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A NOVEL METHOD FOR SOLVING FULLY FUZZY SOLID TRANSPORTATIONS PROBLEMS

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Abstract. In this paper, we proposes a new method for solving solid transportation problem under uncertainty environments. The fully fuzzy solid transportation problem has been formulated. To reduce the model into crisp equivalent, we have used existing method for approximation of fuzzy numbers by interval numbers and its arithmatics. A simplex method and existing method for solving Interval Linear Programming problems are used for solving solid transportation problem with fuzzy parameters and decision variables. Furthermore, for illustration, some numerical examples are used to demonstrate the correctness and usefulness of the proposed method. The proposed algorithm is flexible, easy and reasonable.

1. Intrduction

Transportation problem is an important network structured linear programming problem that arises in several contexts and received a great deal of attention in the literature. Transportation problem can be used for a wide variety of situations such as production, investment, plant location, inventory control, employment scheduling and many others. A solid transportation problem **(STP)** is a generalization of classic transportation problem proposed by Shell [19] in which the constraints are

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defined on three items namely, supply, demand and conveyance. In many industrial problems, a homogeneous product is delivered from an origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, and ships, etc. Effective algorithms have been developed to solve the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some unmanageable factors. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [1] and Zadeh [21] introduced the impression of fuzziness.

Bit et al. [2] developed the fuzzy programming model based on Zimmermann method [23] to solve a multi-objective solid transportation problem. Halay [5] described a solution procedure of a solid transportation problem based on Modi-method. Li et al. [15] used a neural network approach to solve a multicriteria solid transportation problem. Jimenez and Verdegay [8] obtained a solution procedure for two types of uncertain solid transportation problem in which the considered data are interval numbers and fuzzy numbers, respectively. Again in [9], Jimenez and Verdegay developed a parametric approach for solving fuzzy solid transportation problems by an evolutionary algorithm.

In this paper, we attempt to develop the solving Fully Fuzzy Solid Transportation Problems involving triangular fuzzy numbers via Interval Linear Programming problems by converting it to two classical solid transportation Problems. The rest of the paper is organised as follows. In Section 2, we have reviewed some very important basic definitions and the arithmetic operations involved in interval and fuzzy numbers. The model for solid transportation problem with fuzzy parameters and fuzzy variables decisions is presented. In Section 3, we propose a simple method for solving fully fuzzy solid transportation problems. In Section 4, a numerical illustration is given to clearly understand the applicability of the proposed method. Advantages of the presented method over the existing methods are discussed in Section 5. Finally, some conclusions and future scope of the method are drawn in the Section 6.

2. Materials and Methods

In this section, some basic definitions, arithmetic operations for closed Intervals numbers and of linear programming problems involving interval numbers are presented [11, 16, 17].

2.1. **Arithmetic Operations on intervals numbers.**

In this subsection, some arithmetic operations for two intervals are presented [6, 7, 18].

2.1.1. *Definition of intervals numbers.* A interval numbers $\bar{a} = [a^p, a^q]$ is set denoted $\bar{a} = \{a \in \mathbb{R} \backslash a^p \leq a \leq a^q\}$ where a^p et a^q are respectly, the lower limit and the upper limit of $\bar{a}$.

If $\bar{a} = a^p = a^q = a$, then $\bar{a} = [a, a] = a$ is a real number (or degenerated interval). The center of an interval numbers $\bar{a}$ is defined by $m(\bar{a}) = \frac{a^p + a^q}{2}$. The radius of an interval of numbers $\bar{a}$ is defined by $w(\bar{a}) = \frac{a^q - a^p}{2}$.

An interval can also be expressed in terms of its center and its radius

$$\bar{a} = [a^p, a^q] = \langle m(\bar{a}), w(\bar{a}) \rangle = \{a \in \mathbb{R} \backslash m(\bar{a}) - w(\bar{a}) \leq a \leq m(\bar{a}) + w(\bar{a})\}.$$

2.1.2. *A new interval arithmetic.* For any two intervals $\bar{a} = \langle m(\bar{a}), w(\bar{a}) \rangle$ and $\bar{b} = \langle m(\bar{b}), w(\bar{b}) \rangle$, the arithmetic operations on $\bar{a}$ et $\bar{b}$ are defined as:

