, \phi_2)$ that is positive definite and its derivative $\frac{dU(q_1,q_2,\phi_1,\phi_2)}{dt}$ along the system (1.2) under consideration is negative definite. When these properties of $U(q_1, q_2, \phi_1, \phi_2)$ and $\frac{dU(q_1,q_2,\phi_1,\phi_2)}{dt}$ are shown to be satisfied according to Lyapunov's theory ([2], [4], [10]), then the behavior of the state variable describing the system of two coupled circuits is known. The analysis here is of theoretical importance but the practical interpretation will not be considered and the result obtained enable us to characterize the behavior of the system of two coupled circuit and its response to external conductive connection or element as well as eliminate the instability in the system due to self-excitations by providing less restrictive conditions that are implementable at the development stage.

2. STABILITY ANALYSIS

2.1. **Assumptions.** In addition to the basic assumption imposed on the elements $R_1, R_2, R_3, r_1, r_2,$ and D_1, D_2 in (1.2), we suppose also that $M > 0$ such that the followings hold:

(i): $R_2 > R_3$, $R_2 > R_1R_3$;

(ii): $R_2 > D_1$, $R_2 > D_2$, $D_1, D_2 < 1$;

(iii): $r_1 - r_2 > R_1$, $r_1 > r_2$, $r_1r_2 > 1$;

(iv): $|V_1 - V_2| \leq M$.

In order to prove our results, we use the following scalar function $U = U(q_1, q_2, \phi_1, \phi_2)$ defined by

$$2U(q_1,q_2,\phi_1,\phi_2) = \frac{R_1+r_1+r_2}{D_1(R_1+r_1+r_2)+1}\frac{q_1^2}{C_1} + \frac{R_1+r_1+r_2}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}\frac{\phi_1^2}{L_1} + \frac{R_1+r_1+r_2}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}\frac{\phi_2^2}{L_1} + \frac{R_1+r_1+r_2}{D_2(R_1+r_1+r_2)+1}\frac{q_2^2}{C_2}. \tag{2.1}$$

From (2.1), we see that $U(0, 0, 0, 0) = 0$. It is obvious that the function U defined in (2.1) is a positive definite function, that is,

$$2U(q_1, q_2, \phi_1, \phi_2) \geq 0.$$

Hence, there is a positive constant K_1 such that

$$U(q_1, q_2, \phi_1, \phi_2) \geq K_1(q_1^2 + q_2^2 + \phi_1^2 + \phi_2^2), \tag{2.2}$$

where

$$K_1 = \min\left\{\frac{1}{C_1}\frac{R_1+r_1+r_2}{D_1(R_1+r_1+r_2)+1}; \frac{1}{L_1}\frac{R_1+r_1+r_2}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}; \frac{1}{L_2}\frac{R_1+r_1+r_2}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}; \frac{1}{C_2}\frac{R_1+r_1+r_2}{D_2(R_1+r_1+r_2)+1}\right\}.$$

Now, we consider the case where (1.2) is homogeneous that is, $F(t) = 0$. The derivative of function $U(q_1, q_2, \phi_1, \phi_2)$ in (2.1) along system (1.2) with respect to t

after simplification gives:

