STEM Education Notes **1** (2022), no.1, 1–7
ISSN: 2955-1838 (electronic version)
https://doi.org/10.37418/stem.1.1.1

THE ROLE OF MATHEMATICS IN THE STEM STUDIES

Nikola Tuneski

ABSTRACT. Mathematics plays an important role in each STEM studies. Here, certain observations and conclusions about the role of mathematics in the STEM studies will be described and some specifics will be pointed out. Modern trends in teaching mathematics for "non-mathematicians" are described and the content of corresponding mathematical courses are given.

"Book of Nature is written in the Language of Mathematics."
Galileo Galilei (1564-1642) in his well-known book "Il Saggiatore"

1. THE RELATIONSHIP BETWEEN STEM AND MATHEMATICS

1.1. **Basic definitions and history remarks.**

In order to make comprehensive analysis of the relationship between STEM and mathematics some basic definitions and history remark are needed.

How can mathematics, engineering and technology be defined? There are many definitions in the literature today and some of them that are widely recognized are given below.

[1] *corresponding author*
Key words and phrases. STEM studies, mathematics, role, core courses.
Submitted: 27.04.2022; *Accepted:* 13.05.2022; *Published:* 22.05.2022.

The word **"mathematics"** comes from the Greek $\mu\alpha\theta\eta\mu\alpha$ (mÃąthÄŞma) meaning learning, study, science, and $\mu\alpha\theta\eta\mu\alpha\tau\kappa o\varsigma$ (mathÄŞmatikÃşs) meaning fond of learning. Mathematics is study of all conceivable abstract patterns and relationships, usually dealing with concepts as quantity, structure, space and change.

Engineering is the discipline and profession of applying technical and scientific knowledge for utilizing natural laws and physical resources in order to design and implement materials, structures, machines, devices, systems, and processes that safely realize a desired objective and meet specified criteria.

Technology is a broad concept that deals with a species' usage and knowledge of tools and crafts, and how it affects a species' ability to control and adapt to its environment. Technology is a term with origins in the Greek "technologia", "$\tau\epsilon\chi\nu o\lambda o\gamma\iota\alpha$" - "techne", "$\tau\epsilon\chi\nu\eta$" ("craft") and "logia", "$\lambda o\gamma\iota\alpha$" ("saying").

All of the above, mathematics, engineering, and technology are human enterprises. However, according to historians, technology far precedes the others. Namely, first recorded technology dates about 2.4 million years ago, from the Koobi fora site in northern Kenya where evidences are found that humans created primitive tools using the process of chipping the ages away from stones.

In mean time we became more than a tool makers. Humans pursue scientific knowledge since the Stone Age (5000–100000 years BC), first by pragmatic experimentation characterized as discovery by trial and error, then by the invention of the formal logical system (ancient Greek philosophers) and as a final step of the evolution of science we have the discovery of the possibility to find out casual relationship by systematic experiment (Renaissance period).

Evolution of technology is delivered through engineering. Engineering has its formal beginning about 3000 years BC with the building of many temples, tombs and pyramids along the valley of the Nile by the Egyptians. The word "engineer" was created in the Middle Ages when the builders of "engines" of war were called "ingeniators" by Latin writers.

Mathematics dates about 10000 years ago when humans began to count when the nomadic Stone Age hunters became farmers because of the retreating glaciers. Modern mathematics is based on the work of the ancient Greek philosophers and dates from the Middle Ages. Since then the development of engineering sciences goes along and is brought by with the discoveries in mathematical sciences.

1.2. **The synergy of STEM and mathematics.**

As stated in the beginning of this article Galileo Galilei (1564-1642) in his well known book "Il Saggiatore" with the statement *"Book of Nature is written in the Language of Mathematics"* illustrated in brilliant way the relationship between mathematics and nature which means also the relationship between mathematics and sciences studying the man made changes of nature.

Further analysis is based on the following:

- Mathematics is abstract, involved with patterns and their relationships and concerned mainly with providing solutions to theoretical problems.
- Science, technology and engineering are very specifically involved with utilizing the materials and forces of nature for the benefit of mankind, concerned with the improvement of the environment.

Lot of mathematics is ‘embedded’ in the practice of a STEM professional. If you ask a skilled engineer ‘do you ever use mathematics?’, a typical response might be, ‘no, I haven’t used it for 20 years’. But if you probe further, you can identify mathematical elements of their expertise which they no longer think of as mathematics, it’s just ‘doing engineering’. The question then is where does that embedded mathematics come from? It must come from doing explicit mathematics in the formation phase which becomes implicit in practice. The process of embedding depends critically on the student’s experience of learning mathematics and STEM disciplines.

Main reasons why mathematics is important for STEM professionals can be formulated as:

- The laws of nature (e.g., Maxwell’s equations for electromagnetics, Kirchhoff’s Rules for circuit analysis) are mathematical expressions. Mathematics is the language of physical science and engineering.
- Mathematics is more than a tool for solving problems; it also develops intellectual maturity. It is critical that an engineer can visualize abstract concepts.
- Numerical simulation on a digital computer is a powerful and effective tool that is being used by an increasing number of engineers. However, computers do not make traditional mathematical analysis obsolete.