Addition:

$$\bar{a} + \bar{b} = [a^p + b^p, a^q + b^q] = \langle m(\bar{a}) + m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle \tag{2.1}$$

Subtraction:

$$\bar{a} - \bar{b} = [a^p - b^q, a^q - b^p] = \langle m(\bar{a}) - m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle \tag{2.2}$$

Multiplication:

$$\bar{a} \times \bar{b} = [\text{Min}\,(a^p b^p, a^p b^q, a^q b^p, a^q b^q), Max\,(a^p b^p, a^p b^q, a^q b^p, a^q b^q)]$$

or

$$\begin{cases} \langle m(\bar{a})m(\bar{b}) + w(\bar{a})w(\bar{b}), m(\bar{a})w(\bar{b}) + m(\bar{b})w(\bar{a}) \rangle & \text{if} \quad a^p \geq 0, b^p \geq 0 \\ \langle m(\bar{a})m(\bar{b}) + m(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) + w(\bar{b})w(\bar{a}) \rangle & \text{if} \quad a^p < 0, b^p \geq 0 \\ \langle m(\bar{a})m(\bar{b}) - w(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) - m(\bar{a})w(\bar{b}) \rangle & \text{if} \quad a^q < 0, b^p \geq 0 \end{cases} \tag{2.3}$$

Division:

$$\frac{\bar{a}}{\bar{b}} = [\text{Min}\,(\frac{a^p}{b^p}, \frac{a^p}{b^q}, \frac{a^q}{b^p}, \frac{a^q}{b^q}), \text{Max}\,(\frac{a^p}{b^p}, \frac{a^p}{b^q}, \frac{a^q}{b^p}, \frac{a^q}{b^q})] \tag{2.4}$$

2.2. **A new Arithmetic Operations on fuzzy numbers.**

2.2.1. *Fuzzy numbers.* Let X a classical reference set. A fuzzy subset $\tilde{A}$ in the reference set X is defined by an application: $\mu_{\tilde{A}}(x) : X \longrightarrow [0,1]$. ‘ The Support of a fuzzy set $\tilde{A}$, denoted Supp($\tilde{A}$) is the set of elements of X whose membership function $\mu_{\tilde{A}}(x)$ is non zero: Supp($\tilde{A}$) $= \{x \in X/\mu_{\tilde{A}}(x) \neq 0\}$.

The kernel of a fuzzy set $\tilde{A}$, denoted $Noy(\tilde{A})$ is a set of elements of X whose membership function $\mu_{\tilde{A}}(x)$ is equal to 1: Ker($\tilde{A}$) $= \{x\ inX/\mu_{\tilde{A}}(x) = 1\}$.

The height of a fuzzy set $\tilde{A}$, denoted $High(\tilde{A})$ is the greatest membership degree of $\tilde{A}$: $H(\tilde{A}) =$ Sup$\mu_{\tilde{A}}(x)$.

A fuzzy set $\tilde{A}$ is said to be normalized if there exists $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

A fuzzy set $\tilde{A}$ is said to be convex if $\forall(x_1, x_2) \in \mathbb{R}^2$ and $\lambda \in [0,1]$, we have: $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$.

An α-cut $\tilde{A}_\alpha$ of $\tilde{A}$ is a subset of X of level α, defined by:$cut_\alpha(\tilde{A}) = \tilde{A}_\alpha = \{x \in X/ \mu_{\tilde{A}}(x) \geq \alpha; \alpha \in [0,1]\}$.

A fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is a fuzzy number if its membership function $\mu_{\tilde{A}}(x) : \mathbb{R} \longrightarrow [0,1]$ has the following properties:

(i) $\mu_{\tilde{A}}(x)$ is convex i.e. $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq\ min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \forall\lambda \in [0,1]$ and $\forall x_1, x_2 \in \mathbb{R}$

(ii) $\mu_{\tilde{A}}(x)$ is normal i.e. there exists $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x)$ =1

(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

In this article we inform readers that we are interested in regular fuzzy numbers, the most basic of which are triangular fuzzy numbers and trapezoidal fuzzy numbers. Regular fuzzy numbers are named according to their order n. We write the fuzzy numbers of order n as follows:

$$\tilde{a} = (a^1, a^2, a^3, \ldots, a^{n_f}) \text{ with } a^1 \leq a^2 \leq a^3 \leq ... \leq a^n.$$

2.2.2. *Decomposition of fuzzy numbers into intervals numbers.* Basically, the concept of decomposition of fuzzy numbers follows from the decomposition theorem as formulated for regular fuzzy numbers, in a dimension of fuzzy subsets in [22].