$$
\begin{aligned}
\frac{dU}{dt} = &-\Bigg\{1+\frac{R_1+r_2}{D_1(R_1+r_1+r_2)+1}-\frac{R_1+r_2}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}\\
&+\frac{r_2}{D_1(R_1+r_1+r_2)+1}+\frac{r_2}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}\\
&+\frac{1}{D_1(R_1+r_1+r_2)+1}+\frac{1}{D_2(R_1+r_1+r_2)+1}\Bigg\}\frac{q_1^2}{C_1^2}\\
&-\Bigg\{1+\frac{R_1+r_2}{D_1(R_1+r_1+r_2)+1}-\frac{R_1+r_2}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}\\
&+\frac{r_1}{D_2(R_1+r_1+r_2)+1}+\frac{r_1}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}\Bigg\}\frac{\phi_1^2}{L_1^2}\\
&-\Bigg\{1+\frac{r_2}{D_1(R_1+r_1+r_2)+1}+\frac{r_2}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}\\
&+\frac{R_1+r_1}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}+\frac{R_1+r_1}{D_2(R_1+r_1+r_2)+1}\Bigg\}\frac{\phi_2^2}{L_2^2}\\
&-\Bigg\{1+\frac{r_1}{D_2(R_1+r_1+r_2)+1}+\frac{r_1}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}\\
&+\frac{R_1+r_1}{D_2(R_1+r_1+r_2)+1}+\frac{R_1+r_1}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}\\
&+\frac{1}{D_1(R_1+r_1+r_2)+1}+\frac{1}{D_2(R_1+r_1+r_2)+1}\Bigg\}\frac{q_2^2}{C_2^2}\\
&-\Bigg\{\frac{R_1+r_2}{D_1(R_1+r_1+r_2)+1}-\frac{R_1+r_2}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}\Bigg\}\left(\frac{q_1}{C_1}+\frac{\phi_1}{L_1}\right)^2\\
&-\Bigg\{\frac{r_2}{D_1(R_1+r_1+r_2)+1}+\frac{r_2}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}\Bigg\}\left(\frac{q_1}{C_1}-\frac{\phi_2}{L_2}\right)^2\\
&-\Bigg\{\frac{r_1}{D_2(R_1+r_1+r_2)+1}+\frac{r_1}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}\Bigg\}\left(\frac{q_2}{C_2}+\frac{\phi_1}{L_1}\right)^2\\
&-\Bigg\{\frac{R_1+r_1}{D_2(R_1+r_1+r_2)+1}+\frac{R_1+r_1}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}\Bigg\}\left(\frac{q_2}{C_2}-\frac{\phi_2}{L_2}\right)^2\\
&-\Bigg\{\frac{2+(D_1+D_2)(R_1+r_1+r_2)}{[D_1(R_1+r_1+r_2)+1][D_2(R_1+r_1+r_2)+1]}\Bigg\}\left(\frac{q_1}{C_1}-\frac{q_2}{C_2}\right)^2.
\end{aligned}
$$

If (i), (ii) and (iii) of Assumption 2.1 obtained by the Lyapunov functional (2.1) hold, we have that

$$\frac{dU(q_1,q_2,\phi_1,\phi_2)}{dt} \le -K_2(q_1^2+q_2^2+\phi_1^2+\phi_2^2), \tag{2.3}$$

for some $K_2 > 0$. It follows that

$$\frac{dU(q_1,q_2,\phi_1,\phi_2)}{dt} \le 0.$$

Thus, the analyzed state variables q_1, q_2, ϕ_1, ϕ_2 describing the system (1.2) in Figure 1 are asymptotically stable as $t \to \infty$.

3. Boundedness Analysis

As in Section 2, the boundedness analysis of the state variables q_1, q_2, ϕ_1, ϕ_2 describing the system (1.2) in Figure 1 depends on the scalar differentiable Lyapunov function $U(q_1,q_2,\phi_1,\phi_2)$ defined in (2.1).

Here, we consider the heterogenous case where $F(t) \neq 0$ in (1.3). In view of (2.3),

$$\begin{aligned}\frac{dU(q_1,q_2,\phi_1,\phi_2)}{dt} &\le -K_2(q_1^2+q_2^2+\phi_1^2+\phi_2^2) + \left\{i_1 + \frac{|V_1-V_2|}{D_1(R_1+r_1+r_2)+1}\right\}\left|\frac{q_1}{C_1}\right| \\ &+ \left\{i_2 + \frac{|V_1-V_2|}{D_2(R_1+r_1+r_2)+1}\right\}\left|\frac{q_2}{C_2}\right| \\ &+ \frac{1}{R_2(R_1+r_1+r_2)+r_1(R_1+r_2)}\left\{(R_1+r_2)V_1 - r_1V_2\right\}\left|\frac{\phi_1}{L_1}\right| \\ &+ \frac{1}{R_3(R_1+r_1+r_2)+r_2(R_1+r_1)}\left\{-r_2V_1 + (R_1+r_1)V_2\right\}\left|\frac{\phi_2}{L_2}\right|,\end{aligned}$$

since $\dot{U}_{(2)} \le 0$ for all q_1, q_2, ϕ_1, ϕ_2.