2. Modern trends in teaching mathematics for "non-mathematicians"

Modern trends in teaching mathematics for STEM professionals faces the following challenges:

- Mathematics education for STEM professionals, with some exceptions, has typically been about techniques, which were needed in the past to perform practical calculations, but today, nearly all of those techniques have disappeared to be replaced by computer software. What has not been replaced is the need to understand modeling and now days the need is shifting in that direction.
- Many students believe that the way to solve a problem is to search for the proper formula, and then substitute numbers into the formula. This may be all right for solving quadratic equations, but this is not a good general attitude.
- Helping students to understand that "doing maths" helps them to develop a logical thought process, a discipline of problem solving that is essential for solving engineering problems of many kinds.
- Few problems can be solved immediately. It is critical that STEM students develop persistence at solving problems. Often the "best" way does not come instantly or even easily; one must try various methods and see what happens. The experience of working large numbers of homework problems, of diverse kinds, seems to build a personal collection of approaches and tools, and add to an understanding of mathematics.
- Many students need more practice in how to start solving a problem, including translating "word problems" into mathematical expressions.

Extensive use of mathematical software (Maple, Mathcad, Matlab ...) should be introduced since there are so many students with weak algebraic skills. Not only that: the software is also extremely powerful for exploring mathematical modeling. Never the less one should be aware that:

- Computer programs contain mathematical relations; understanding and fluency with manipulation of these relations is still necessary.
- Knowledge of traditional mathematical analysis is essential for validating computer programs. One of the best ways to validate a program is to

compare the computer simulation of simple situations to the analytical solution for the same situation.

- It is relatively easy to write brute-force computer code that requires a long runtime and produces significant error, owing to accumulation of errors from the limited resolution of machine numbers. Great increases in both speed and accuracy can be obtained by using analytical solutions for parts of the problem, or by careful development of appropriate algorithms. Knowledge of traditional mathematics is highly relevant to this task.

Finally, we can firmly point out that for three year STEM studies in the first cycle, two core mathematical courses are needed. They should include: basics of linear and vector algebra, differential and integral calculus of functions of one and several variables, ordinary differential equations and basics of applied statistics and optimization.

Additionally, depending on the field of study, more mathematical courses can and should be included.

Detailed contents of these two core courses are given in Appendices A and B of this paper.

APPENDIX A

Content of Mathematics 1 course within STEM Curriculum.

(1) Matrices and determinants.
  1.1 Manipulations with matrices.
  1.2 Evaluation of determinants.
  1.3 Finding the inverse of a non-singular square matrix.
  1.4 Solving systems of simultaneous linear equations using matrices.
(2) Vector algebra and its applications.
  2.1 Operations with vectors.
  2.2 Scalar and dot product of a vectors.
  2.3 Lines and planes in space.
(3) Sequences of real numbers.
  3.1 Increasing, decreasing and bounded sequences.

3.2 Convergence of sequences.
(4) Properties of functions of one variable.
4.1 Ways of defining a function.
4.2 Graphs of some elementary functions.
4.3 The limit and the continuity of function.
4.4 Horizontal, vertical and slant asymptotes.
(5) The derivative of a function.
5.1 The definition of the derivative of a function.
5.2 Geometric and kinematical interpretation of the derivative.
5.3 Formulas for differentiation.
5.4 The derivative of a composite function – the chain rule.
5.5 Maxima and minima of functions. The first derivative test.
5.6 Higher derivatives. Concavity and the second derivative test.
5.7 Comprehensive graphing of a function.
5.8 Application of the derivative.
(6) Integral calculus of one variable.
6.1 The definite integral.
6.2 The indefinite integral.
6.3 Techniques of integration.
6.4 Application of the definite integral.

APPENDIX B

Content of Mathematics 2 course within STEM Curriculum.

(1) Multivariable functions.
1.1 Functions of two variables. Level curves.
1.2 Functions of three variables. Level surfaces.
1.3 Quadric surfaces.
1.4 Cylindrical and spherical coordinates in space.
(2) The differential calculus of multivariable functions.
2.1 Limits and continuity.
2.2 Partial derivatives.
2.3 The chain rule.

   2.4 Application of partial derivatives.
(3) The integral calculus of multivariable functions.
   3.1 Double integrals.
   3.2 Surface area.
   3.3 Triple integrals.
   3.4 Application of triple integrals.
(4) Ordinary differential equations.
   4.1 Homogeneous differential equations.
   4.2 Exact differential equations.
   4.3 Linear first-order differential equations.
   4.4 Linear second-order differential equations: homogeneous and nonhomogeneous.
(5) Introduction to applied statistics.
   5.1 Population. Sample. Random variables.
   5.2 Presenting and summarizing the data.
   5.3 Point and interval estimation of data parameters.
   5.4 Parametric test of hypotheses.
(6) Optimization.
   6.1 Linear programming. Simplex algorithm.
   6.2 Formulation and optimization of a transportation problem.

References

[1] G. James et al.: *Modern Engineering Mathematics*, Prentice-Hall, 2000.
[2] J.D. Faires, B.T. Faires: *Calculus*, Random House, 1989.
[3] K.A. Stroud, J.B. Dexter: *Engineering Mathematics*, Palgrave Macmillan, 2007.

Department of Mathematics and Informatics
Faculty of Mechanical Engineering
Ss. Cyril and Methodius University in Skopje
Karpoš II b.b., 1000 Skopje
Republic of North Macedonia.
Email address: nikola.tuneski@mf.ukim.edu.mk