He states that any fuzzy set $\tilde{A}$ can only be represented by the sequence of its associates an alpha cut by the formula: $\mu_{\tilde{A}}(x) = \text{Sup} \quad \alpha.\mu_{\tilde{A}_\alpha}(x)$, $\alpha \in [0,1]$.

To the same extent, this theorem is valid for any fuzzy number as a special case of a fuzzy set and can be rewritten in the form: $\mu_{\tilde{a}}(x) = \text{Sup} \quad \alpha.\mu_{\tilde{a}_\alpha}(x)$, $\alpha \in [0,1]$.

To make this decomposition theorem usable for practical applications, we make a discretization of the function membership $\mu_{\tilde{a}}$ by subdividing the interval $[0,1]$ in intervals of length m. The discrete values are then given by: $\mu_k = \frac{k}{m}$, $k = 0, \cdots, m$.

Thus the application of the decomposition theorem to a fuzzy number of order n $\tilde{a} = (a^1, a^2, \cdots, a^n)$ allows it to be rewritten in its decomposed form by the set

$$\tilde{a} = (X^{(n_I-1)}, X^{(n_I-2)}, \cdots, X^1, X^0)$$

of n_I intervals.

Here:

$$n_I = \tfrac{n+1}{2}, \text{ if } n \text{ is odd and } n_I = \tfrac{n}{2}, \text{ if } n \text{ is even,}$$

and

$$X^k = [a^p, a^q] = coupe_{\mu_k}(\tilde{a}),$$

with $\mu_k = \frac{k}{n_I-1}$, $p = 1+k$, $q = n-k$, $k = 0, \cdots, n_I - 1$.

2.2.3. *A new fuzzy number arithmetic.* Let $\tilde{a}$ and $\tilde{b}$ two fuzzy numbers. Based on the concept of the decomposition of fuzzy numbers, the arithmetic operations between these two fuzzy numbers are defined by:

$$\tilde{a} * \tilde{b} = cut_\alpha(\tilde{a}) * cut_\alpha(\tilde{b}) = ([a^p, a^q] * [b^p, b^q]).$$

Here $*$ refers to $(+, -, \times, \div)$ the usual arithmetic operations between two intervals of classical numbers.

2.3. **Formulation of transportation problem with the interval numbers parameters.** In this subsection, a solution procedure for solving the transportation problems involving interval numbers is developed in the following steps [12–14] We consider the transportation problem involving interval numbers as follows ([4]):

$$(2.5)\quad \begin{cases} \bar{Z}^{pq}(\bar{x}^{pq}) \approx \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} \bar{c}_{ij}^{pq}\bar{x}_{ij}^{pq} \longrightarrow \text{Min} \\ \\ \qquad \text{subject to the constraints:} \\ \displaystyle\sum_{j=1}^{n} \bar{x}_{ij}^{pq} \approx \bar{a}_i^{pq}, \quad \text{for} \quad i = 1, 2, \cdots, m \\ \displaystyle\sum_{i=1}^{m} \bar{x}_{ij}^{pq} \approx \bar{b}_j^{pq}, \quad \text{for} \quad j = 1, 2, \cdots, n \end{cases}$$

Applying the property et a new arithmetic of intervals numbers the problem (2.5) is equivalent to the following problem:

$$(2.6)\quad \begin{cases} \bar{Z}^{pq}(\bar{x}^{pq}) \approx \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} \langle m(\bar{c}_{ij}^{pq}\bar{x}_{ij}^{pq}), w(\bar{c}_{ij}^{pq}\bar{x}_{ij}^{pq})\rangle \longrightarrow \text{Min} \\ \\ \qquad \text{subject to constraints} \\ \displaystyle\sum_{j=1}^{n} \langle m(\bar{x}_{ij}^{pq}), w(\bar{x}_{ij}^{pq})\rangle \approx \langle m(\bar{a}_i^{pq}), w(\bar{a}_i^{pq})\rangle \quad \text{for} \quad i = 1, 2, \cdots, m \\ \displaystyle\sum_{i=1}^{m} \langle m(\bar{x}_{ij}^{pq}), w(\bar{x}_{ij}^{pq})\rangle \approx \langle m(\bar{b}_j^{pq}), w(\bar{b}_j^{pq})\rangle \quad \text{for} \quad j = 1, 2, \cdots, n. \end{cases}$$