It follows that

$$\begin{aligned}\frac{dU(q_1,q_2,\phi_1,\phi_2)}{dt} &\le -K_2(q_1^2+q_2^2+\phi_1^2+\phi_2^2) \\ &\quad + \left\{K_3|q_1| + K_4|q_2| + K_5|\phi_1| + K_6|\phi_2|\right\}|V_1-V_2|,\end{aligned}$$

where

$$K_3 = \frac{1}{C_1}\left(i_1 + \frac{1}{D_1(R_1+r_1+r_2)+1}\right),$$

$$K_4 = \frac{1}{C_2}\left(i_2 + \frac{1}{D_2(R_1 + r_1 + r_2) + 1}\right),$$
$$K_5 = \frac{1}{L_1}\left(\frac{1}{R_2(R_1 + r_1 + r_2) + r_1(R_1 + r_2)}\right)max\left\{(R_1 + r_2), r_1\right\},$$
$$K_6 = \frac{1}{L_2}\left(\frac{1}{R_3(R_1 + r_1 + r_2) + r_2(R_1 + r_1)}\right)max\left\{(R_1 + r_1), r_2\right\}.$$

By noting (iv) of Assumption 2.1, we have that

$$\frac{dU(q_1, q_2, \phi_1, \phi_2)}{dt} \leq -K_2(q_1^2 + q_2^2 + \phi_1^2 + \phi_2^2) + K_7\left(|q_1| + |q_2| + |\phi_1| + |\phi_2|\right)M,$$

where $K_7 = max\{K_3, K_4, K_5, K_6\}$.

It follows that

$$\frac{dU(q_1, q_2, \phi_1, \phi_2)}{dt} \leq K_7\left(|q_1| + |q_2| + |\phi_1| + |\phi_2|\right)M,$$

and using the fact that

$$| q_1 |< 1 + q_1^2, \quad | q_2 |< 1 + q_2^2, \quad | \phi_1 |< 1 + \phi_1^2, \quad | \phi_2 |< 1 + \phi_2^2,$$

we get

$$\frac{dU(q_1, q_2, \phi_1, \phi_2)}{dt} \leq K_7\left(4 + q_1^2 + q_2^2 + \phi_1^2 + \phi_2^2\right)M.$$

From (2.2), where $q_1^2 + q_2^2 + \phi_1^2 + \phi_2^2 \leq K_1^{-1}U(q_1, q_2, \phi_1, \phi_2)$, we have that

$$\frac{dU(q_1, q_2, \phi_1, \phi_2)}{dt} \leq K_7\left(4 + K_1^{-1}U(q_1, q_2, \phi_1, \phi_2)\right)M.$$

Thus,

$$\frac{dU(q_1, q_2, \phi_1, \phi_2)}{dt} - \frac{K_7M}{K_1}U(q_1, q_2, \phi_1, \phi_2) \leq 4K_7M. \tag{3.1}$$

Multiplying both sides of (3.1) by the integrating factor $\exp(-\frac{K_7M}{K_1}t)$ gives

$$\frac{d}{dt}\left(U\exp(-\frac{K_7M}{K_1}t)\right) \leq 4K_7M\exp(-\frac{K_7M}{K_1}t). \tag{3.2}$$

Integrating both sides of the inequality (3.2) from 0 to t, yields

$$U\exp(-\frac{K_7M}{K_1}t) \leq U(0) + 4K_7M\int_0^t \exp(-\frac{K_7M}{K_1}s)ds, \tag{3.3}$$

since $U(0) = U(0,0,0,0)$.