Note that:

(i) $\displaystyle\sum_{j=1}^{n} \bar{x}_{ij}^{pq} = \bar{a}_i^{pq}$ if and only if $\displaystyle\sum_{j=1}^{n} w(\bar{x}_{ij}^{pq}) = w(\bar{a}_i^{pq})$, for $i = 1, 2, \cdots, m$.

(ii) $\displaystyle\sum_{j=1}^{n} \bar{x}_{ij}^{pq} \neq \bar{a}_i^{pq}$ if and only if $\displaystyle\sum_{j=1}^{n} w(\bar{x}_{ij}^{pq}) \neq w(\bar{a}_i^{pq})$, for $i = 1, 2, \cdots, m$.

(iii) $\displaystyle\sum_{i=1}^{m} \bar{x}_{ij}^{pq} = \bar{b}_j^{pq}$ if and only if $\displaystyle\sum_{i=1}^{m} w(\bar{x}_{ij}^{pq}) = w(\bar{b}_j^{pq})$, for $j = 1, 2, \cdots, n$.

(iv) $\displaystyle\sum_{i=1}^{m} \bar{x}_{ij}^{pq} \neq \bar{b}_j^{pq}$ if and only if $\displaystyle\sum_{i=1}^{m} w(\bar{x}_{ij}^{pq}) \neq w(\bar{b}_j^{pq})$, for $j = 1, 2, \cdots, n$.

Therefore the problem (2.6) is decomposed into two problems:

$$\text{(2.7)} \qquad \begin{cases} m(\bar{Z}^{pq}(\bar{x}^{pq})) = \sum_{i=1}^{m}\sum_{j=1}^{n} m(\bar{c}_{ij}^{pq}\bar{x}_{ij}^{pq}) \longrightarrow \text{Min} \\ \\ \qquad\qquad \text{subject to the constraints:} \\ \sum_{j=1}^{n} m(\bar{x}_{ij}^{pq})) = m(\bar{a}_i^{pq}),\ for\ i = 1, \cdots, m \\ \sum_{i=1}^{m} m(\bar{x}_{ij}^{pq}) = m(\bar{b}_j^{pq}),\ for\ j = 1, \cdots, n \end{cases}$$

having $m(\bar{x}_{ij}^{pq})$ as optimal solution, and

$$\text{(2.8)} \qquad \begin{cases} w(\bar{Z}^{pq}(\bar{x}^{pq})) = \sum_{i=1}^{m}\sum_{j=1}^{n} w(\bar{c}_{ij}^{pq}\bar{x}_{ij}^{pq}) \longrightarrow \text{Min} \\ \\ \qquad\qquad \text{subject to the constraints:} \\ \sum_{j=1}^{n} w(\bar{x}_{ij}^{pq}) = w(\bar{a}_i^{pq}),\ for\ i = 1, \cdots, m \\ \sum_{i=1}^{m} w(\bar{x}_{ij}^{pq}) = w(\bar{b}_j^{pq}),\ for\ j = 1, \cdots, n. \end{cases}$$

having $w(\bar{x}_{ij}^{pq})$ as optimal solution.

The interval optimal solution of the (2.5) according to the choice of the decision maker with minimum uncertainty is: $\bar{x}_{ij}^{pq} = [m(\bar{x}_{ij}^{pq}) - w(\bar{x}_{ij}^{pq}), m(\bar{x}_{ij}^{pq}) + w(\bar{x}_{ij}^{pq})]$ with the condition $w(\bar{x}_{ik}^{pq}) \geq w(\bar{x}_{il}^{pq})]$ if $c_{ik}^{q} \leq c_{il}^{q}$ for $j = 1, 2, \cdots, m.$

2.4. **Formulation of fully fuzzy solid transportation problems.** The following notations are used in the formulation of the fully fuzzy solid transportation model:

- m: total number of sources,
- n: total number of destinations,
- r: total number of conveyances
- $\tilde{a}_i$: fuzzy amount of the product available at the source i,
- $\tilde{b}_j$: fuzzy demand of the product of the destination j,
- $\tilde{e}_k$: fuzzy transportation maximal capacity of conveyance k,

- $\tilde{c}_{ijk}$: fuzzy unit cost of transportation from source i to destination j by conveyance k, and
- $\tilde{x}_{ijk}$: fuzzy amount of product transported from source i to destination j by conveyance k

$$(2.9)\quad \begin{cases} \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r}(\tilde{c}_{ijk}.\tilde{x}_{ijk}) \longrightarrow \text{Min} \\ \text{Subject to constraints:} \\ \sum_{j=1}^{n}\sum_{k=1}^{r}\tilde{x}_{ijk} \approx \tilde{a}_i, \ for \quad i = 1, 2, \cdots, m \\ \sum_{i=1}^{m}\sum_{k=1}^{r}\tilde{x}_{ijk} \approx \tilde{b}_j, \ for \quad j = 1, 2, \cdots, n \\ \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{x}_{ijk} \approx \tilde{e}_k, \ for \quad k = 1, 2, \cdots, r. \end{cases}$$

3. Main results

In this section, we will describe our method for solving fully fuzzy solid transportation problem to overcome the shortcomings of the existing method. The algorithm for solving fully fuzzy solid transport problems with fuzzy parameters and decision variables is composed of the following steps:

Step1. Formulate the fully fuzzy solid transportation problem, and then convert it into a balanced one if it is not.

Step2. Convert the problem constructed in **Step1.** into corresponding fully interval problems based on interval decomposition of the fuzzy number according to its type, and then into equivalent classical solid transport problems based on interval arithmetic.

Step3. Solve the classic solid transport problems found in **Step2.** by the simplex method.

Step4. Determine $w(\bar{x}_{ijk}^{pq})$ for each means of transport $1 \leq k \leq r$ fixed according to the following cases:

Case 1. If $\sum_{i=1}^{m}\sum_{j=1}^{n}|x_{ijk}^{pq}-x_{ijk}^{(p+1)(q-1)}|+w(\bar{e}_k^{(p+1)(q-1)}) \leq w(\bar{e}_k^{pq})$ then $\sum_{x_{ijk}^{pq}\neq 0} w(\bar{x}_{ijk}^{pq}) = w(\bar{e}_k^{pq})$ with $\bar{x}_{ijk}^{pq} = \left[x_{ijk}^{p}, x_{ijk}^{q}\right] = \left[x_{ijk}^{pq} - w(\bar{x}_{ijk}^{pq}), x_{ijk}^{pq} + w(\bar{x}_{ijk}^{pq})\right]$.

Case 2. If $\sum_{i=1}^{m}\sum_{j=1}^{n}|x_{ijk}^{pq}-x_{ijk}^{(p+1)(q-1)}|+w(\bar{e}_k^{(p+1)(q-1)}) > w(\bar{e}_k^{pq})$, then $w(\bar{e}_k^{pq}) = |x_{ijk}^{pq}-x_{ijk}^{(p+1)(q-1)}| + w(\bar{e}_k^{(p+1)(q-1)})$ with $\bar{x}_{ijk}^{pq} = \left[x_{ijk}^{p}, x_{ij}^{q}\right] = \left[x_{ijk}^{pq} - w(\bar{x}_{ijk}^{pq}), x_{ijk}^{pq} + w(\bar{x}_{ijk}^{pq})\right]$.