By Gronwall's-Reid inequality, (3.3) becomes

$$U(q_1, q_2, \phi_1, \phi_2) \leq (U(0) + 4K_7M)\exp(\frac{K_7M}{K_1}) = K_8 < \infty, \tag{3.4}$$

where $K_8 > 0$. In view of the inequality (2.2) and (3.4), we have

$$q_1^2 + q_2^2 + \phi_1^2 + \phi_2^2 \leq K_1^{-1}U(q_1, q_2, \phi_1, \phi_2) \leq K_9, \tag{3.5}$$

where $K_9 = K_8K_1^{-1}$. Inequality (3.5) implies that

$$\begin{aligned} \mid q_1(t) \mid &\leq \sqrt{K_9} \\ \mid q_2(t) \mid &\leq \sqrt{K_9} \\ \mid \phi_1(t) \mid &\leq \sqrt{K_9} \\ \mid \phi_2(t) \mid &\leq \sqrt{K_9} \\ \sqrt{K_9} &= K, \ \ \forall t \geq t_0 \geq 0, \end{aligned}$$

where K depends on the elements L_1, L_2, C_1, C_2, R_1, R_2, R_3, r_1, r_2. It shows that the state variables q_1, q_2, ϕ_1, ϕ_2 describing the system of two coupled circuits in Figure 1 are bounded by a constant K if (i), (ii), (iii) and (iv) of the Assumption 2.1 hold.

Thus, this implies that every bounded input produces a bounded output if the stated assumptions are satisfied.

3.1. **Stability and Boundedness Analysis of System** (1.2).

(1) The plot of $q_1(t)$, $q_2(t)$, $\phi_1(t)$, $\phi_2(t)$ of equation (1.2) which are the state variables characterizing the system of two coupled circuits in Figure 1 is shown in Figure 2 and Figure 3. It is very clear from Figure 2 and Figure 3 that (i), (ii) and (iii) of Assumptions 2.1 obtained by the Lyapunov functional (2.1) are satisfied and $q_1(t)$, $q_2(t)$, $\phi_1(t)$, $\phi_2(t)$ are asymptotically stable as $t \to \infty$.

(2) In Fig. 4, Figure. 5, Figure. 6 and Figure. 6 respectively, as (i), (ii), (iii) and (iv) of Assumptions 2.1 are satisfied, $q_1(t)$, $q_2(t)$, $\phi_1(t)$, $\phi_2(t)$ are bounded for all $t \geq 0$. The boundedness of $q_1(t)$, $q_2(t)$, $\phi_1(t)$, $\phi_2(t)$ obviously depend on L_1, L_2, C_1, C_2, R_1, R_2, R_3, r_1, r_2.. Our result shows that

$q_1(t)$, $q_2(t)$, $\phi_1(t)$, $\phi_2(t)$ are bounded by a single constant if the (i), (ii), (iii) and (iv) of Assumptions 2.1 are satisfied for all $t \geq 0$. This means that the state variables $q_1(t)$, $q_2(t)$, $\phi_1(t)$, $\phi_2(t)$ describing the the system (1.2) do not exhibit unbounded increase or decrease and remain within a reasonable limit as $t \to \infty$.

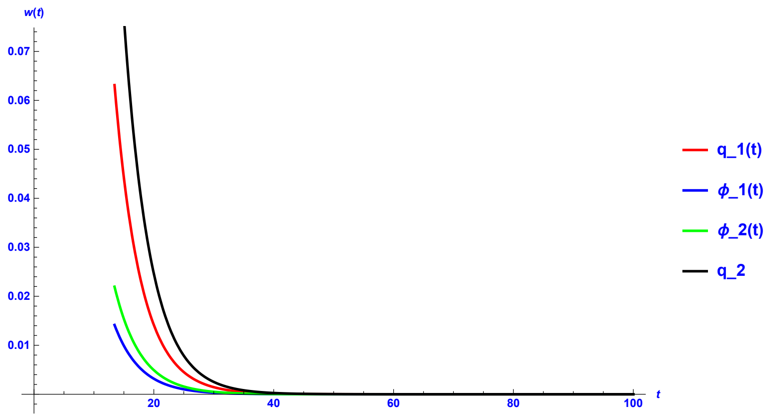

FIGURE 2. *The graph of state variables of system* (1.2) $q_1(t)$ *in (red),* $\phi_1(t)$ *in (blue),* $\phi_2(t)$ *in (green) and* $q_2(t)$ *in (black) on the same axes satisfying (i),(ii), (iii) of Assumptions 2.1 tend to* 0 *as* $t \to \infty$.