Step5. The current fuzzy optimal solution according to the choice of the decision maker is in **Step4.** is $\tilde{x}$

3.1. **Our method for Solving Fully Fuzzy Solid Transportation (FFSTP) problems with triangular fuzzy numbers.** In this section, a method to find a fuzzy optimal solution of fully fuzzy solid transportation**(FFSTP)** problems involving triangular fuzzy numbers is presented.

$$(3.1)\qquad \begin{cases} \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r}(c_{ijk}^1, c_{ijk}^2, c_{ijk}^3)(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3) \longrightarrow \text{Min} \\ \\ \qquad\qquad \text{Subject to be constraints:} \\ \sum_{j=1}^{n}\sum_{k=1}^{r}(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3) \approx (a_i^1, a_i^2, a_i^3), \quad for \quad i = 1, 2, \cdots, m \\ \sum_{i=1}^{m}\sum_{k=1}^{r}(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3) \approx (b_j^1, b_j^2, b_j^3), \quad for \quad j = 1, 2, \cdots, n \\ v\sum_{i=1}^{m}\sum_{j=1}^{n}(x_{ijk}^1, x_{ijk}^2, x_{ijk}^3) \approx (e_k^1, e_k^2, e_k^3), \quad for \quad k = 1, 2, \cdots, r \end{cases}$$

The steps of our proposed method for solving fully fuzzy solid transportation problem involving triangular fuzzy numbers as follows:

Step 1. Check the given FFSTP is balanced. If not, change into it.

Step 2. Using the decomposition of fuzzy numbers to interval numbers, the problem (3.1) is equivalent to:

$$
(3.2)\quad \begin{cases} \tilde{Z}(\tilde{x}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r}(c_{ijk}^{2};\bar{c}_{ijk}^{13})(x_{ijk}^{2};\bar{x}_{ijk}^{13}) \longrightarrow \text{Min} \\ \text{Subject to be constraints:} \\ \sum_{j=1}^{n}\sum_{k=1}^{r}(x_{ijk}^{2};\bar{x}_{ijk}^{13}) \approx (a_i^2;\bar{a}_i^{13}), \quad for \quad i=1,2,\cdots,m \\ \sum_{i=1}^{m}\sum_{k=1}^{r}(x_{ijk}^{2};\bar{x}_{ijk}^{13}) \approx (b_j^2;\bar{b}_j^{13}), \quad for \quad j=1,2,\cdots,n. \\ \sum_{i=1}^{m}\sum_{j=1}^{n}(x_{ijk}^{2};\bar{x}_{ijk}^{13}) \approx (e_k^2;\bar{e}_k^{13}), \quad for \quad k=1,2,\cdots,r. \end{cases}
$$

and the problem (3.2) is decomposed to the two following problems. For $p = q = 2$, we have:

$$
(3.3)\quad \begin{cases} Z^2(x^2) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r} c_{ijk}^{2}x_{ijk}^{2} \longrightarrow \text{Min} \\ \text{subject to be constraints:} \\ \sum_{j=1}^{m}\sum_{k=1}^{r} x_{ijk}^{2} = a_i^2 \quad for \quad i=1,2;\cdots,m \\ \sum_{i=1}^{n}\sum_{k=1}^{r} x_{ijk}^{2} = b_j^2 \quad for \quad j=1,2;\cdots,n \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{2} = e_k^2 \quad for \quad k=1,2;\cdots,r \end{cases}
$$

For $p = 1$ and $q = 3$, we have:

$$
\bar{Z}^{13}(\bar{x}^{13}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r} \bar{c}_{ijk}^{13}\bar{x}_{ijk}^{13} \longrightarrow \text{Min}
$$

subject to be constraints:

$$(3.4)\qquad \begin{cases} \displaystyle\sum_{j=1}^{n}\sum_{k=1}^{r}\bar{x}_{ij}^{13} \approx \bar{a}_i^{13}, \quad for \quad i=1,2,\cdots,m \\ \displaystyle\sum_{i=1}^{m}\sum_{k=1}^{r}\bar{x}_{ij}^{13} \approx \bar{b}_j^{13}, \quad for \quad j=1,2,\cdots,n \\ \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{x}_{ijk}^{13} \approx \bar{e}_k^{13}, \quad for \quad k=1,2,\cdots,r \end{cases}.$$

Using the properties of interval numbers the problem (3.4) is equivalent to the problem: for $p=1$ and $q=3$, we have:

$$(3.5)\qquad \begin{cases} \bar{Z}^{13}(\bar{x}^{13}) \approx \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r} m(\bar{c}_{ijk}^{13})(\bar{x}_{ijk}^{13}) \longrightarrow \text{Min} \\ \qquad\qquad \text{subject to be constraints:} \\ \displaystyle\sum_{j=1}^{n}\sum_{k=1}^{r} m(\bar{x}_{ij}^{13}) \approx m(\bar{a}_i^{13}), \quad for \quad i=1,2,\cdots,m \\ \displaystyle\sum_{i=1}^{m}\sum_{k=1}^{r} m(\bar{x}_{ij}^{13}) \approx m(\bar{b}_j^{13}), \quad for \quad j=1,2,\cdots,n \\ \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} m(\bar{x}_{ijk}^{13}) \approx m(\bar{e}_k^{13}), \quad for \quad k=1,2,\cdots,r \end{cases}.$$

Step 3. Applying the simplex method to determine the optimals solutions of the problems (3.3) and (3.5).

Step 4. Determine $w(\bar{x}_{ijk}^{13})$ for each means of transport $1 \leq k \leq r$ fixed according to the following cases:

Case 1. If $\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}|x_{ijk}^{13} - x_{ijk}^{2}| + w(\bar{e}_k^{2}) \leq w(\bar{e}_k^{13})$ then $\displaystyle\sum_{x_{ijk}^{13}\neq 0} w(\bar{x}_{ijk}^{13}) = w(\bar{e}_k^{13})$ with $\bar{x}_{ijk}^{13} = \left[x_{ijk}^{1}, x_{ijk}^{3}\right] = \left[x_{ijk}^{13} - w(\bar{x}_{ijk}^{13}), x_{ijk}^{13} + w(\bar{x}_{ijk}^{13})\right]$.

Case 2. If $\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}|x_{ijk}^{13} - x_{ijk}^{2}| + w(\bar{e}_k^{2}) > w(\bar{e}_k^{13})$,
then $w(\bar{e}_k^{13}) = |x_{ijk}^{13} - x_{ijk}^{2}| + w(\bar{e}_k^{2})$ with $\bar{x}_{ijk}^{13} = \left[x_{ijk}^{1}, x_{ij}^{3}\right] = \left[x_{ijk}^{13} - w(\bar{x}_{ijk}^{13}), x_{ijk}^{13} + w(\bar{x}_{ijk}^{13})\right]$.

4. Numerical examples

Consider the following solid transportation prob+lem:

TABLE 1. Table description

	D_1	D_2	Supply $\tilde{a}_i$	Capacity $\tilde{e}_k$
S_1	(22, 31, 34) (20, 29,32)	(15,19,29) (13, 17, 27)	(150,201,246)	(100,200,240)
S_2	(30, 39, 54) (28, 37, 52)	(8,10, 12) (6,8, 10)	(50, 99, 154)	(100, 100, 160)
Demand $\tilde{b}_j$	(100, 150, 200)	(100, 150, 200)		$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Step1. $\sum_{i=1}^{m} \tilde{a}_i = \sum_{i=1}^{m} \tilde{b}_j = \sum_{k=1}^{r} \tilde{e}_k$, the problem is balanced.

Now, using the **Step 2** to **Step 4** we have the following:

For $p = q = 2$

$$Z^2(x^2) = 31x^2_{111} + 29x^2_{112} + 19x^2_{121} + 17x^2_{122} + 39x^2_{211} + 37x^2_{212} + 10x^2_{221} + 8x^2_{222} \longrightarrow \text{Min}$$

Subject to be constraints:

$$\begin{gathered}
x^2_{111} + x^2_{112} + x^2_{121} + x^2_{122} = 201 \\
x^2_{211} + x^2_{212} + x^2_{221} + x^2_{222} = 99 \\
x^2_{111} + x^2_{112} + x^2_{211} + x^2_{212} = 150 \\
x^2_{121} + x^2_{122} + x^2_{221} + x^2_{222} = 150 \\
x^2_{111} + x^2_{211} + x^2_{121} + x^2_{221} = 200 \\
x^2_{112} + x^2_{122} + x^2_{212} + x^2_{222} = 100
\end{gathered}$$

The optimal solution is: $x^2_{111} = 149$, $x^2_{112} = 1$, $x^2_{121} = 51$, $x^2_{122} = 0$, $x^2_{211} = 0$, $x^2_{212} = 0$, $x^2_{221} = 0$ et $x^2_{222} = 99$