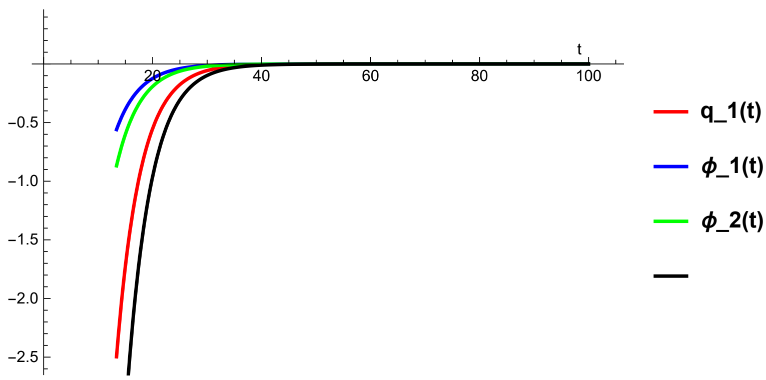

FIGURE 3. *The graph of state variables of system* (1.2) $q_1(t)$ *in (red),* $\phi_1(t)$ *in (blue),* $\phi_2(t)$ *in (green) and* $q_2(t)$ *in (black) on the same axes satisfying (i),(ii), (iii) of Assumptions 2.1 tend to* 0 *as* $t \to \infty$.

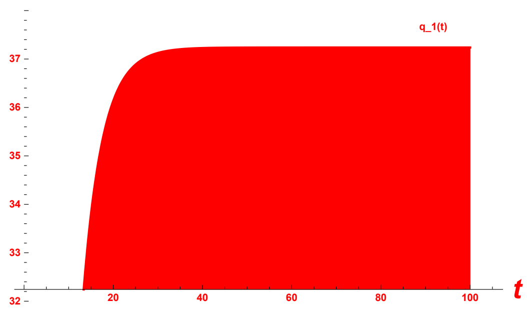

FIGURE 4. *The boundedness of $q_1(t)$ satisfying (i), (ii), (iii) and (iv) of Assumptions 2.1*

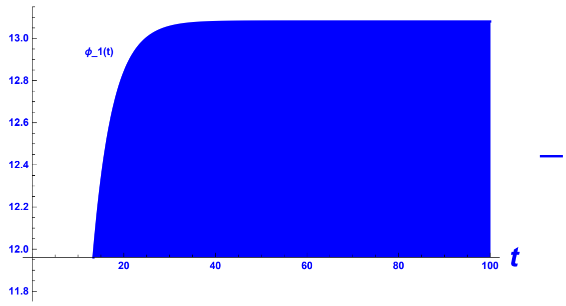

FIGURE 5. *The boundedness of $\phi_1(t)$ satisfying (i), (ii), (iii) and (iv) of Assumptions 2.1*

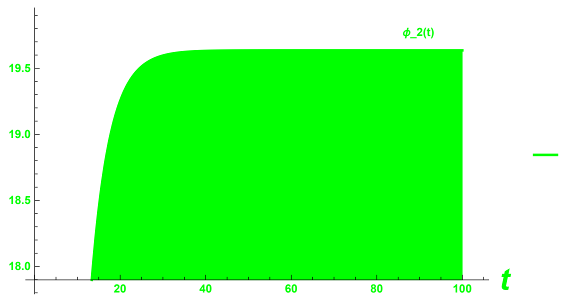

FIGURE 6. *The boundedness of $\phi_2(t)$ satisfying (i), (ii), (iii) and (iv) of Assumptions 2.1*

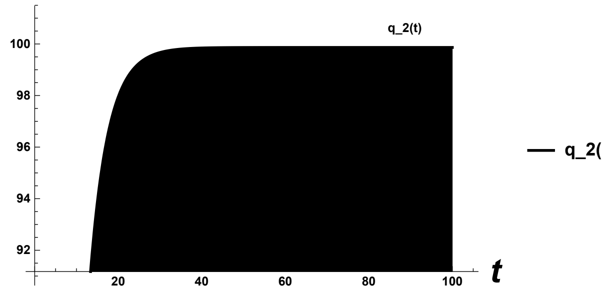

FIGURE 7. *The boundedness of $q_2(t)$ satisfying (i), (ii), (iii) and (iv) of Assumptions 2.1*

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