For $p = 1$, $q = 3$:

$$\begin{aligned}\bar{Z}^{13}(\bar{x}^{13}) \approx [22,34]\bar{x}^{13}_{111} + [20,32]\bar{x}^{13}_{112} + [15,29]\bar{x}^{13}_{121} + [13,27]\bar{x}^{13}_{122} + [30,54]\bar{x}^{13}_{211} \\ + [28,52]\bar{x}^{13}_{212} + [8,12]\bar{x}^{13}_{221} + [6,10]\bar{x}^{13}_{222}\end{aligned} \longrightarrow \text{Min}$$

Subject to be constraints:

$$\bar{x}^{13}_{111} + \bar{x}^{13}_{112} + \bar{x}^{13}_{121} + \bar{x}^{13}_{122} = [150,246]$$
$$\bar{x}^{13}_{211} + \bar{x}^{13}_{212} + \bar{x}^{13}_{221} + \bar{x}^{13}_{222} = [50,154]$$
$$\bar{x}^{13}_{111} + \bar{x}^{13}_{112} + \bar{x}^{13}_{211} + \bar{x}^{13}_{212} = [100,200]$$
$$\bar{x}^{13}_{121} + \bar{x}^{13}_{122} + \bar{x}^{13}_{221} + \bar{x}^{13}_{222} = [100,200]$$
$$\bar{x}^{13}_{111} + \bar{x}^{13}_{211} + \bar{x}^{13}_{121} + \bar{x}^{13}_{221} = [100,240]$$
$$\bar{x}^{13}_{112} + \bar{x}^{13}_{122} + \bar{x}^{13}_{212} + \bar{x}^{13}_{222} = [100,160].$$

The optimal solution is: $\bar{x}^{13} = ([87,157];[1,55];[13,83];[0,0];[0,0];[0,0];[0,0];[100,104])$

$$\bar{x}^{3}_{111} = [87,157]$$
$$\bar{x}^{3}_{112} = [1,55]$$
$$\bar{x}^{3}_{121} = [13,83]$$
$$\bar{x}^{3}_{122} = [0,0]$$
$$\bar{x}^{3}_{211} = [0,0]$$
$$\bar{x}^{3}_{212} = [0,0]$$
$$\bar{x}^{3}_{221} = [0,0]$$
$$\bar{x}^{3}_{222} = [99,105]$$

Step5: Depending on the decision maker's choice, the overall fuzzy optimal solution to this problem is:

$$\text{Min } \tilde{Z}^* \approx (3826;[2723,10555]) = (2723,3826,10555)$$

where

$$\tilde{x}_{111} = (149;[87,157]) = (87,149,157)$$
$$\tilde{x}_{112} = (1;[1,55]) = (1,1,55)$$
$$\tilde{x}_{121} = (51;[13,83]) = (13,51,83)$$

$$\tilde{x}_{122} = (0; [0, 0]) = (0, 0, 0))$$
$$\tilde{x}_{211} = (0; [0, 0]) = (0, 0, 0)$$
$$\tilde{x}_{212} = (0; [0, 0]) = (0, 0, 0)$$
$$\tilde{x}_{221} = (0; [0, 0]) = (0, 0, 0)$$
$$\tilde{x}_{222} = (99; [99, 105]) = (99, 99, 105)$$

5. Interpretation of results

Our method consists of giving a fuzzy value of the minimum total transport cost, that is to say a set of values around the exact value of the minimum total cost. The further we move away from this set, the greater the chance of finding the exact value of the minimum cost total decreases. Thus our result obtained being $(2723, 3826, 10555)$ can be interpreted physically as follows:

(i) the lowest of the minimum total transport cost is 2723,
(ii) the safest amount of minimum total transport cost is 3826,
(iii) the highest of the minimum total transportation cost is 10555.

Thus the minimum total transport cost will always be greater than 2723, less than 10555, and the probable value of the minimum total transport cost will be 3826.

6. Conclusion

We introduced the notation of primal Fully Fuzzy Solid Transportation Problems **(FFSTP)** involving fuzzy numbers. We presented a new method for solving fully fuzzy single-objective linear solid transport problems in which all parameters and decision variables are fuzzy numbers. We first formulated the model, wrote the algorithm for solving said problem. The proposed method made it possible to find the optimal fuzzy solution to the fully fuzzy and balanced single-objective linear solid transport problems.

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